

## Effect of a Neighbouring Vibrating Cylinder on a Circular Cylinder Wake

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### Abstract

This work aims to investigate experimentally the effect of a neighbouring vibrating cylinder on a circular cylinder near-wake. In the present investigation, two side-by-side tubes of identical diameter  $d = 10$  mm were mounted in a water tunnel; one was stationary and the other forced to vibrate in the lateral direction, up to a vibration amplitude  $A = 0.5d$ . The dye-marked vortex streets behind the tubes, illuminated by a thin laser sheet, were recorded using a digital video camera recorder. Two values of  $T/d$ , i.e. 2.2 and 3.5 were investigated, where  $T$  is the cylinder centre-to-centre spacing. The Reynolds number  $Re$  ranges from 150 to 1000. The effect of  $A/d$ ,  $T/d$  and  $f_e / f_s$  ( $f_s$  is the vortex shedding frequency of an isolated stationary cylinder and  $f_e$  is the structural vibration frequency) is examined on the vortex shedding and the wake structure. Specific attention was given to the occurrence of 'lock-in', that is, the vortex shedding frequency of the vibrating cylinder synchronizes with  $f_e$ . It has been found that the shedding frequency associated with not only the vibrating but also the stationary cylinder can be modified as  $f_e / f_s$  approaches unity. Significant influence of these parameters has been observed on the flow behind the cylinders in terms of the predominant vortex patterns and interactions between vortices. The results are also compared with those of an isolated vibrating cylinder.

### Introduction

The wake of a vibrating structure has received considerable attention in the literature because of its practical significance. One of the primary concerns is that a structure subjected to a cross flow vibrating at its natural frequency may coincide with the vortex-shedding frequency, resulting in the so-called 'lock-in' phenomenon, which amplifies structural vibration amplitude and leads to a reduced life span of the structure or even early failure. Furthermore, the wake of one or more vibrating structures is of relevance to the prediction of forces on downstream structures.

In their early studies of a single forced oscillating cylinder in a cross flow, Bishop & Hassan [2] observed that when the cylinder oscillating frequency  $f_e$  approached the vortex shedding frequency  $f_s$  of a stationary cylinder, the two sets of forces were synchronized, and the natural shedding frequency was lost. Within the synchronization range, lift and drag forces varied in phase and amplitude with the imposed frequency. Koopmann [7] noted that, the 'lock-in' range, over which the vortex shedding frequency was dictated by  $f_e$ , was dependent on amplitude  $A/d$  and, to some extent, on the Reynolds number  $Re$ . The vortex shedding frequency could vary up to  $\pm 25\%$  for the same  $Re$  ( $< 300$ ). Griffin *et al.* [4,5] found that the 'lock-in' occurred for  $f_e = 0.8 \sim 1.2 f_s$ . They further observed that, due to the forced vibration effect, the lateral spacing between vortices decreased as  $A/d$  increased but the longitudinal spacing was unchanged. Williamson & Roshko [11] discussed various spatial modes of vortices, generated by a vibrating cylinder at different  $A/d$  and  $f_e / f_s$ . Ongoren & Rockwell [8,9] investigated at various  $f_e / f_s$  the phase shift, recovery and mode competition in the near-wake for an oscillating cylinder of different cross sections (circular, square

and triangular). They noted that when  $f_e / f_s \approx 1$ , the vortex formation timing switched phase by approx.  $180^\circ$  over a very narrow range of  $f_e$ . Numerical studies have also been conducted. For example, using a primitive-variable formulation on a spectral element spatial discretization, Blackburn & Henderson [3] simulated a two-dimensional flow past a circular cylinder that was either stationary or in simple harmonic cross-flow oscillation at  $Re = 500$ ,  $A/d = 0.25$  and  $f_e / f_s = 0.875 \sim 0.975$ . They showed that the change in phase of vortex shedding was not a unique function of the oscillation frequency.

Previous studies have greatly improved our understanding of the flow behind a vibrating cylinder. In engineering, however, we are frequently faced with the problem of multiple vibrating cylinders instead of an isolated one. Numerous investigations have been conducted to understand the cylinder wake in the presence of a neighbouring cylinder. It is now well known that, when the cylinder centre-to-centre spacing ratio  $T/d$  is greater than 2, two distinct vortex streets occur behind the cylinders. The two streets are predominantly symmetrical about the flow centreline, i.e. in antiphase mode (Ishigai *et al.* [6]). Nevertheless, vortex streets anti-symmetrical about the flow centreline or in phase are also observed from time to time. (e.g. Bearman & Wadcock [1]; Williamson [12]; Zhou *et al.* [13]). At  $1.5 < T/d < 2.0$ , the gap flow between the cylinders is deflected, forming one narrow and one wide wake (e.g. Bearman & Wadcock [1]; Sumner *et al.* [10]). The deflected gap flow may change over intermittently from one side to another and is bi-stable. For very small spacing ratio, i.e.  $T/d < 1.2$ , vortices are alternately shed from the free-stream side of the two cylinders, generating one single vortex street. However, there is a dearth of information on the flow when the neighbouring cylinder is vibrating.

This work aims to investigate the effect of a neighbouring vibrating cylinder on a circular cylinder wake based on laser-induced flow visualization. Great attention was given to the occurrence of 'lock-in', when the vortex shedding frequency of the vibrating cylinder synchronizes with the vibrating frequency. Various parameters are investigated, including  $A/d$ ,  $T/d$  and  $f_e / f_s$ . The results are also compared with the data of an isolated cylinder.

### Experimental Setup

Experiments were carried out in a water tunnel with a square working section (150mm  $\times$  150mm) of 0.5m long. The water tunnel is a re-circulating single reservoir system. A centrifugal pump delivers water from the reservoir to the tunnel contraction. A honeycomb is used to remove any large-scale irregularities prior to the contraction. The flow speed, controlled by a regulator valve, is up to a maximum of about 0.32m/s in the working section. The working section is made up of four 20mm thick Perspex panels.

Two side-by-side acrylic circular tubes of an identical diameter of 0.01m, were horizontally mounted 0.20m downstream of the exit plane of the tunnel contraction and placed symmetrically about the mid-plane of the working section (Figure 1). The cylinders were cantilevered; there was a 1mm gap between the cylinder tip and the tunnel wall. The resulting blockage was about 13%. Two transverse spacing ratios were used, i.e.,  $T/d = 3.50$  and  $2.20$ , respectively. The upper cylinder vibrated laterally at  $A/d = 0.1$  and  $0.5$ . The vibrating frequency,  $f_e$ , measured by a tachometer (Max.  $\pm 5\%$ ), was adjusted so that  $f_e / f_s$  varied

between 0.74 and 1.44. In present investigations,  $f_s$  denotes the vortex shedding frequency of an isolated stationary cylinder, while  $f_{s1}$  and  $f_{s2}$  are the vortex shedding frequencies of the vibrating and the stationary cylinder, respectively. For each cylinder, dye (Rhodamine 6G 99%, which has a faint red color and will become metallic green when excited by laser) was introduced through one injection pinhole located at the mid-span of the cylinder and distributed around  $90^\circ$ , both clockwise and anti-clockwise, respectively, from the forward stagnation point. A thin laser sheet, which was generated by laser beam sweeping, provided illumination vertically at the mid-plane of the working section. A Spectra-Physics Stabilite 2017 Argon Ion laser with a maximum power output of 4 watts was used to generate the laser beam. A digital video camera recorder (Sony DCR-PC100E) was used to record, at a framing rate of 25 frames per second, the dye-marked vortex streets. Flow-visualization was carried out in the range of  $Re = 150\sim 1000$  over  $0 \leq x/d \leq 8$ . At larger  $Re$  and  $x/d$ , the dye diffused too rapidly to be an effective marker of vortices.

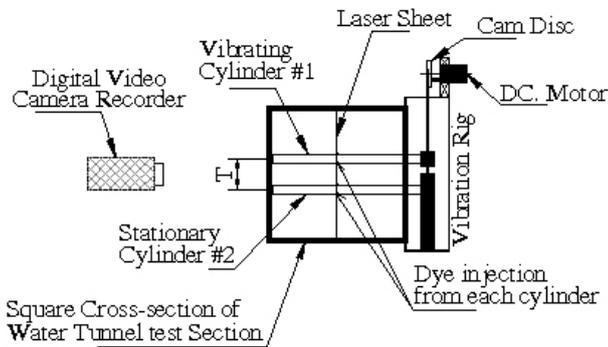


Figure 1. Experimental arrangement: assembly of cylinders

### Presentation of Results

Vortex frequencies were estimated by the playback of recorded data and counting consecutive vortices (about 100 pairs) at  $x/d \approx 2$  for a certain period. The experimental uncertainty was estimated to be about 2%. The results are summarised in Table 1 for  $A/d = 0.5$ , where the tick indicates the occurrence of  $f_e = f_{s1} = f_{s2}$  or  $f_e = f_{s1}$ , while the cross means inequality. It has been presently confirmed that, within the range of  $f_e/f_s \approx 0.8 \sim 1.2$ , lock-in always occurs for a single cylinder. A number of observations can be made based on the table.

$Re$	$T/d = 3.5$			$T/d = 2.2$		
	$f_e/f_s$	$f_e = f_{s1}$	$f_e = f_{s1} = f_{s2}$	$f_e/f_s$	$f_e = f_{s1}$	$f_e = f_{s1} = f_{s2}$
150	0.83	✓	✗	1.00	✓	✓
	1.00	✓	✓	1.03	✓	✓
	1.22	✓	✓	1.28	✓	✓
300	0.94	✓	✓	0.95	✗	✗ <sup>1</sup>
	1.19	✓	✗	1.20	✗	✗ <sup>2</sup>
	1.41	✓	✗	1.44	✓	✓
500	0.74	✓	✓	0.76	✓	✗ <sup>3</sup>
	0.96	✓	✓	0.97	✓	✓
	1.12	✓	✗	1.15	✓	✓
1000	0.80	✓	✓	0.93	✓	✗ <sup>4</sup>
	1.00	✓	✓	1.11	✓	✓
	1.20	✓	✓	1.21	✓	✓

Table 1: Relationship between  $f_e, f_{s1}$  and  $f_{s2}$ ,  $T/d = 3.5$  and  $2.2$ ,  $A/d = 0.5$

<sup>1</sup>  $f_e = 0.51$  Hz,  $f_{s1} = 0.77$  Hz,  $f_{s2} = 0.82$  Hz

<sup>2</sup>  $f_e = 0.64$  Hz,  $f_{s1} = 0.75$  Hz,  $f_{s2} = 0.78$  Hz

<sup>3</sup>  $f_e = 0.83$  Hz,  $f_{s1} = 0.83$  Hz,  $f_{s2} = 1.05$  Hz

<sup>4</sup>  $f_e = 1.92$  Hz,  $f_{s1} = 1.92$  Hz,  $f_{s2} = 2.07$  Hz

Firstly, at  $T/d = 3.5$ , the vortex shedding frequency  $f_{s1}$  of the vibrating cylinder is always equal to  $f_e$  for the  $f_e/f_s$  range investigated, that is,  $f_{s1}$  is modified and ‘‘locked in’’ with  $f_e$ , as the isolated cylinder case. This is not always the case for  $A/d = 0.1$  since the lock-in range dwindles as  $A/d$  decreases (Koopmann [7]). However, as  $T/d$  reduces to 2.2,  $f_{s1}$  does not always follow  $f_e$ ; instead, it may approach  $f_{s2}$ . The observation suggests that because of the close proximity between the two cylinders,  $f_{s1}$  is not only influenced by  $f_e$  but also by  $f_{s2}$ .

Secondly, the vortex shedding frequency  $f_{s2}$  of the stationary cylinder is not necessarily affected by  $f_e$ . However, when  $f_e/f_s$  approaches unity, in particular, for  $A/d = 0.5$ ,  $f_{s2}$  is inclined to follow the change of  $f_{s1}$ , resulting in  $f_e = f_{s1} = f_{s2}$ . It is likely that the effect of ‘resonance’, when  $f_e$  coincides with  $f_{s1}$ , is strongest at  $f_e/f_s \approx 1$  so that  $f_{s2}$  is also modified and ‘locked’ in with  $f_{s1}$  or  $f_e$ .

Finally, it is pertinent to mention that, for  $A/d = 0.1$ ,  $f_{s1}$  is not always equal to  $f_e$  (not shown) since the ‘lock-in’ range is likely shrinks as  $A/d$  reduces (Koopmann [7]). Furthermore,  $f_{s2}$  is not likely modified by  $f_e$  or  $f_{s1}$  even  $f_e/f_s$  approaches unity because of diminishing the effect of ‘resonance’ for smaller  $A/d$ .

At  $T/d = 3.5$ , the interference between the two streets is relatively small in the near-wake. This is illustrated in Figure 2, where the two vortex streets behind two cylinders are compared with those behind isolated vibrating and stationary cylinders, respectively. The upper street (Figure 2a) behind the vibrating cylinder appears rather similar to that (Figure 2b) behind the isolated vibrating cylinder, and the lower street behind the stationary cylinder resembles that (Figure 2c) behind the isolated stationary cylinder. But the vibration effect of cylinder 1 on the flow is appreciable. Figures 3a, b and c indicate a growing wake width as  $A/d$  increases. This is reasonable since the vibration increases the effective spacing between cylinders. In general, the two streets behind the cylinders are predominantly symmetrical about the flow centreline, irrespective of the vibration amplitude, as the stationary cylinder case (e.g. Zhou *et al.* [13])

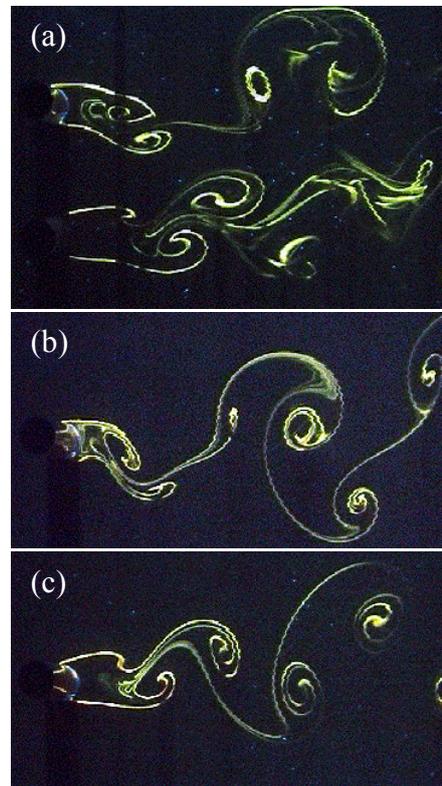


Figure 2. Comparison between the vortex streets behind two cylinders and those behind isolated vibrating and stationary cylinders: (a)  $T/d = 3.5$ ,  $A/d = 0.5$ ,  $f_e/f_s = 0.83$ ; (b) an isolated vibrating cylinder,  $A/d = 0.5$ ,  $f_e/f_s = 0.83$ ; (c) an isolated stationary cylinder.  $Re = 150$ .

As  $T/d$  reduces to 2.2, the interaction between the two streets is expected to intensify, especially for large  $A/d$ . For stationary cylinders (Figure 3d), two distinct streets are evident and predominantly symmetrical about the flow centreline or in antiphase. Present data also show the occurrence of anti-symmetrical or in-phase vortex streets, as previously reported (e.g. Bearman & Wadcock [1]). The two configurations for vortex streets are also observed when cylinder 1 vibrates. The vibration effect of cylinder 1 is dependent on the configurations of vortex streets. For symmetrical vortex streets, the vibration acts to destabilize the two streets. The two streets quickly break up (Figures 3e, f). On the other hand, the two streets, if anti-symmetrical about the flow centreline, typically start to merge into one at a distance rather near the cylinders (Figure 4). In order to obtain more details of the merging process of the two anti-symmetrical vortex streets, Figure 5 presents sequential photographs for  $Re = 150$  and  $A/d = 0.1$ . The two opposite-signed vortices  $A$  and  $B$  generated by the vibrating cylinder approach each other (Figures 5a ~ 5d). The approaching counter-rotating vortices are likely to generate a low-pressure region between. The low-pressure region is probably responsible for drawing in the inner vortex  $C$  shed from the stationary (lower) cylinder, thus resulting in the amalgamation of the three vortices (Figures 5b ~ 5d). The merging of the three vortices subsequently leads to the formation of a single vortex street downstream, which is probably asymmetrical. The observation bears resemblance to that behind two stationary cylinders. In the asymmetrical flow regime, i.e.  $T/d = 1.5 \sim 2.0$ , where a combination of one wide and one narrow wake occurs, Zhou *et al.* [13] showed the amalgamation of the two cross-stream vortices in the narrow wake with the gap vortex in the wide wake. Subsequently the two streets merge into one. It seems plausible that the vibration of one cylinder could act to reduce the 'effective' spacing between the cylinders. Therefore, the amalgamation of three vortices occur at a  $T/d$  value greater than 2.0.

### Conclusions

The effect of a neighbouring vibrating cylinder on a circular cylinder wake has been investigated based on the laser-illuminated flow visualization. The following conclusions may be drawn.

1. For  $T/d = 3.5$ , the vortex shedding frequency  $f_{s1}$  of the forced vibrating cylinder is always 'locked in' with the vibration frequency  $f_e$  as  $f_e / f_s$  ranges between 0.8 and 1.2, the same as the single cylinder case (Griffin *et al.* [4,5]). But  $f_{s2}$  may not be affected. When  $f_e / f_s$  approaches unity, the vortex shedding frequency  $f_{s2}$  of the stationary cylinder may also be modified, resulting in  $f_e = f_{s1} = f_{s2}$ .
2. When  $T/d$  reduces to 2.2, the two streets do not seem to be stable as a result from the neighbouring vibrating cylinder. The vibration effect depends on the mode of vortex streets. For in-antiphase mode streets, the cross-stream inner vortices interact strongly and consequently break up quickly. A single street probably emerges further downstream. For in-phase mode streets, the two vortices behind the vibrating cylinder appear pairing. The pairing vortices are likely to induce a relatively low-pressure region between, thus drawing in the inner vortex generated by the stationary cylinder. The coalescence of the three vortices leads to the formation of a single vortex street downstream, which is probably asymmetrical. The observation bears resemblance to the asymmetrical flow regime behind two stationary cylinders ( $T/d = 1.5 \sim 2.0$ ). Evidently, the vibration of one cylinder acts to enlarge the asymmetrical flow regime.



Figure 4. Downstream evolution of in-phase vortex streets as  $T/d = 2.2$ ,  $Re = 150$ ,  $f_e / f_s = 1.22$ ,  $f_e = f_{s1} = f_{s2}$ ,  $A/d = 0.1$ .

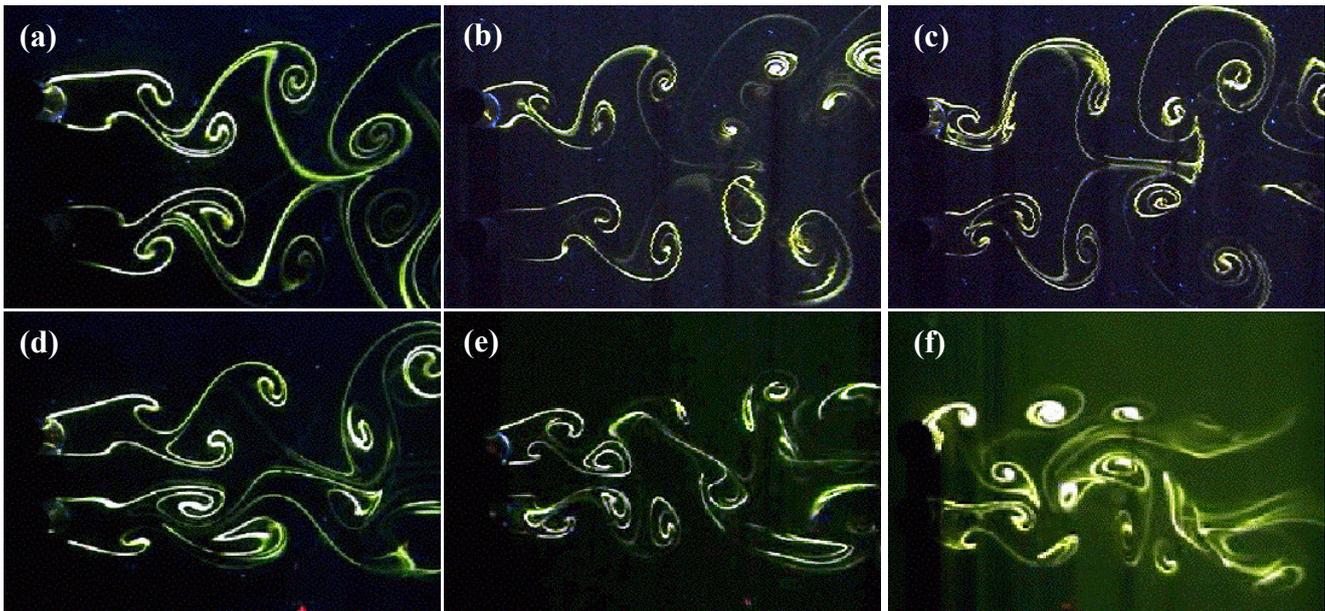


Figure 3 Comparison of flow patterns between different vibration amplitude  $A/d$ : (a)  $T/d = 3.5$ ,  $A/d = 0$ ,  $f_e / f_s = 0$ ; (b) 3.5, 0.1, 1.28; (c) 3.5, 0.5, 1.22; (d) 2.2, 0, 0; (e) 2.2, 0.1, 0.89; (f) 2.2, 0.5, 1.28.  $Re = 150$  and  $f_{s1} = f_{s2}$ .

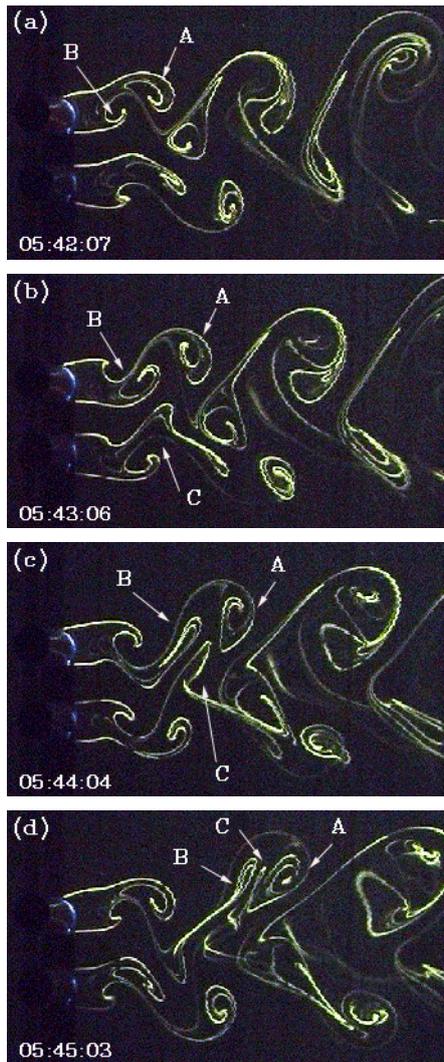


Figure 5. Sequential photographs of flow visualization: merging of the two streets into one.  $T/d = 2.2$ ,  $Re = 150$ , the upper cylinder is vibrating at  $A/d = 0.1$ ,  $f_e / f_s = 0.89$ ,  $f_e / f_{s1} = 1.06$  and  $f_e / f_{s2} = 1.17$ .

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