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Some Aspects of the Aerodynamics of Membrane Wings

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Abstract

The primary aim of this paper is to explore some of the consequences of membrane stiffness for real flying membrane wings, with emphasis on the structural stiffness due to the initial shape as well as the elasticity of the membrane itself. Earlier work has concentrated on the effect of membrane stretch on lift, but since this analysis is set in the context of a flying wing supporting an aircraft or animal the effect of membrane stiffness on the pitching moment is also presented. The constraint of fixed lift force is then shown to make significant changes to the apparent effect of membrane deformation. The paper develops the work of Johnston [6], who investigated the gliding flight of the dinosaur *Pteranodon ingens*.

Introduction

Since the shape of a membrane wing may alter significantly when subjected to an aerodynamic loading, the analysis of such wings requires simultaneous solution of the coupled aerodynamic and structural problems. Three aspects of stiffness need to be considered for membrane wings. The first is the elasticity of the membrane itself, which causes changes in the twist and camber of a wing when aerodynamic loading is applied. The relative importance of the in-plane membrane deformation is measured by the non-dimensional aerodynamic *stiffness number* of the membrane, defined by Jackson [3] as

$$Æ = \frac{k}{qc}. \qquad (1a)$$

Here k is the stiffness of the membrane sheet (the product of the elastic modulus of the sheet and its thickness), q is the dynamic pressure and c is the reference chord length.

What has not been noted before is that for a particular flying wing this stiffness number is not fixed, since it is a function of the dynamic pressure and therefore must change with the flight speed. A more suitable stiffness number is therefore

$$\Xi = \frac{kS}{Wc} = \frac{Æ}{C_L}, \qquad (1b)$$

where S is the wing area and W the weight supported. This number, which we call the *specific stiffness*, is a constant for a given aircraft or animal with a membrane wing and therefore provides a more appropriate measure of the effect of membrane stretch on its aerodynamic performance.

The second contribution to wing stiffness is the so-called *geometric stiffness* inherent in the unloaded wing shape - this depends upon intrinsic measures of its curvature and the manner in which the membrane is attached. A membrane with a surface of zero Gaussian curvature (a developable surface) can be deformed into other shapes of zero curvature without in-plane deformation of the membrane. A wing of conical section, for example, can deform to an infinite range of other conical sections if is restrained only along its generators, as recently discussed by Jackson [5]. For wings of this kind the precise initial shape of the membrane is undefined and the limit of infinite Æ does not correspond to that of a rigid wing of the same initial shape. Conversely, wings with non-zero Gaussian curvature must suffer some in-plane deformation of the membrane in order to change shape. The magnitude of this deformation then depends not only upon the applied load and elastic stiffness of the membrane, but also upon the magnitude of the initial curvature. Generally a decrease in initial camber will be associated with an increase in membrane stresses and consequently in membrane displacement.

The third consideration of stiffness is that membranes cannot sustain a compressive stress. This means that an inextensible membrane with non-zero Gaussian curvature, which would otherwise be effectively rigid, may still be readily deformed to other shapes which include regions of wrinkling. The effects of this characteristic are not explored here, but all the results include an appropriate model for wrinkling.

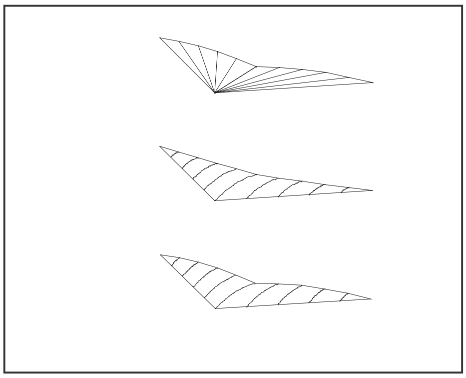

Figure 1. Test wings with sweep angle γ = 39.8°. Wing A at the top has its apex at the centre, while Wing B has apexes at the tips and a NACA 64 section. Wing C, at the bottom, has compound curvature.

Requirements for Equilibrium and Stability

For an aircraft or animal of fixed weight in level flight or at a shallow glide angle, equilibrium of vertical forces and pitching moment requires

$$C_L = \frac{W}{qS}, \quad h_{cg} = -\frac{C_M}{C_L}, \qquad (2)$$

where h_{cg} is the dimensionless distance from the leading edge to the centre of gravity, and C_M is the nose-up moment at the leading edge. From the argument presented above, the lift and moment coefficients of a membrane wing of given unloaded shape must be functions of the angle of attack and stiffness number; $C_L = C_L(\alpha, Æ)$, $C_M = C_M(\alpha, Æ)$. For a given design of fixed Ξ, we may then eliminate Æ as an independent variable, so that $C_L = C_L(\alpha)$ and $C_M = C_M(C_L)$, as we should expect.

For a rigid wing the relationship between C_M and C_L is linear, but this is not necessarily the case for a flying membrane wing because the wing shape changes with C_L. However the relationship may still be linearised in the neighbourhood of any given angle of attack, and this permits a conventional analysis of longitudinal stability. While this analysis is well known (Etkin [1]) it is included here for completeness. The region of interest is that near the balance point for longitudinal moment equilibrium where equations (2) are satisified. If near this point the moment is approximated by $C_M = C_{M0} - h_{ac}C_L$ then a small change in angle of attack leads to a moment about the centre of gravity of $C_M + h_{cg}\, C_L = C_{Mo} + (h_{cg} - h_{ac})(C_{Lo} + \Delta C_L)$. From the balance requirement $C_{Mo} = (h_{ac} - h_{cg})C_{Lo}$, and for static stability the change in moment must attempt to restore the original angle of attack. This requires

$$h_{ac} > h_{cg} \quad \text{and so} \quad C_{Mo} > 0 \qquad (3)$$

The length h_{ac} gives the distance to the aerodynamic centre (the point about which the moment does not vary with angle of attack), which is considered fixed for a rigid wing but here may vary with lift and speed because these change the wing shape.

Calculation of the Flying Shapes

The remainder of the paper demonstrates some of the effects of initial wing shape and membrane stiffness on the aerodynamic performance of membrane wings. The approach employed here was developed by Johnston[6] and is similar to that used by a number of previous authors (e.g. Jackson [3], Schoop [11], Muttin [7]) and so we do not dwell on the details here.

The method of solution for the aerodynamics employs the well-known panel method using quadrilateral doublet panels (Hunt [2]). The finite element formulation follows that developed by Oden and Sato [9] for the analysis of large displacements and finite strains in elastic membranes, and employs triangular elements having a linear displacement field. The primary point of departure of the present method from those of earlier studies is that the elements used in the aerodynamic and structural analyses do not coincide. Although using the same elements is numerically simpler, a more accurate solution is obtained by utilising the most efficient arrangement of node and collocation points for each of the two methods independently. The coupling of the two independent numerical methods was achieved by using smooth representations of the surface geometry, mean velocity distribution and potential jump. This was also used by Jackson and Fiddes [4] is very similar to the more recent work of Schoop et al.[10].

A full justification and demonstration of the accuracy and convergence of the method is given by Johnston[6]. In the examples below, 256 doublet panels and 324 finite elements are used with the overall criterion for convergence requiring that the last iteration performed by the aeroelastic model contributes less than 0.5% to the total nodal displacements of the wing surface. The model then normally took 4 to 7 iterations to converge.

Wing A: a free developable surface

As noted above, membrane wings having initial shapes which are developable surfaces can be deformed to new shapes with no in-plane deformation of the membrane, if the boundary conditions permit. Here we study a wing of initially conical shape which is fully restrained along the centreline and its straight leading edges, as shown in Figure 1. Because these edges are generators, even with no membrane stretch the trailing edge of the wing may take on an infinite range of shapes which preserve the overall length of the trailing edge. In this study the trailing edge is straight in planform but has an initial vertical displacement prescribed by a sinusoid of given maximum displacement, *t*. The only remaining parameters needed to describe the initial shape fully are the aspect ratio and leading edge sweep angle. As a reference wing we took a shape with sweep angle 39.8° (= $\tan^{-1}5/6$), stiffness number 4096, and spanwise camber on the trailing edge of $t/c = 0.05$. The other wings are variations on the reference wing, all having an aspect ratio of 6 and no dihedral.

Figure 2 shows the initial shape of the trailing edge and the corresponding equilibrium shapes for the typical range Æ = 512, 1728, 4096, and 8000. Figure 3 shows performance data in a standard format. The left side shows the lift coefficient, the induced drag parameter C_D/C_L^2 and the moment coefficient C_M, all versus angle of attack, while the right side shows moment versus lift. The 'rigid' data are for the initial undeformed wing.

As expected the lift curve at high membrane stiffness does not correspond to that of the initial shape, but shows slightly higher lift at a given angle of attack. This is because the equilibrium wing shape has less twist inboard and more twist at the tip than the initial shape, as shown in Figure 2, so that the main in-board part of the equilibrium wing is at a higher angle of attack than that of the initial wing. However as the stiffness number decreases, the increasing stretch in the membrane causes the twist to increase again until the lift falls below the initial value. The slope and intercept of the C_L/α curve therefore vary with Æ, although the non-linearity is slight and most evident at low angles of attack.

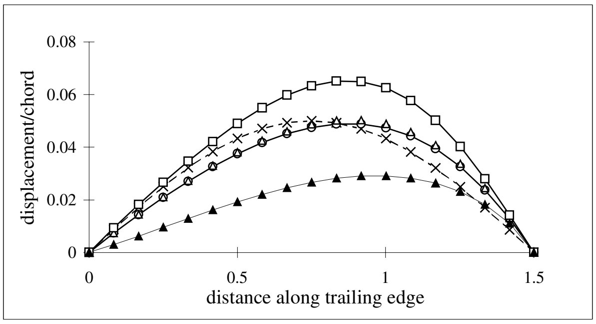

Figure 2 Trailing edge shapes for Wing A . (X, □, Δ, O, key as for Fig 3; with no initial camber and Æ = 4096 ,–▲ –)

Because of its initial spanwise camber this membrane has a high geometric stiffness and so there is little membrane stretch until the membrane stiffness number falls to quite low values. For this particular test case the distribution of stress in the membrane (not shown) shows that the stress is fairly uniform in magnitude and direction, and directed more-or-less spanwise. This indicates that the strain is also fairly uniform and therefore that the equilibrium membrane also maintains a more-or-less developable shape. As Figure 2 suggests, increasing the stretch then changes the magnitude of the displacements rather than the surface shape.

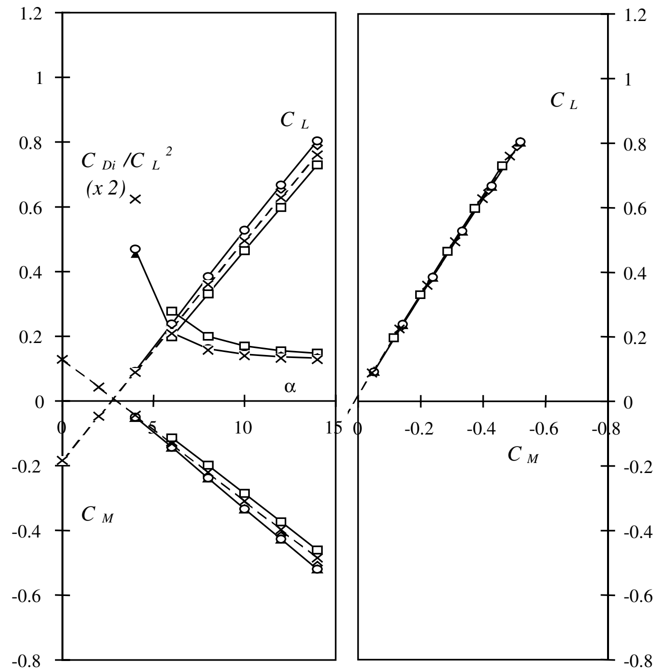

Figure 3. The aerodynamic performance of Wing A for various values of aerodynamic stiffness.(rigid, – –x– –: Æ = 512, –□–; 1728,✧; 4096, Δ; 8000, –o–)

By contrast the C_M/C_L curves for the equilibrium wing shapes barely differ from that for the initial shape, so the membrane stiffness has little effect on trim or stability. Because all the points lie close to the same curve and this curve has a very small C_{Mo}, the wing is nearly neutral in stability and may fly at a wide range of lift coefficients with very little change of the centre of gravity position. The induced drag parameter shown in Figure 3 indicates that, as should be expected, the distribution of the loading is far from elliptic and changes significantly with incidence, particularly for values less than about 6 degrees, with the effect of stretch being to increase the drag at any given lift.

The effect of geometric stiffness was explored by altering the degree of spanwise camber in the reference wing. These tests were made with the initial camber of the trailing edge set to *t/c* = 0.00, 0.025, 0.05 and 0.075. The results are not shown here but, as expected, the lift at a given angle reduced as this in-built twist was increased. However the lift slope increased as membrane stiffness increased, because the shapes with lower spanwise camber generated much greater in-plane loads at the same lift and therefore the extra twist induced by the membrane stretch was higher. This can be seen in Figure 2, where the equilibrium shape for Wing A with no initial spanwise camber (*t/c = 0*) is shown for *Æ* = 4096; for this case the deflection of the trailing edge from its initial position can be seen to be very much higher than that of the original reference Wing A with the same *Æ* .

Wing B: a locked developable shape

A second class of wing may start with zero Gaussian curvature, but be 'locked' into a particular shape by the boundary conditions. One such example is a wing which is conical along straight lines radiating from the tips, restrained along its leading edge but also fixed along its central chord-line. Because of this second boundary condition the wing cannot change shape without in-plane deformation of the membrane, which requires either stretching of the membrane or compressive wrinkling. This wing has the same aspect ratio and sweep as that used above, but with no initial deflection of the trailing edge. The initial geometric stiffness must then arise from chordwise camber, which here was set by specifying the sections to the camberline of a NACA 64 airfoil, the resulting shape being shown in Figure 1.

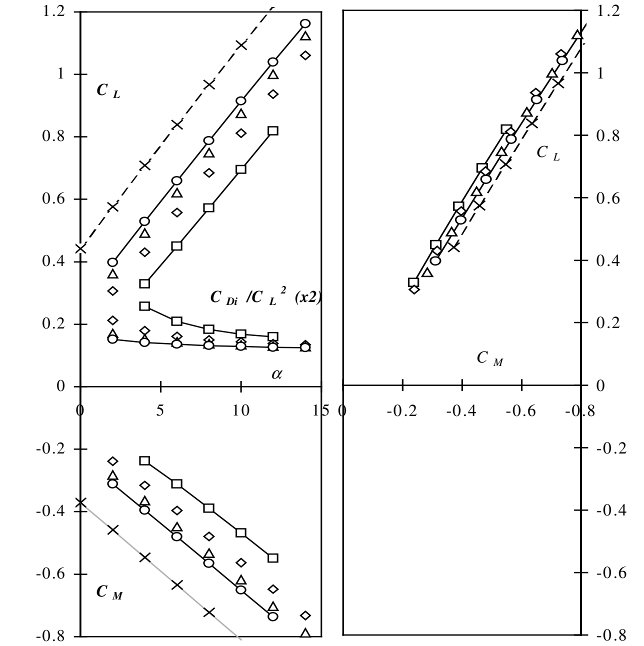

Figure 4 The aerodynamic performance of Wing B for various values of aerodynamic stiffness (key as for Fig 3).

Figure 4 shows the wing behaviour for the same range of stiffness numbers as before. The effect of membrane stiffness is quite dramatically different from the results shown in Figure 3, primarily because the initial wing shape now has no geometric stiffness along its unsupported trailing edge. It therefore develops large stresses there, and while the associated membrane strain is still small it leads to a significant degree of deflection in the trailing edge (again, because the edge is initially straight) and therefore to significant wing twist. As a result, the lift of the initial rigid wing shape is always far higher than the equilibrium membrane shape, even at high levels of membrane stiffness. The strong effect of Æ on C_L is also reflected in the C_M/α curves, with a lift reduction leading to a corresponding loss of moment, but the C_L/C_M curves show that this is also associated with a much more pronounced effect on stability. The intercept C_{Mo} increases (becomes less negative) as Æ decreases so membrane stretch is stabilising in this case, presumably because it adds washout to the wing.

However in order to find the aerodynamic behaviour of a wing supporting a fixed weight these results must be converted from curves of constant aeroelastic number to curves of constant specific stiffness, by interpolating to find lines of constant $\Xi = Æ/C_L$. This was done here by an appropriate curve-fitting process, producing the results shown in Figure 5 for a range of values of Ξ. The C_L/α curves are again much as one would expect, but with a steeper slope than for the rigid wing shape. The effect of membrane stiffness on stability is now *much* less pronounced than Figure 4 would suggest, but the flexible wing is still slightly more stable than a rigid wing of the same initial shape. The slope of the C_L/C_M curves changes for different values of Ξ at the same lift coefficient, and also changes slightly with C_L for a fixed value of Ξ. This means that for a particular wing the membrane stretch may cause both the aerodynamic centre and the stability margin to change with flying speed.

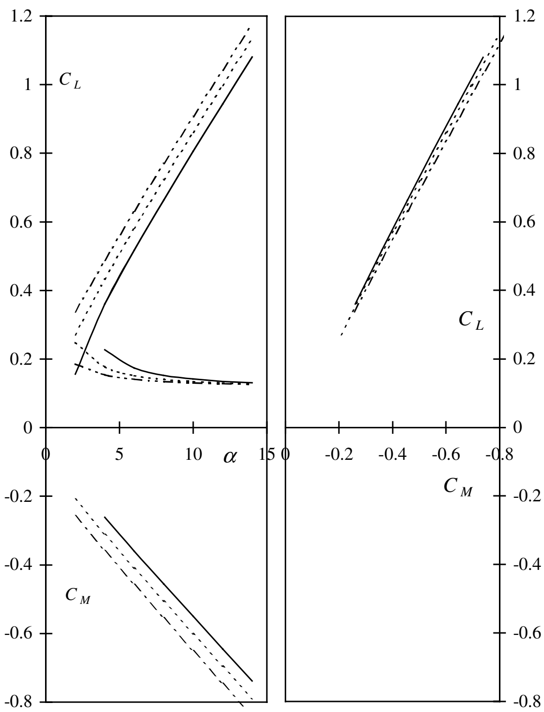

Figure 6 The aerodynamic performance of Wing B for various values of specific stiffness. (Ξ = 2000,——; 4000, – – – –, ; 8000, - - - - - -)

Wing C: a surface with compound curvature

The final wing presented is a combination of the first two; the chordwise camber is as for Wing B, while built-in twist is added by using the same trailing edge shape as for Wing A. This wing therefore has non-zero Gaussian curvature and so cannot change shape without in-plane deformation, with the curved trailing edge adding significant geometric stiffness. The membrane is again restrained along its leading edge and the centreline chord.

The added stiffness due to spanwise camber changes the stress field (not shown) to one of greatly reduced magnitude and more even distribution than that of Wing B. As expected the lift of the rigid wing is much less than that for Wing B due to the initial deflection of the trailing edge but the effect of the membrane stretch is dramatically less for the same reason. Because *Æ* has relatively little effect on this wing, it follows that neither does Ξ. Apart from a small improvement in stability, this membrane wing has much the same behaviour as a rigid wing of the same shape until the aerodynamic stiffness falls to relatively low values.

Conclusions

It has been shown that the appropriate non-dimensional number for the aerodynamics of aircraft or animals with elastic membrane wings should be formed using the fixed wing loading and the membrane modulus, giving a new specific stiffness for the wing. For the wings examined here, the effect of this specific stiffness on lift and drag was much the same as that of the membrane stiffness number used previously, but its effect on longitudinal stability was markedly less in one case.

The three wings examined were chosen to demonstrate the concepts of elastic and geometric stiffness and to explore the ways in which these can interact. For Wing A with a developable initial shape all the equilibrium flying shapes were very similar with the membrane stiffness affecting only the magnitudes of the membrane displacement. Because this wing has an initial curvature in the direction spanning its fixed supports it has a high geometric stiffness, so that the induced membrane stresses are relatively low and the membrane stiffness therefore has little effect on its aerodynamics.

By contrast the initial shape of Wing B is not curved between its supports and so has low initial geometric stiffness and is not capable of sustaining load without deforming the membrane. The subsequent strains cause a significant twist in the wing which therefore has much less lift than the initial rigid shape even at high levels of membrane stiffness. However the membrane stretch also induces washout, by the same mechanism, and therefore improves the longitudinal stability of this wing.

In the final wing the initial shape has non-zero Gaussian curvature. The increased geometric stiffness results in it having a much lower and more uniform stress distribution than Wing B, so the resulting small strains mean that membrane stretch has little effect on the wing. Consequently this wing has much the same behaviour as a rigid wing of the same shape, except at very low values of the aerodynamic stiffness.

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