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FLOW FIELD CALCULATION IN A TURBULENT LUBRICATION FILM

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Abstract
A simple computational technique to calculate the velocity field in a laminar-turbulent hydrodynamic lubricant film is presented. A modified eddy viscosity model to express the Reynolds stresses was used in the computation. There are two physical aspects of lubricant flow to be considered in the model. Firstly, the flow in a lubrication film occurs in a very thin film, and as such unless the bearing's Reynolds number is very large, a velocity profile governed by the universal law of wall will not be established. The eddy viscosity model must be able to cover this region. Secondly, favourable and adverse pressure gradients are always present in the lubrication film. The effect of these pressure gradients is accounted for in the model in the computational procedure. Using the technique, the effect of various typical hydrodynamic lubrication flow conditions such as Reynolds numbers and pressure gradients is presented for a long bearing case.

Introduction
The use of a low viscosity lubricant and high operating speeds are common in many bearing applications. As a result of these practices, many bearings are now operated beyond the laminar flow regime. Turbulent theories have been proposed by Constantinescu [1], Ng and Pan [2] and Hirs [3]. Their theories are the most widely accepted ones. Bearing performance characteristics predicted by these theories have shown well agreement with experimental data for both journal and thrust bearing operated in turbulent regime. However it is difficult for an engineer to determine if a flow in a thin film is laminar, transition or fully turbulent before any of the mentioned theories can be applied because of varying lubrication local film thickness and hydrodynamic pressures. This means that some section of the film operate in laminar regime and some operate in turbulent regime. The velocity profiles in the film will adopt its shape according to the local flow regime and pressure gradient. Therefore the study of the velocity field in the hydrodynamic lubrication film is very important especially in a journal bearing and a thrust bearing. In this paper velocity field in the lubricant film are calculated using a Reynolds stress model. Some efforts to calculate Reynolds stresses in thin lubrication film were made by the authors [4] using the modified Van Driest mixing length model. Using the same approach but with an improved eddy viscosity model of Antonia et. al. [5] a computational technique is developed for thin lubrication film flow in this paper. The technique computes the two components of flow, ie. Poiseuille and Couette components, and superimpose them to obtain the true flow field in a lubricant film.

The computational procedure provides an engineer with a tool for calculating the flow field in lubricant flow based on the eddy viscosity model.

Reynolds Stress Modelling
The total shear stress in turbulent planar flows is given by[6].

$$\tau = \mu \frac{du}{dy} - \rho \overline{u'v'} = \mu\left(1 + \frac{\nu_t}{\nu}\right)\frac{du}{dy} \tag{1}$$

In the wall layer, the characteristic velocity is given in Eq.(2).

$$u_\tau = \sqrt{\tau_w / \rho} \tag{2}$$

Non-dimensionalising u and y with Eq. (2) the total shear stress expression now becomes,

$$\tau^+ = \frac{du^+}{dy^+} + \nu_t^+ \frac{du^+}{dy^+} \tag{3}$$

The mean streamwise velocity gradient can be obtained from Eq. (3),

$$\frac{du^+}{dy^+} = \frac{\tau^+}{1+\nu_t^+} \tag{4}$$

The Eddy viscosity ν_t^+ is given by Eq. 4 which is the form proposed by Antonia et. al. [5].

$$\nu_t^+ = \kappa y^+ \tau^+ \left[1 - e^{-\left(\frac{y^+}{\lambda^+}\right)^2} + A_1 \left(e^{-\left(\frac{y^+}{\lambda^+}\right)^3} - e^{-A_2\left(\frac{y^+}{\lambda^+}\right)^3} \right) \right] \tag{5}$$

Where A_1=0.06 and A_2=70.

The damping coefficient λ^+ is a semi empirical function of shear stress gradient, $d\tau^+/dy^+$. For thin channel flow as in a lubricant film the shear stress gradient $d\tau^+/dy^+$ can be related to pressure gradient.

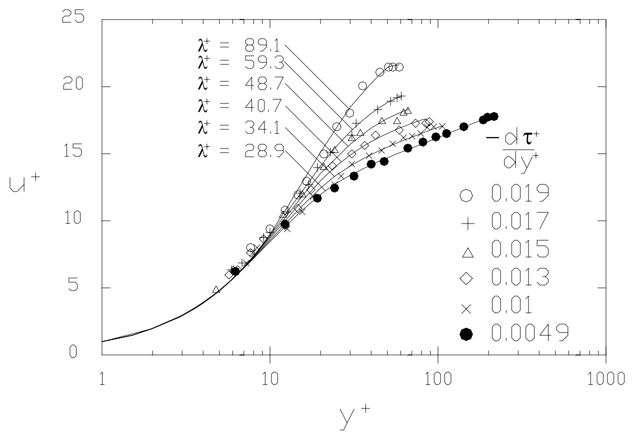
$$\frac{d\tau^+}{dy^+} \approx p^+ \tag{6}$$

To determine the empirical relation between λ^+ and - $d\tau^+/dy^+$ the law of wall profile is established via integration of Eq. (4) using the eddy viscosity model of Eq. (5). The optimum λ^+ that gives the best fit to the experimental data has been determined by means of an error minimization least square fitting method to the channel flow data of [4,7,8].

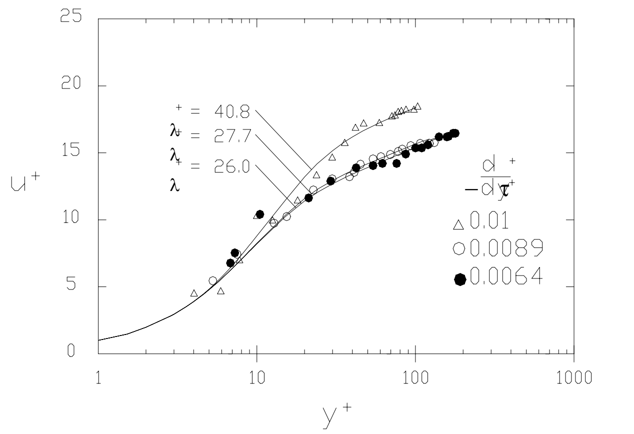
The result of the fitting process is shown in Figure 1. In the figures the law of wall profiles for various channel flow conditions is depicted. The profiles shown cover $-d\tau^+/dy^+$ from 0.0049 to 0.019. These values correspond to Reynolds numbers based on the boundary layer thickness h^+ from 204 to 53. The fits are generally good and the declining asymptotic trend of h^+ will shift the law of wall profiles to that of a linear viscous sublayer ie. $U^+ = y^+$. For a channel flow, it is more convenient to express the relation between λ^+ and $-d\tau^+/dy^+$ in terms of h^+ rather than p^+

although $-d\tau^+/dy^+$ is also related to p^+. For a channel flow the boundary layer thickness is the channel half width. The relation between $-d\tau^+/dy^+$ and h^+ is given by Eq. (7).

$$\frac{d\tau^+}{dy^+} = -\frac{1}{h^+} \tag{7}$$



(a)



(b)

Figure 1. Law of wall profiles. Experimental data (a) Patel & Head [8]. (b) Tieu & Kosasih [4].

It is interesting to note from Figure 2., when $h^+ \rightarrow \infty$, the dependence of λ^+ on h^+ diminishes and λ^+ stays at 26. The velocity profile in the wall layer will then follow the universal law of the wall profile. On the opposite side, when $h^+ \rightarrow h_c^+$, $\lambda^+ \rightarrow \infty$. This means there is a critical wall shear stress where the wall layer will be completely dominated by viscous layer. The determination of h_c^+ warrants further investigation.

For the purpose of modelling, we propose to use the relation given in Eq.(8).

$$\left(\frac{26}{\lambda^+}\right)^2 = 1 - \frac{A}{h^+} \tag{8}$$

The choice of constant A is governed by the value of critical wall shear stress. From confined flow analysis we found that a value of A = 25 is suitably chosen. Based on this value from Eq. (8) it can be inferred that h_c^+ is 25. For $h^+ < 25$, λ^+ is ∞. When this is substituted into Eq. (5), $\nu_t^+ \rightarrow 0$.

The Reynolds stress distributions in channel flow is shown in Figure 3. The figure shows the effect of decreasing h^+ on Reynolds stress. One clear indication is that with decreasing h^+ the proportion of viscous sublayer over the channel width grows. Comparison with Direct Numerical Simulation taken from [5] is shown in Figure 4. The correlations with DNS results are very good.

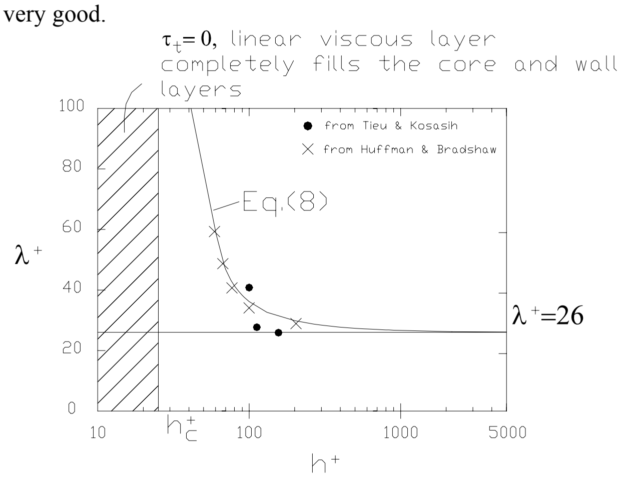


Figure 2. Variation of damping coefficient λ^+ with h^+

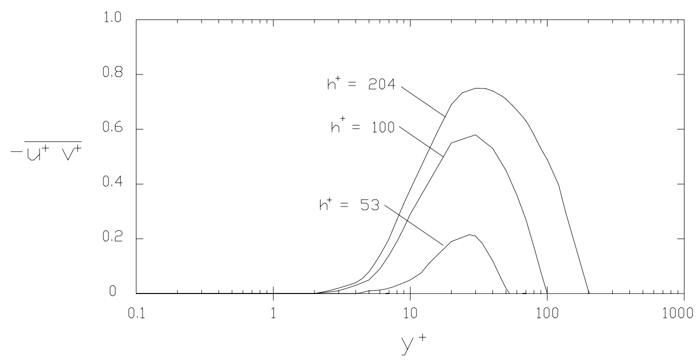


Figure 3. Reynolds stress distribution in channel flow

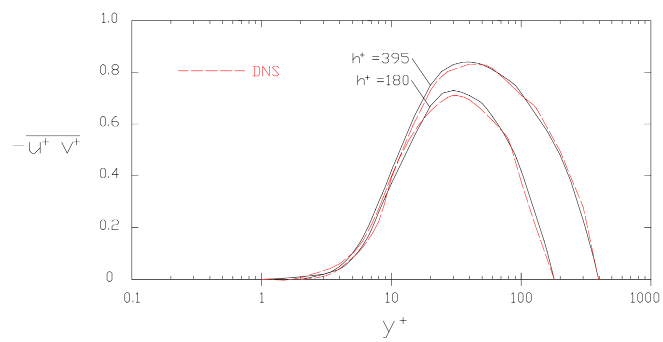


Figure 4. Reynolds stress distribution in duct flow, ------, DNS [5]

Application of the Eddy viscosity model for lubrication flow

For planar flow, the velocity profiles can be obtained upon integration of Eq. (4). However, the flow in typical lubrication film will be a combination of Couette and Poiseuille flow (CPF). The velocity distribution for CPF is normally expressed in dimensionless form with respect to the sliding velocity of the moving surface for the velocity and with respect to the film thickness for the transverse y-coordinate. From Eq. (1) the velocity gradient can be written as.

$$\frac{d\bar{u}}{d\bar{y}} = \frac{H\left(\tau_{wc} + \tau_{wp} + \frac{dp}{dx}y\right)}{V\rho\nu\left(1+\nu_T^+\right)} \tag{9}$$

Eq. (9) can then be separated into two parts $\frac{d\bar{u}_c}{d\bar{y}}$ and $\frac{d\bar{u}_p}{d\bar{y}}$.

$$\frac{d\bar{u}_c}{d\bar{y}} = \frac{\eta_{Hc}^2}{Re\left(1+\nu_T^+\right)}$$

(10)

Also, the Poiseuille component is expressed.

$$\frac{d\bar{u}_p}{d\bar{y}} = \frac{\eta_{Hp}^2\left(1-a_x y\right)}{Re\left(1+\nu_T^+\right)} \tag{11}$$

The relation between η_{Hc} and η_{Hp} can be expressed as in Eq. (12).

$$\eta_H^2 = \eta_{Hc}^2 + \eta_{Hp}^2 \tag{12}$$

η_{Hp}^2 is dependent on the pressure gradient $\frac{dp}{dx}$. Using the dimensionless form η_{Hp1}^2 is obtained.

$$\eta_{Hp1}^2 = \frac{Bx\,Re}{2} \tag{13}$$

η_{Hp2}^2 is equal to η_{Hp1}^2 but of opposite sign. Noting that $\eta_{Hc1}^2 = \eta_{Hc2}^2$, it yields,

$$\eta_{H1}^2 - \eta_{H2}^2 = Bx\,Re \tag{14}$$

Velocity field is calculated via integration of Eqs. (10) and (11). The flow field is divided into 2 equal thicknesses, $0 \le \bar{y} \le \bar{y}_n$ and $1-\bar{y}_n \le \bar{y} \le 1$ as shown in Figure 3.

Using non-dimensional terms eddy viscosity can now be written as follows,

$$\nu_T^+ = \kappa\bar{y}\eta_H\left[1-e^{-\left(\frac{\bar{y}\eta_H}{\lambda^+}\right)^2} + 0.06\left(e^{-\left(\frac{\bar{y}\eta_H}{\lambda^+}\right)^3} - e^{-70\left(\frac{\bar{y}\eta_H}{\lambda^+}\right)^3}\right)\right] \tag{15}$$

In Eq. (15) , the damping coefficient is obtained by Eq. (8), which is a function of the wall shear stress and the boundary layer thickness or half film thickness of the concerned half of the film. λ^+ is given by Eq. (7) and re-written using the present notation.

$$\left(\frac{26}{\lambda^+}\right)^2 = 1 - \frac{a}{\eta_h} \tag{16}$$

where;

$$\eta_{h1} = \eta_{H1}\bar{y}_n \qquad \text{for field 1} \tag{17}$$

$$\eta_{h2} = \eta_{H2}\left(1-\bar{y}_n\right) \qquad \text{for field 2} \tag{18}$$

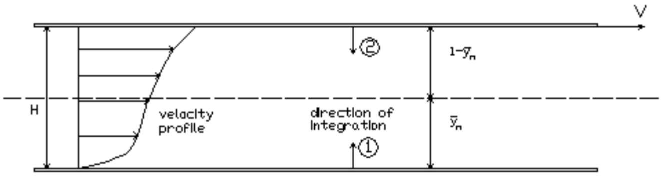


Figure 3. Shear stress and velocity profile in planar thin lubrication film

Given the values of pressure gradient parameter Bx and Reynolds number Re, the iterative process is initiated by assuming a value of total wall shear stress for field 1 expressed as η_{H1}^2 and $\bar{y}_n$. From this assumption, η_{Hc1}^2 is calculated from Eq. (12). Once η_{Hc1}^2 is known, $\bar{u}_{c1}$ is calculated by Eq. (10). $\bar{u}_{p1}$ is calculated by Eq. (11). In the similar manner $\bar{u}_{c2}$ and $\bar{u}_{p2}$ are obtained. And the search for correct η_{H1}^2 is stopped when $\bar{u}_1$ and $\bar{u}_2$ which corresponds to the velocity at $\bar{y} = \bar{y}_n$ must be equal within the set tolerance. Typical velocity profiles in planar lubrication film are shown in Figures 4 and 5.

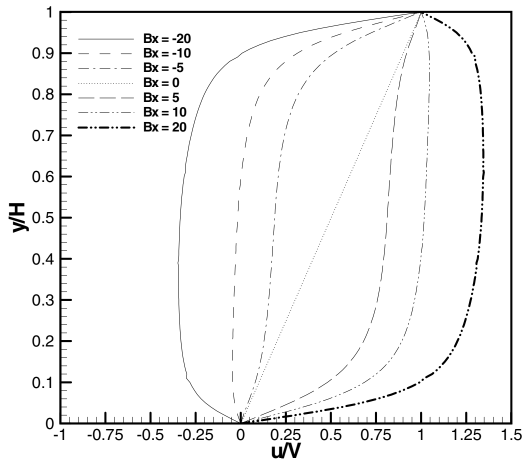


Figure 4. Velocity profiles in lubrication film for Re = 2000 and various pressure gradient.

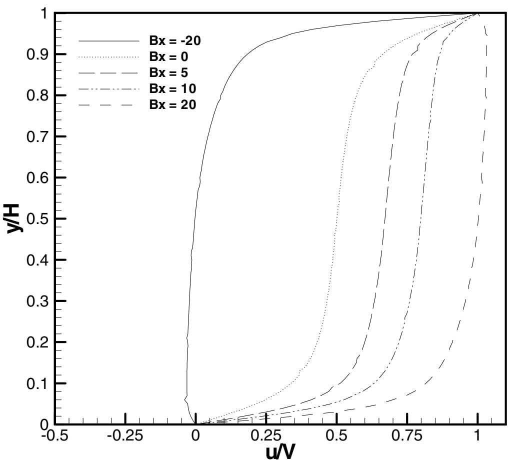


Figure 5. Velocity profiles in lubrication film for Re = 5000 and various pressure gradient.

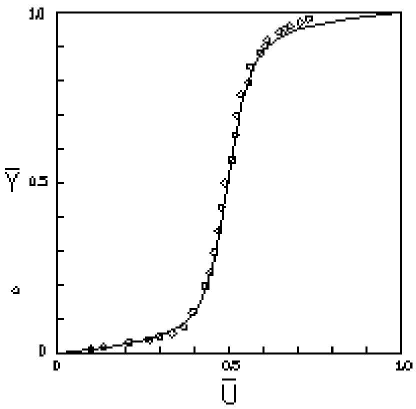


Figure 6. Comparison of theoretical and experimental couette velocity profile [9] Re = 9524.

Figure 6. shows quantitative comparison of theoretical couette velocity profile with experimental profile.

Conclusion

A computational technique for flow field analysis in lubrication film has been presented. The computation procedure uses transition-turbulent model based on eddy viscosity with damping coefficient. It is shown that this damping coefficient can be related to the term $h\,u_\tau / \nu$ or wall Reynolds number. It is shown that the transition from stable laminar flow to turbulence is initiated when the term hu_τ / ν reachs a certain value.

Acknowledgement

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Nomenclature

a_x $= \dfrac{Bx\,Re}{\eta_H^2}$

Bx = dimensionless pressure gradient, $-\dfrac{H^2}{\mu V}\dfrac{dp}{dx}$

H = channel thickness

h = half film thickness / channel thickness

h^+ = Wall reynolds number , $h\,u_\tau / \nu$

Re = Reynolds number , HV / ν

l = mixing length, $l^+ = l\dfrac{u_\tau}{\nu}$

u',v' = fluctuating velocity component

u = stream wise velocity component,

$\bar{u} = u/V$, $u^+ = \dfrac{u}{u_\tau}$

u_τ $= \sqrt{\dfrac{\tau_w}{\rho}}$

$u_{\tau c}$ $= \sqrt{\dfrac{\tau_{wc}}{\rho}}$, $u_{\tau p} = \sqrt{\dfrac{\tau_{wp}}{\rho}}$

y = y-coordinate, $\bar{y} = y/H$, $y^+ = y\dfrac{u_\tau}{\nu}$

Symbols

η_H $= H\dfrac{u_\tau}{\nu}$, $\eta_{Hc} = H\dfrac{u_{\tau c}}{\nu}$, $\eta_{Hp} = H\dfrac{u_{\tau p}}{\nu}$

η_h $= \bar{y}\,\eta_H$

κ = constant , 0.44

λ^+ = damping coefficient

τ = shear stress, $\tau^+ = \dfrac{\tau}{\tau_w}$

τ_w = wall shear stress

τ_{wc} = wall shear stress due to Couette flow

τ_{wp} = wall shear stress due to Poiseuille flow

μ = dynamic viscosity

ν_T = eddy viscosity, $\nu_T^+ = \dfrac{\nu_T}{\nu}$

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