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# Influence of Electromagnetic Field on the Capillary Flow between Parallel Plates

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### Abstract

The influence of electromagnetic field on the surface tension driven capillary flow between two parallel plates is investigated with special reference to MEMS. Analytical solutions are obtained for different values of the non-dimensional parameters that represent the physics of the problem. It is inferred that the application of electric current in the presence of relatively low magnetic field strength may be used to optimize the fluid flow characteristics. Thus one may overcome partial curing of liquid epoxy during the underfill encapsulation process and dry-out phenomena occurring in micro-channel cooling devices used especially in space thermal management.

### Introduction

Modeling and understanding fluid flows in micro-channels are important to design and control of various micro-electrical-mechanical systems (MEMS), that are used nowadays in the modern electronic industry for high rates of heat removal and to enhance the reliability of flip-chip interconnects. Recently surface tension driven flow between parallel plates was investigated by a few [4, 5] referring it either to underfill epoxy encapsulation process between the chip and substrate, or rewetting phenomena that often occur during micro-thermal management in space applications. In these applications the time required for the fluid to flow in the micro-gap is so crucial as they can lead to partial curing or dry-out of the fluid before the flow is complete. It is found from the survey of literature that investigations on the optimization of the fluid flow characteristics in micro-channels, especially when driven by surface tension forces, is lacking. Therefore in this paper we propose to study the influence of electromagnetic field on the fluid flow characteristics in the micro-gap formed between parallel plates and driven by surface tension forces. More investigations, especially on the silicon filler particle migration during underfill encapsulation, are on going and will be discussed on the dais.

### The Model

Consider a one-dimensional fully-developed incompressible, laminar and electrically conducting fluid flow between two parallel plates, separated from each other by a distance $d$ (depth), and driven by surface tension forces under the action of constant electrical and magnetic fields as shown in Fig. 1.

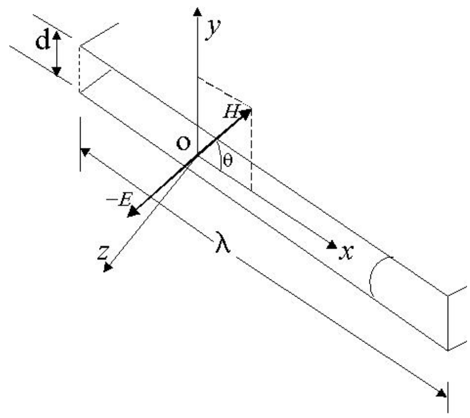


Figure 1. Schematic diagram of the channel

Let $u$ be the axial component of the fluid velocity and $p$ the pressure along the flow is assumed to a function of $x$ only. An unperturbed electrical field $\vec{E} = (0,0,E)$ is supplied and magnetic field $\vec{H} = (H\cos\theta, H\sin\theta, 0)$, is assumed to exist everywhere in the fluid layer with $\theta$ being the inclination angle of the magnetic field. Then the equations together with the boundary conditions that govern the above assumptions may be written in the dimensionless form as

$$-\frac{dp}{dx} = -\frac{1}{\beta \mathrm{Re}}\frac{d^2u}{dy^2} + \frac{Ha^2 \sin^2\theta}{\beta \mathrm{Re}}u - \frac{\alpha Ha^2 \sin^2\theta}{\beta \mathrm{Re}} - \mathrm{R}, \quad (1)$$

$$u\left(\pm\frac{1}{2}\right) = 0, \qquad p_{|x=0} = 0, \qquad p_{|x=L} = -\frac{\Delta p}{\rho_o c_o{}^2}. \quad (2)$$

Since both sides of (1) are independent of each other, the partial differential equation can be separated into two ordinary differential equations:

$$\frac{dp}{dx} = -k, \tag{3}$$

$$\frac{d^2u}{dy^2} - Ha^2 \sin^2\theta u + \alpha Ha^2 \sin\theta + 3 = -k\beta \,\mathrm{Re}, \tag{4}$$

where $k$ is to be determined. Solving (3) subject to the boundary conditions (2) gives

$$u = \left(\frac{D_1 + D_2 + D_3}{A}\right)\left[1 - D_4 \cosh(\sqrt{A}\, y)\right], \tag{5}$$

where

$$D_1 = \alpha\, Ha^2 \sin\theta, \quad D_2 = 3, \quad D_3 = k\beta\,\mathrm{Re}, \quad D_4 = \frac{2e^{\frac{\sqrt{A}}{2}}}{e^{\sqrt{A}} + 1}$$

and $A = Ha^2 \sin^2\theta$

The speed of the fluid front is equal to total average flow velocity and is given by

$$\frac{dL}{dt} = \int_{-1/2}^{1/2} u\, dy. \tag{6}$$

Substituting (5) into (6), integrating and solving for $k$ gives

$$\frac{dp}{dx} = -k = -\frac{A}{D_3}\left(1 - \frac{2D_4}{\sqrt{A}}\sinh\left(\frac{\sqrt{A}}{2}\right)\right)\frac{dL}{dt} - \left(\frac{D_1 + D_4}{D_3}\right). \tag{7}$$

The Laplace-Young equation of capillarity states that the pressure drop across the liquid-vapor interface, from the concave side to the convex side, is inversely proportional to the radius of curvature of the free surface [2,3] and is given by, $\Delta p = (2\gamma \cos\varphi)/d$, where $\gamma$ is the surface tension at the liquid-vapor interface. Now solving (7) for the pressure, $p$, subject to the boundary conditions (2) using the above relation for $\Delta p$, determines $k$ and is given by

$$k = -\frac{2\gamma \cos\varphi}{\rho_o c_o^2 dL} = -\frac{We}{L}, \tag{8}$$

where $We$ is the Weber number describing the influence of surface tension forces.

**Results and Discussions**

To present the results in brief we discuss only the effect of electromagnetic field on velocity distribution across the channel. The values of the parameters are chosen to be $Re = 10^{-1}$, $We = 10, \theta = 45^o, \beta = 10^{-2}$. Figures 2-4 represents the fluid velocity variations for different combinations of electric field parameter $\alpha$ and the Hartmann number $Ha$.

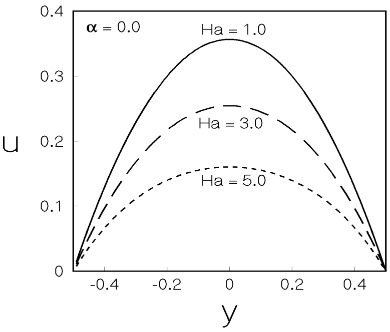


Figure 2. Velocity distribution at different $Ha$ for $\alpha = 0.0$

For $\alpha = 0$, the results reveal the classical Hartmann-Poiseuille

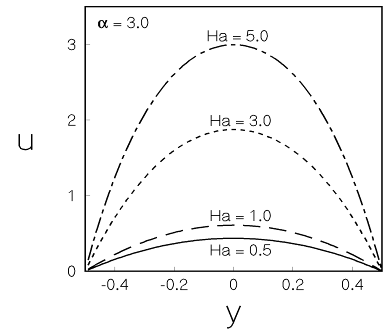


Figure 3. Velocity distribution at different $Ha$ for $\alpha = 3.0$

flow. The magnitude of the velocity decreases with increase of

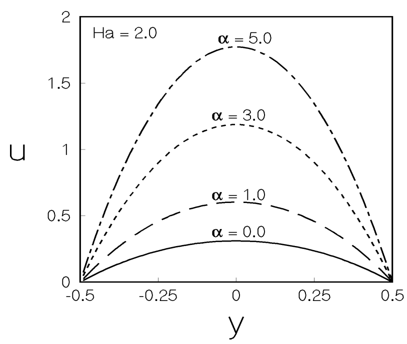


Figure 4. Velocity distribution at different $\alpha$ for Ha = 2.0

$Ha$ ((at $\alpha = 0$) see Fig. 2), due to the stabilizing effect of magnetic field, and hence the resulting fluid flow time is expected to increase. The application of electric current together with the magnetic field is accomplished by the increase in the magnitude of the velocity compared to its absence (see Figs. 2 & 3). This in turn results in lesser fill time for the fluid to fill the entire channel. This may be explained as follows. If an electrical current is passed through the fluid with the external magnetic field applied perpendicular to the direction of the electric current, a pressure difference is created in addition to that of the surface tension forces. This results in pumping of the fluid along the axial direction of the channel, thus increasing the fluid flow rate. The AC type of electrical field is assumed to be chosen here as the DC type may lead to bubble generation in the channel which in turn may affect the fluid flow rates, [1]. Figure 4 depicts the velocity profiles for a fixed $\alpha$ and different Ha. It is clear from this figure that higher fluid flow rates in the channel may be achieved even at relative low magnetic field strength.

## Conclusion

A new way of influencing the capillary flow driven by surface tension forces, by way of an electromagnetic field, has been investigated here. It is found that the fluid flow rate between parallel plates can be significantly increased by the application of electromagnetic field, when compared to its absence. The application of both the electric and magnetic fields may further be employed to optimize the flow characteristics in micro-channels. The past history of underfill flow studies, our present on-going numerical investigations on the migration of filler particles (suspended in the underfill liquid) into the channel, and the scope of future studies will be discussed in detail on the dais.

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## Nomenclature

$A$ – parameter in Equation (5)
$c_o$ – Nusselt velocity
$d$ – channel depth
$D_1, D_1, D_3, D_4$ – parameters in Equation (5)
$E$ – electric field
$H$ – magnetic field
$Ha$ – Hartmann number
$g$ – gravitational acceleration
$k$ – constant defined in Equations (3) & (4)
$L$ – fluid flow length
$R$ – parameter appearing in Equation (1)
$Re$ – Reynolds number
$p$ – pressure
$t$ - time
$u$ – velocity
$We$ – Weber number
$(x,y,z)$ – co-ordinates

Greek Symbols

$\alpha$ - electric field parameter
$\beta$ – dispersion parameter
$\nu$ – kinematic viscosity ($\mu/\rho_o$)
$\phi$ – inclination angle of the channel
$\sigma$ – surface tension
$\varphi$ – contact angle
$\lambda$ – length
$\mu$ – dynamic viscosity
$\rho_o$ – reference density
$\theta$ – inclination angle of magnetic field