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Modelling of Bubble Dynamics in a Venturi Flow with a Potential Flow Method

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Abstract

A simple model for cavitation bubble dynamics has been developed using a combination of boundary element methods and one-dimensional bubble dynamic equations. Each bubble is assumed to be spherical and is modelled using a potential source or sink. The strength of the source or sink is governed by one-dimensional bubble dynamic equations so that the velocity of the growing or decaying bubble at the interface between the vapour and liquid is correctly represented. The model has been implemented into computer program to study the growth, collapse, and interaction of bubbles as they flow through a venturi. Interactions between a bubble starting as a nucleus of gas, the surrounding liquid and the venturi boundaries are described. Although this is a simple model, surprisingly complex interactions can be studied with short computational times and limited computer resources. Thus, insights have been gained which otherwise would have been extremely difficult to obtain. These are described in terms of the bubble history, instantaneous velocity maps and instantaneous stream function contours.

Introduction

Multi-fluid systems are important in many natural and industrial processes, in particular in cavitation and boiling. Despite recent developments in the visualisation and measurement of fluid flows, in numerical simulations and in the power of computers for modelling fluid flows, difficulties remain when dealing with two-phase flow, because the characteristics of interest are the result of complicated interactions of the two phases.

In order to make progress in the numerical modelling of two phase liquid flow in which evaporation and condensation occur, two approaches have been adopted. The first involves the study of the phenomena associated with a single, isolated vapour bubble [1]. The accurate numerical prediction of two-phase flow processes at the micro-scale, even without heat and mass transfer, is a major problem in itself [2], so that the inclusion of micro-physical phenomena in codes for the calculation of macro-phenomena is beyond the means of most researchers and designers.

The second approach is therefore usually based on the assumption that a liquid-vapour mixture exists whose properties can be approximated by correlations or theories based on single bubble results. Once the properties have been established the equations of motion are solved, with or without the inclusion of heat and mass transfer [3]. The problem is made more difficult by the fact that very often two-phase flow problems are also unsteady. Numerical studies of cavitation or boiling are therefore extremely expensive in terms of computer resources and in particular computer time.
A number of authors have attempted to reduce the computing time requirements, by assuming that the liquid and vapour are inviscid (eg. [4]). They obtained results, which gave reasonable agreement with experimental data, but, more significantly, resulted in important insights into phenomena in two phase flow [6]. It appears therefore that it might be possible to represent two-phase flow by modelling a sufficient number of individual bubbles in an inviscid liquid.

A technique for such a model of cavitation in a venturi handling a cavitating liquid is described. The flow in the venturi is generated by a panel method. Whereas other workers have used boundary elements to represent bubbles, thereby limiting the number which could be studied simultaneously, moving sources or sinks are used here. Growth or decay of bubbles, assumed to remain spherical, is determined from the Rayleigh-Plesset equation. The strength of each of the sources can then be calculated by matching the velocity of the vapour-liquid interface, as determined from the Rayleigh-Plesset equation, with the velocity generated at the same radius by a source or sink. This results in a simple code, which runs very fast, and is able to handle a large number of bubbles simultaneously.

Theory

Figure 1 shows the dimensions and uniform velocity distribution at the inlet of the two dimensional venturi. The walls of the venturi are sinusoidal and are modelled using lumped vortex boundary elements with a constant streamfunction boundary condition [5]. That is, on the walls the streamfunction, ψ, is given by:

$$\psi_w = U_\infty h, \tag{1}$$

for the upper wall, and $\psi_w = -U_\infty h$, for the lower wall. Here, h is the half height of the inlet (see Figure 1) and U_∞ is the magnitude of the uniform velocity at the inlet.

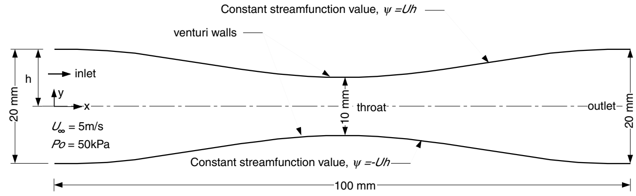


Figure 1 : Definition sketch of venturi.

The walls are divided into a number of boundary elements with a single potential vortex positioned at the centre of each element, which are positioned in such away that there is a higher concentration at the throat of the venturi. At the centre of each element the following conditions is satisfied:

$$\sum_{i=0}^{i=n} \psi_{vortex_{ij}} = \psi_w - \psi_{fs_j} - \sum_{k=0}^{k=m} \psi_{bubble_{kj}}, \tag{2}$$

in which,

$$\psi_{vortex_{ij}} = \frac{\Gamma_i \Delta_i}{2\pi} \ln \sqrt{\left(x_j - x_i\right)^2 + \left(y_j - y_i\right)^2}\,, \tag{3}$$

where $\psi_{vortex_{ij}}$ is the streamfunction of the vortex i at the centre element j, Γ_i is the vortex strength, Δ_i length of the element i, n is the total number of vortices on the wall. In eq. (2)

$$\psi_{fs_j} = U_\infty y_j \tag{4}$$

is the streamfunction of the freestream at the centre of the boundary element j. The velocity field induced by a bubble on the venturi flowfield is modelled using a two-dimensional point source when the bubble is growing and a sink when the bubble is collapsing. In eq. (2)

$$\psi_{bubble_{kj}} = \frac{\sigma_k}{2\pi} \tan^{-1}\left(\frac{y_j - y_k}{x_j - x_k}\right) \tag{5}$$

is the streamfunction of bubble k, m is the total number of bubbles and σ is the source strength or sink which is governed by the radius and growth rate of the bubble k. The bubble, which is assumed to remain spherical, has radius R at time t, and its growth rate, dR/dt, is determined by solving the Rayleigh-Plesset equation [6] which with the assumption that the temperature variations are very small, may be written as

$$\frac{P_v - P(t)}{\rho_l} + \frac{P_{Go}}{\rho_l}\left(\frac{R_o}{R}\right)^3 = R\frac{d^2R}{dt^2} + \frac{3}{2}\left(\frac{dR}{dt}\right)^2 + \frac{4\upsilon_l}{R}\frac{dR}{dt} + \frac{2S}{\rho_l R}\,, \tag{6}$$

in which, P_{Go} is the initial gas partial pressure in a bubble of initial radius R_0, P_v is the vapour pressure, $P(t)$ is the pressure at the bubble centre at time t, ρ_l is the liquid density, ν_l is the liquid viscosity and S is the coefficient of surface tension. This non-linear differential equation is solved using second order finite differencing. $P(t)$ is calculated from the unsteady Bernoulli equation [5],

$$P(t) = P_o - \frac{1}{2}\rho_l V^2 - \frac{\partial \phi}{\partial t}, \tag{7}$$

in which, P_o is the total pressure, V is the bubble velocity and ϕ is the velocity potential.

Once R and dR/dt have been determined, σ can be calculated. Since the radial velocity field is given by

$$v_r = \frac{\sigma}{2\pi r} \tag{8}$$

when $r = R$, and $v_r = dR/dt$, the source or sink strength is $\sigma = 2\pi R\ dR/dt$.

A set of simultaneous equations is constructed and solved for Γ_i. In matrix notation, this is written as

$$\left[\Gamma_i\right] = \left[K_{ij}\right]^{-1}\left[\psi_w - \psi_{fs_j} - \sum_{k=0}^{k=m}\psi_{bubble_{kj}}\right], \tag{9}$$

in which,

$$K_{ij} = \frac{\Delta_i}{2\pi} \ln \sqrt{\left(x_j - x_i\right)^2 + \left(y_j - y_i\right)^2} \tag{10}$$

The problem is now fully defined, unfortunately no analytic solution exists, so that a computer program has to be written in order to obtain the necessary solutions.

Numerical Procedure

The method of solving the whole time dependent flow-field is firstly, to solve for the vortex strengths from which, together with the stream function of each of the bubbles, the velocity field and velocity potential can be calculated everywhere in the region of interest. The pressure at the position of the bubble can then be determined and the Rayleigh-Plesset equation solved using second order discretisation in time. Now, R, dR/dt, and σ, can be evaluated. The new bubble positions are calculated from the local velocity for the time step, δt. This is repeated until all bubbles have exited from the venturi.

Results

A study of the motion and growth histories of a single bubble and two bubbles is presented here in order to illustrate the capabilities of the program. The program is capable of handling multiple bubbles. In all cases bubbles were released at the entrance to the venturi. The bubbles were initially filled with air and the liquid was water at 25^oC. The time-step in the solutions in this paper $50ns$.

As bubbles move downstream and the pressure decreases in the convergent part of the venturi, they initially grow by the expansion of air. When the pressure falls below the vapour pressure, due to evaporation, there is a significant increase in the growth rate. The maximum bubble size is reached downstream of the venturi throat and the bubbles collapse further downstream.

Single Bubble

Figure 2 shows the history of the growth and motion as well as the flow when a bubble, with an initial radius R_0, of $50\mu m$, is released on the centre line. At t=7.5ms the bubble has reached the throat, Figure 2(a), it continues to grow, Figure 2(b), until it reaches its maximum size at t=8.5ms, Figure 2(c), a full millisecond after its passage trough the throat. Now, the collapse begins and the bubble radius reduces, Figure 2(d). Finally at time t=9.5ms the bubble has become so small that it cannot be seen in Figure 2(e).

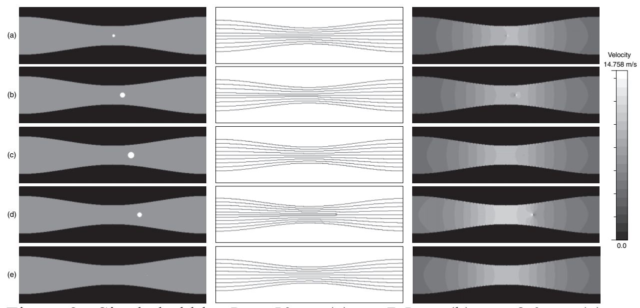
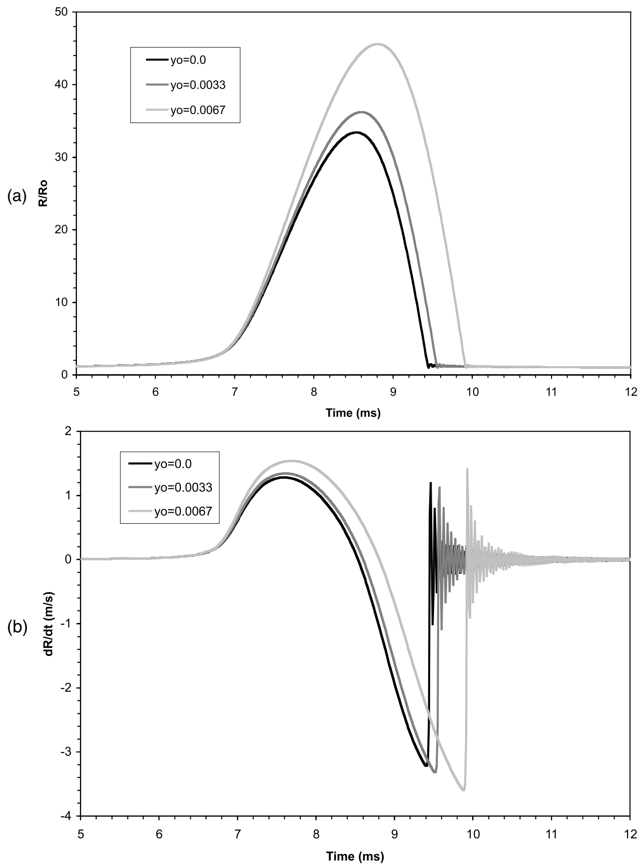


Figure 2 : Single bubble: $R_0 = 50\mu m$ (a) $t = 7.5ms$, (b) $t = 8.0ms$, (c) $t = 8.5ms$, (d) $t = 9.0ms$,and (e) $t = 9.5ms$. Left: bubble image, centre: streamlines, and the right: velocity map.

The streamlines in the centre of Figure 2 illustrate that there is a wake (Figures 2 (a-b)), caused by the dividing streamline with the total flow rate out of the venturi greater than the flow into it. Because of the presence of the moving source there is no violation of continuity, but it might be unphysical. It may be seen in Figure 2(b) that the source strength is increased over that in Figure 2(a) and that at the point of maximum growth, Figure 2(c) the source strength is zero. In Figure 2(d) the sink indicates that the bubble is collapsing and the inflow into the venturi is increased, which, in this case, is physically meaningful. The liquid velocity distribution is shown on the right hand side of Figure 2. The effect of the presence of the bubble is most clearly seen in Figure 2(d) in which the velocity near the throat has been increased relative to those shown in the other Figures in this group. Since the bubble is collapsing, liquid is required to fill the "void" created, so that this is physically correct, as mentioned above. Similarly to the moving source, a sink in a moving fluid results in a dividing streamline, which encloses the source and stretches to the entrance. Since the flow within this enclosing streamline is absorbed by the source the inflow into the venturi is increased, despite the fact that the velocity at the entrance has been set to a constant value.

Figure 3 illustrates the effect of releasing a $50\mu m$ bubble at different initial positions (y_0) above the centreline. The history of the motion is similar for all bubbles, however, the larger the y_0 the greater is R_{max}. The wall forces the fluid between it and the bubble to accelerate, thereby lowering the pressure near it and enhancing their growth. Bubbles away from the centreline are able to grow for a longer time, Figure 3(a), have a higher maximum interface velocity, Figure 3(b), and their motion is more effected by the walls, Figure 3(c). A bubble released on the centreline remains on that line, whereas bubbles released closer to the walls move away from their starting streamline (Figure 3(d)). A bubble released at y_0=0.0033m drifts towards the wall, whereas a bubble released at y_0=0.0067m moves towards the centre.



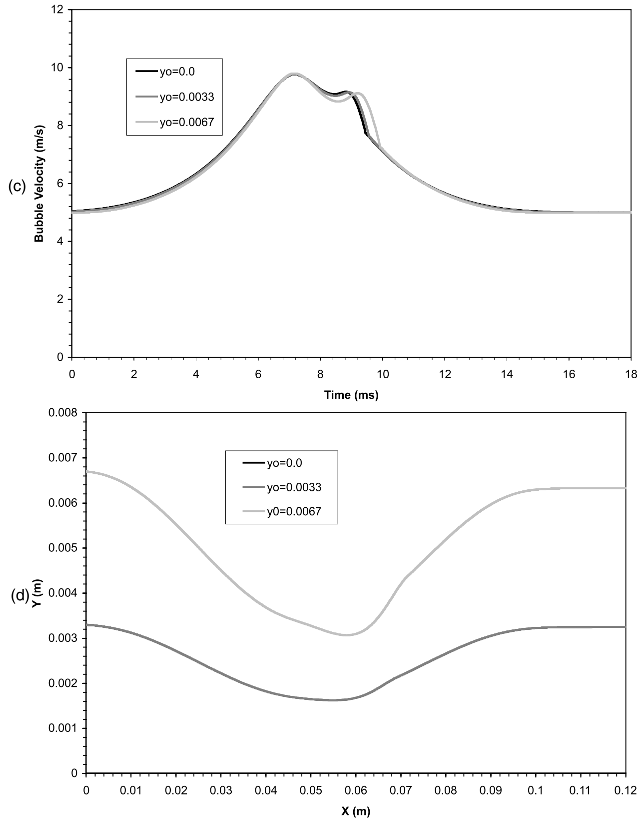


Figure 3: Single bubble R_0 = 50 μm: y_o = 0.0, 0.0033 & 0.0067m (a) R/R_0 versus time, (b) dR/dt versus time, (c) Velocity of the bubble versus time, and (d) Path traced by the bubble.

Two Horizontally Separated Bubbles

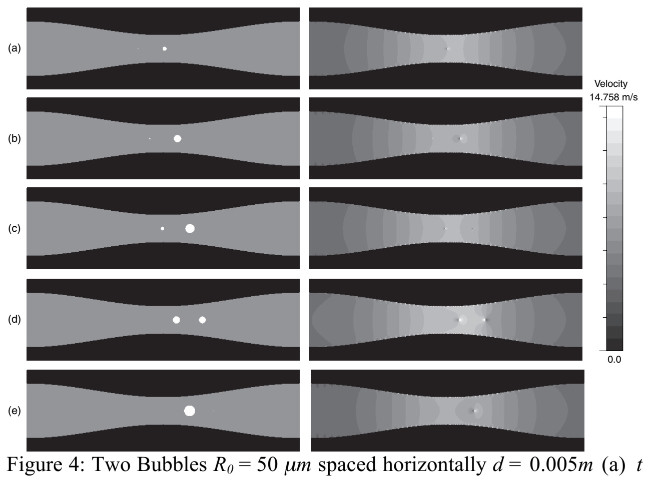


Figure 4: Two Bubbles R_0 = 50 μm spaced horizontally d = 0.005m (a) t = 7.5ms, (b) t = 8.0ms, (c) t = 8.5ms, (d) t = 9.0ms, and (e) t = 9.5ms. Left: bubble image; Right: velocity map.

The effect of releasing two bubbles each with an initial radius of $50\mu m$, on the centreline, one 0.005m behind the other, is shown in Figure 4. Figures 4(c) and Figure 4(e), show that the second bubble grows to a larger size than the first, Figure 5(a). However, a comparison of Figures 3(a) and 5(a) show that all first bubbles are only slightly affected by the second.

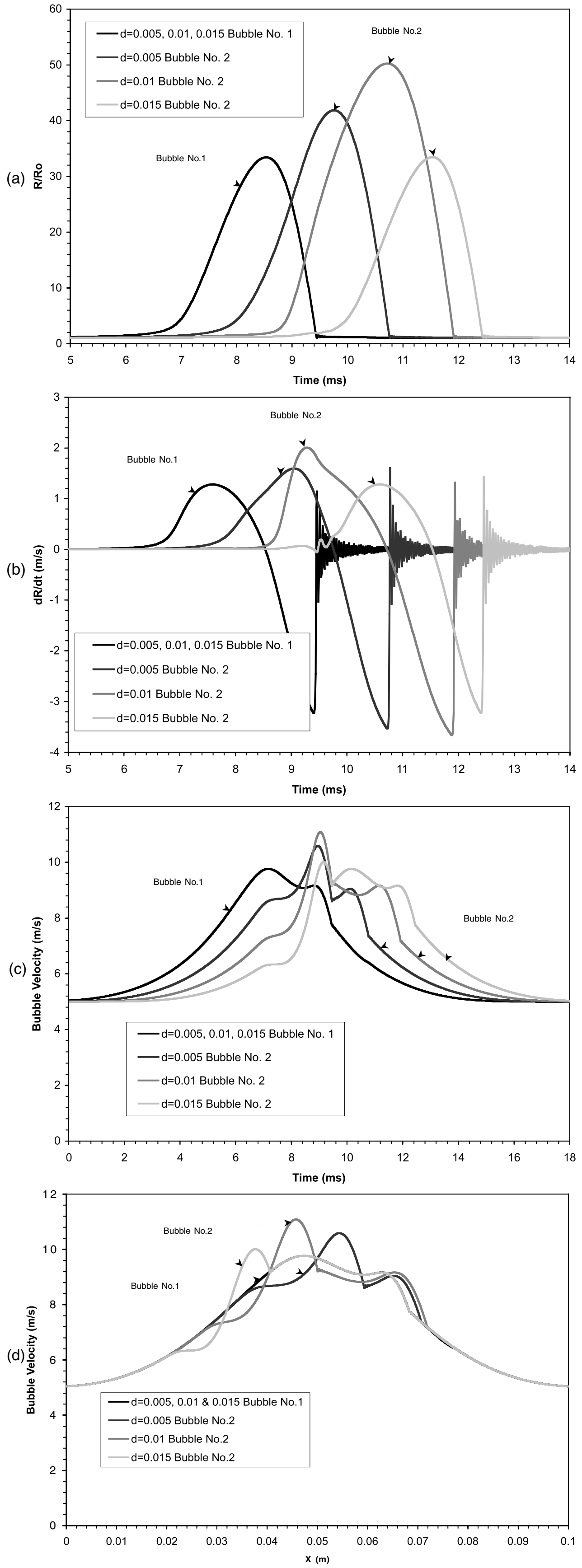


Figure 5: Two Bubbles: $R_0 = 50$ *μm*, spaced horizontally $d = 0.005m$ (a) R/R_0 versus time, (b) dR/dt versus time (c) Velocity of bubble versus time, and (d) Velocity of bubble versus distance.

Unexpectedly, the second bubble, whose initial separation is $200R_0$ from the first bubble, has an R_{max} 1.5 times that the R_{max} of an isolated bubble, while a bubble whose initial separation is $100R_0$ only grows to 1.25 the R_{max} of an isolated bubble. Similarly, the maximum bubble growth rate (Figure 5(b)) and the maximum bubble velocities (Figures 5(c and d) are greater for the second bubble whose initial separation is $200R_0$ than when the separation is $100R_0$. This may be explained by the fact that, if the bubbles are close together, the fluid between the two bubbles needs to be displaced while the bubbles grow, so that the forces required are such that the pressure rises and inhibits the growth of both bubbles. The pressure rises less when the bubbles are further apart and the effect is negligible at $d=300R_0$.

Superficially all the bubble velocity histories of the second bubble look similar (Figure 5 (c)), with the maximum bubble velocity occurring at approximately the same time for all three bubbles studied. However, the maximum bubble velocity occurs at very different positions as may be seen in Figure 5 (d).

Conclusions

A theory for the motion, growth and decay of vapour bubbles in an inviscid liquid flowing in a venturi was developed in this paper. The liquid flow was modelled by the boundary element method and the bubble, assumed to remain spherical, by a moving source whose strength was determined by matching the solution of the Rayleigh-Plesset equation at the bubble interface. Single bubble and two bubbles arranged in various positions, were studied. It has been demonstrated that valuable physical insights were be easily obtained for this complex problem from this very simple model, which takes only a few minutes on a desk top computer.

Acknowledgments

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