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Three Dimensional Flow Development in the Wake of Elongated Bluff Bodies

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Abstract

The results from *Floquet stability analysis* of the flow around nominally two-dimensional long plates with elliptical leading edges and blunt trailing edges are presented, elucidating the early stages of wake transition to turbulent flow. Three modes of instability are found: Mode A, Mode B and a period doubling mode. The first two of these also occur in wake transition for circular cylinders and square cross-sectioned cylinders. For sufficiently large aspect ratio, Mode B is found to be the dominant instability mode. This research indicates the generic turbulence transition scenario suggested for circular two-dimensional bodies does not apply to all two-dimensional bluff bodies.

Introduction

The structure and formation of streamwise vortices in the wake of nominally two dimensional bluff bodies has been the subject of intense study and debate over the past fifty years [Roshko 1955, Gerrard 1966, Williamson 1996]. Over the last 20 years, progress in numerical and experimental technology has allowed researchers to accurately map the parameter space, general geometry and dynamics governing these streamwise wake vortices [Williamson 1996, Barkley and Henderson 1996, Henderson 1997]. It is generally accepted that the inception of streamwise vortex structures in the wake is the first step in the progression to a fully turbulent wake. Indeed, the overall wake structures observed in flow fields of relatively low Reynolds number have been shown to persist in flow fields having much higher Reynolds numbers [Williamson 1996]. Most work has been focused on the flow around a circular cylinder. The research described in this paper aims to extend the field of knowledge to encompass the flow around blunt flat plates of varying aspect ratio.

In an experimental study, Williamson (1996) identified two discontinuous alterations of the flow field in the wake of a circular cylinder as the flow changes from a two- to a three-dimensional state. These changes consist of a periodic distortion of the Karman vortex-street in the cylinder span-wise direction and the generation of streamwise vortex structures in the wake of the flow field. These instabilities, referred to as Mode A and Mode B, occur successively with increasing Reynolds number and may be identified by a discontinuous jump in the Strouhal number as the Reynolds number is increased [Williamson 1988].

For a circular cylinder, the Mode A instability exists with a span-wise periodicity of 3-4 cylinder diameters of the Karman vortex-street. This induces the generation of streamwise vortex structures, formed between successive primary vortex cores. The streamwise vortex structures exhibit an out-of-phase symmetry, implying that the streamwise vortices will change sign between successive primary Karman vortex cores. Williamson (1996) observed that, for a circular cylinder, Mode A existed within the Reynolds number range Re = 180-250; he found the critical Reynolds number of inception varies between 180-194 due to the hysteretic nature of the mode.

In a Reynolds number range 230 to 250 Williamson observed that a new instability (Mode B) replaces Mode A as the dominant span-wise instability. For a circular cylinder, Mode B has a span-wise periodicity of around 1 cylinder diameter wavelength, which does not appear to vary appreciably with increasing Reynolds number. Contrary to Mode A, Mode B exhibits an in phase symmetry, thus the streamwise vortices will not change sign between successive primary Karman vortex cores.

Williamson's (1988, 1996) observations have been verified numerically by a number of authors. Thompson et al. (1994, 1996) conducted (DNS) computational studies of the flow around a circular cylinder using a three-dimensional flow field. Their work verified the existence of both Mode A and Mode B. Barkley and Henderson (1996) conducted a Floquet stability analysis of the flow around a circular cylinder, verifying the span-wise wavelength for both Mode A and Mode B, and the critical Reynolds number of inception for Mode A. These results have also been experimentally verified by Zhang et al. (1995).

To date, very little research has been conducted on the flow field around bluff bodies with cross sectional geometry other than that of a circular cylinder. Zhang et al. (1995) experimentally observed the existence of a "Mode C" instability in the wake of a circular cylinder when a tripwire was placed adjacent to a circular cylinder in a direction transverse to the fluid flow. The Mode C instability was found to have a span-wise wavelength of 1.8 cylinder diameters, and was found to occur when the tripwire was located within 1 diameter of the cylinder. Their results indicate that the suppression of the flow field near the boundary layer results in a Mode C instability occuring in preference to Mode A and Mode B. Numerical calculations performed by Zhang et al. (1995) verified their experimental observations. While their results were corroborated by the numerical Floquet analysis described by Noack et al. (1993 and 1994), who found the three-dimensional instability had a wavelength of 1.8 to 2 cylinder diameters, it now appears that this may have been due to insufficient numerical resolution. (As mentioned previously, the better resolved studies of Barkley and Henderson (1996) give wavelengths consistent with accepted values.)

Robichaux et al. (1999) performed a Floquet stability analysis on a square cross section cylinder. Their model predicted the existence of a third mode of instability, which they denoted "Mode S". While many physical features of this instability mode corresponded to Zhang et al.'s experimental work, Robichaux et al. did not refer to this instability as Mode C as their work did not require the existence of a trip wire. Within the parameter space modelled ($150 < Re < 225$), Mode S was not found to be critical, and would not be observed experimentally. Their research did not indicate if an increase in Reynolds number would cause Mode S to exist experimentally.

Previous research has not adequately answered whether the results obtained from the circular cylinder describe all three-dimensional modes observed for nominally two-dimensional bluff bodies of general geometry. In the two examples above, a slight change in the geometry of interest has altered the wake transition to three-dimensionality.

The research described in this paper simulates the flow around blunt flat plates using a Galerkin Spectral element method coupled with a Floquet stability analysis technique to determine the most unstable span-wise mode(s) as a function of Reynolds number. The Floquet stability analysis effectively investigates the stability of a two-dimensional periodic base flow to three-dimensional spanwise disturbances by determining from the linearised Navier-Stokes equations whether an assumed sinusoidal spanwise disturbance will grow from one base flow period to the next. Owing to the requirement of a known two-dimensional periodic base flow field, the modelling was performed in two stages; initially a two-dimensional periodic base flow field solution (in the form of a Karman vortex street) is determined using the Galerkin spectral element method. In the second stage, the (linearised) spectral element method was used to determine instabilities in the third (spanwise) dimension.

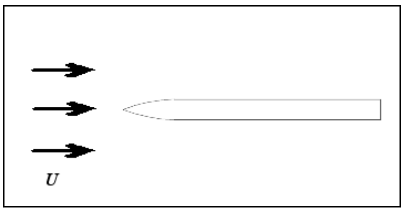
Three modes of instability were found; corresponding to Mode A, Mode B, and a period doubling mode. While Mode A had the same spanwise wavelength as for a circular cylinder, Mode B had a spanwise wavelength of around 2.2 plate thicknesses. These critical wavelengths remained constant across the range of plate aspect ratios considered. For a plate with sufficiently high aspect ratio, Mode B was found to be unstable at a lower Reynolds number than Mode A, contrary to what has been discovered for the flow around a circular cylinder. The period doubling mode was not found to become unstable in the parameter space modelled, and would not be observed experimentally.

The next section outlines the numerical methodology used in modelling the flow past the blunt flat plates. Following this, results are presented, discussed and conclusions drawn.

Numerical Method

Domain Geometry

The bluff body under investigation consists of a flat plate of finite thickness (H) and finite chord (C). Plates of aspect ratio (AR= C/H) ranging from 2.5 to 17.5 have been investigated. In each investigation the plate is modelled as being immersed within a uniform flow field travelling in the positive x direction with constant velocity U. The fluid is considered to be a homogeneous incompressible Newtonian fluid. The leading edge has a streamlined elliptical profile. This profile prevents vortex shedding from the leading edge and thus allows the behaviour of the wake transition to be studied in isolation. Parameters describing the geometry of the leading edge are kept constant between plates. The trailing edges of the plate are square, thus providing a predetermined location from which the trailing vortices are generated, and therefore simplifying the post-processing analysis. An image of the plate geometry is given below (figure 1):



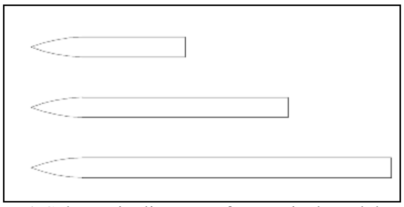


Figure 1. (above) Schematic diagram of numerical model set-up. (below) Relative aspect ratios of plates.

Galerkin Spectral Element Technique

A Galerkin Spectral element scheme was used to determine a periodic two-dimensional flow field in order to apply the Floquet stability technique. The method solves the time-dependent Navier-Stokes and continuity equations in primitive variable form. The spatial domain is subdivided into several 'macro' elements. Within each element, the solution is approximated by high order Lagrangian interpolants. Temporal discretisation is performed using a three-step time splitting scheme. Details of the method may be found in Thompson et al. (1996)

Floquet Stability Analysis

For each plate, the parameter space consists of two variables. These are the Reynolds number based on the plate thickness (Re_H), and the span-wise wavelength applicable to the stability analysis. The Floquet analysis is applied using the following approach. Starting from random perturbation fields for the perturbation velocity components and the perturbation pressure, the linearised Navier-Stokes equations are integrated forward in time. At the end of each period, the perturbation fields are normalised. Using Floquet theory it can be shown that a perturbation field can be expanded in terms of Floquet modes, where for any chosen span-wise wavelength, the different modes have different growth/decay rates. Thus by normalising the perturbation fields at the end of each period, after a long time, effectively only the Floquet mode with the largest growth rate will remain. (All other modes grow less, or decay more, over each base flow period.) This growth rate can then be determined by calculating the amplitude of the mode at the end of a period relative to the initial amplitude. This is called the Floquet multiplier, $|\mu|$, and it plays an important role in determining the stability of the two-dimensional base flow. The Floquet multiplier describes the stability of the two-dimensional flow against the selected span-wise wavelength at the given Reynolds number for the plate aspect ratio of interest. If $|\mu|$ is less than unity, the span-wise wavelength under investigation is stable at the given Reynolds number, and will not be observed experimentally. If $|\mu|$ is greater than unity, the span-wise wavelength under investigation is unstable at the given Reynolds number and should be observed experimentally (since it will grow from background noise). If $|\mu|$ is equal to unity then the Reynolds number under investigation is said to be the critical Reynolds number of inception (Re_{cr}). In experiments, this is the lowest Reynolds number above which the instability will be observed. A detailed description of the Floquet analysis methodology may be found in Ioos and Joseph (1990).

Results

Floquet Analysis Graphs

Displayed below (figure 2) is a diagram depicting the Floquet multiplier as a function of span-wise wavelength for a blunt flat plate with AR = 7.5, for various Reynolds numbers. Inspection of figure 2 points to three distinct modes of instability. (The three different local maxima correspond to different instability modes.) Two of these modes correspond to Mode A and Mode B found for the flow around a circular cylinder. A period doubling mode was also found with a critical wavelength varying between 0.8

and 1.1 plate thicknesses, increasing with increasing Reynolds number. Mode B has a critical wavelength of 2.5 plate thicknesses, which is markedly different to findings for a circular cylinder. In the Reynolds number range modelled for this plate ($400 \leq Re \leq 500$), only Mode A became unstable. Indicating that Mode A is the first mode of instability for the flow around the AR=7.5 plate.

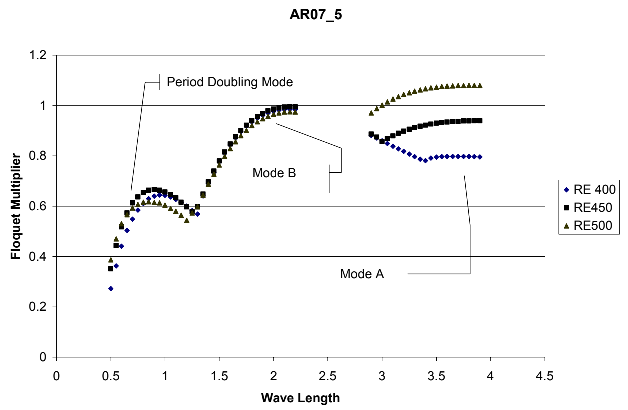


Figure 2. Floquet multiplier versus span-wise wavelength plate AR = 7.5

As the plate AR is increased to 12.5 (figure 3), Mode B was observed to become unstable at a Reynolds number lower than Mode A. Therefore, experimentally the inception of Mode B should be observed at a lower Reynolds number than Mode A. Mode B is found to become critically unstable at a Reynolds number slightly higher than 400, whereas the Re_{cr} for Mode A is about 600. The period doubling mode remains subcritical for the parameter space of interest for this plate.

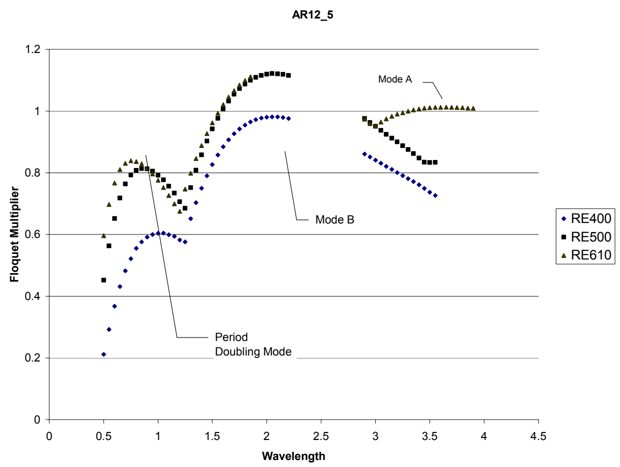


Figure 3. Floquet multiplier versus span-wise wavelength plate AR=12.5

As the plate AR is further increased to 17.5 (figure 4), Mode B clearly becomes the dominant mode, having a critical Re of around $RE_{cr} = 450$. Mode A, by comparison has a RE_{cr} of around 700. As yet little work has been undertaken on the period doubling region of the parameter space for this plate AR.

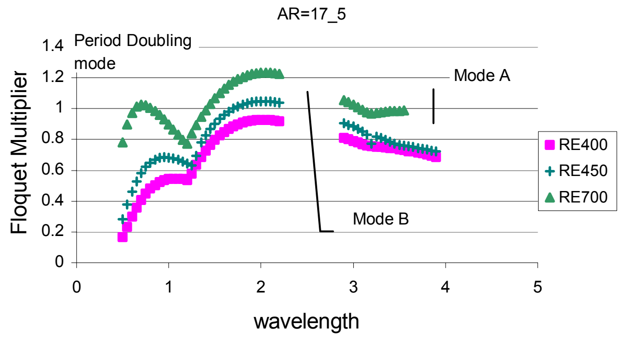


Figure 4. Floquet multiplier versus span-wise wavelength plate AR=17.5

Flow Visualisation

Observation of the above graphs alone does not provide sufficient information to classify the mode of each instability, where each mode has been defined in previous studies (Williamson, 1996). Each mode is a description of the generation and interaction of stream-wise vortex structures formed between the primary Karman vortex cores in the wake of the bluff body in question. To compare modes of instability across bluff bodies of arbitrary geometry, the structure of the stream-wise vortices must be matched.

Floquet stability analysis provides a solution for a three-dimensional disturbance velocity at a predetermined span-wise wavelength. By choosing the critical wavelength for a specific mode (the local maxima in the previous graphs), the stream-wise vortex structures may be calculated for that mode of instability.

Below are several images depicting the stream-wise vortex patterns in the wake flow field, as calculated using the Floquet stability technique. The stream-wise vortices are shaded contour regions, red indicating positive stream-wise vorticity, blue representing negative stream-wise vorticity. The primary Karman vortex cores are also depicted as contour lines. The following figures are representative of a plate with AR = 12.5.

Figure 5 represents the stream-wise vortex pattern for a Mode A instability. As may be observed, the stream-wise vortex structures alternate in sign between each primary Karman vortex core. This out-of-phase symmetry between primary vortex cores is typical of the Mode A type instability structure.

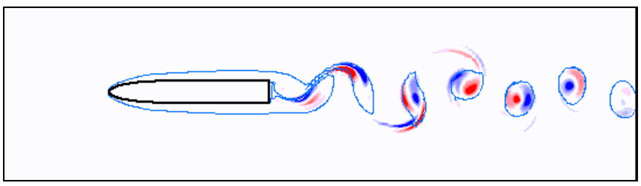


Figure 5. Streamwise vortex structures in the wake flow field, representing a Mode A instability for a plate AR=12.5.

Figure 6 shows the streamwise vortex pattern for a Mode B instability. The stream-wise vortex is observed to maintain its sign between each primary vortex core, consistent with the definition of Mode B instability for a circular cylinder (Williamson, 1996). The negative (blue) stream-wise vortex maintains its form through several primary vortex core regions into the far wake where it is eventually dissipated.

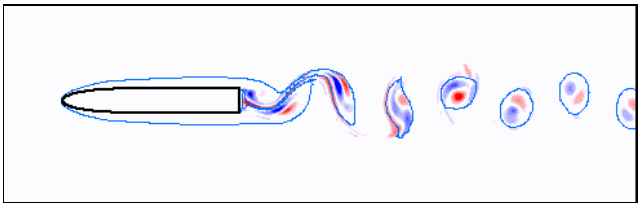


Figure 6. Streamwise vortex structures in the wake flow field, representing a Mode B instability for a plate AR=12.5.

Figure 7 shows the streamwise vortex pattern for the period doubling mode. As may be observed, the streamwise vortex patterns are more intense for every second primary Karman vortex core. (the upper primary vortex cores in figure 7).

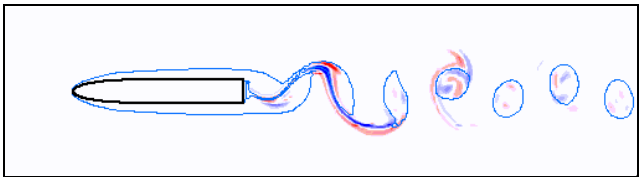


Figure 7. Streamwise vortex structures in the wake flow field, representing a period doubling mode for a plate AR=12.5.

Discussion

The choice of geometry allows a wider parameter space than has been presented in previous studies, namely, the effect of alteration of bluff body aspect ratio to the wake flow field. Previous studies (Roshko, 1955) indicate that all bluff bodies have very similar wake structures.

An increase in the aspect ratio, alters the preferred mode of instability, however the critical Reynolds number of inception remains relatively constant. For a plate AR = 7.5 the initial instability is Mode A with a critical Reynolds number of roughly 475. As the aspect ratio is increased to 12.5, the critical Reynolds number is roughly 450, however Mode B is now the initial mode of instability, in preference to mode A. As the aspect ratio is further increased to 17.5, the critical Reynolds number is around 450, and once again Mode B is the initial instability mode.

As the aspect ratio is increased, the local maxima describing a Mode A instability is less well defined. Further increases in the aspect ratio may result in no appreciable maxima at all. Therefore, it is hypothesised that there is a critical aspect ratio beyond which a Mode A instability would not be observed experimentally.

The wavelength of the Mode A instability compares favourably with the value found for a circular cylinder. Williamson (1996) has provided compelling evidence that a Mode A instability scales on the primary Karman vortex cores. It may therefore be assumed that there is little difference in the primary vortex cores between the geometries presented here, and a circular cylinder. This is in agreement with the work presented by Roshko (1955).

In these studies, Mode B has a markedly different critical wavelength when compared to a circular cylinder. The wavelength found in these studies was equal to 2.2 plate thicknesses, whereas for a cylinder, a Mode B instability has a critical wavelength of 1 cylinder diameter. In this study, the critical wavelength for Mode B was found to be constant across the range of plate AR simulated. Williamson's (1996) studies indicate that the Mode B instability scales on the braid region of the wake flow field. Given this, the choice of leading and trailing edge must be altering the braid region of the wake. From the graphs above, it is clear that the aspect ratio of the plate does not affect the critical wavelength of instability.

The period doubling mode was not found to become critically unstable for the range of Reynolds numbers and aspect ratios observed here. While it may be observed experimentally, a Floquet analysis would not be sufficient to prove this. The three-dimensional wake profile would already be saturated and non-linear prior to the inception of the period doubling mode.

Conclusions

The results from *Floquet stability analysis* of the flow around nominally two-dimensional long plates with elliptical leading edges and blunt trailing edges have been presented. Three modes of instability have been observed: Mode A with a critical wavelength of 4 plate thicknesses, Mode B with a critical wavelength of 2.2 plate thicknesses, and a period doubling mode with a critical wavelength of 1 plate thickness. As the plate aspect ratio is increased, Mode A is less well defined. For sufficiently large aspect ratio, Mode B is found to be the dominant instability mode.

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