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An Effective method of calculating transonic flows and wave drag of axisymmetric and 3-D elongate bodies within framework of transonic equivalence rule.

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Abstract

A theoretical study of steady transonic flows about axisymmetric and 3-D elongate bodies has been carried out within the framework of non-linear small disturbance theory using the transonic equivalence rule. A numerical method for calculating transonic flow about bodies of revolution was used by formulating the Alterating Direction Method together with monotone Engquist-Osher's algorithm for the axisymmetric case. This approach has a good reputation in solving similar two-dimensional problems.

A new effective algorithm to determine wave drag of bodies of revolution in inviscid transonic flow has been developed. As a result, the wave drag value is obtained by velocity jump integration along shocks closing local supersonic regions. This method is more exact then the ordinary method of integration of pressure along the body surface.

The efficiency of a numerical algorithm is demonstrated by calculating transonic flow over thin bodies of revolution. The results concerning calculations of the wave drag of elongate 3-D configurations of hypersonic aircraft in the framework of transonic equivalence rule are also given; these results were derived earlier by A.S. Fonarev and M.A. Naida, whilst working at the Central Aero-hydrodynamic Institute (TsAGI), Moscow, Russia.

Introduction

When transonic flows about thin wings with moderate aspect ratio and their combinations with fuselage, the maximum cross-sections size of which is small in comparison with its length, are considered within the framework of small disturbance theory, the significant problem simplification can be achieved through the use of asymptotic matching expansions technique [1]. As it was shown in [1] there are two regions, flows in which are governed by different boundary-value problems. In the inner (adjacent to body) region the main term of the disturbed potential satisfies Laplace equation in perpendicular to body planes. In the outer region at large distance from the body the solution has axisymmetric pattern and its main term coincides with the solution for the equivalent body of revolution.

Thus, the solution for 3-D flow problem can be obtained by simultaneous solving two boundary-value problems with unknown functions depending on two spatial variables. As a result, this approach is employed basically to determine pressure distribution over the surface of the vehicles with small lift [2].

In the numerical study of problems concerning flow about bodies, the necessary aerodynamic coefficients are usually calculated by integrating the distribution of pressure over the surface of the body. However, it should be noted that the accuracy of this approach might be affected by errors made in the numerical calculation – particularly in the nose and tail regions. Calculations have shown [3] that these errors are negligible in the determination of the lift and moment coefficients and that the traditional method is acceptable for both steady and unsteady problems.

The situation is different in regard to determination of the wave drag. It is known [3,4] that integration errors – connected both with errors in the numerical analysis and with violations of the postulates of the transonic theory of small perturbations at individual points on the body – may produce errors in the final result obtained by the traditional approach. These errors may even lead to negative values for the drag coefficient. Thus, here we use another method to determine the wave drag of bodies – a method that is less sensitive to integration errors. The authors of [4] were the first to propose replacing integration of pressure over the surface of the body by another procedure that does not involve integration at locations where the postulates of the transonic theory may be violated. The alternative method has now been used extensively in studies devoted to finding ways to determine steady and unsteady wave drag in transonic flows.

For example, it was proposed in [3,4] that wave drag in a steady flow past thin airfoils be found by integrating along shock waves enveloping supersonic regions. This method is less sensitive to numerical integration errors connected with specific features of perturbation theory.

A similar approach in determining the steady-state wave drag was used in [5], where it was assumed that the contour integral of the longitudinal component of momentum along the shocks is equal to the contour integral over the surface of the body.

The author of [6] and [7] generalised the method to the case of unsteady transonic flow and obtained the time dependence of aerodynamic characteristics of wing profiles for different transient – such as the interactions of the profile with a wind shear, a moving shock wave, etc.

The value of drag calculated in the above-cited works for different airfoils agree well with the experimental data, which shows the effectiveness and reliability of the method. It is interesting to attempt extend the method to the case of flow about axisymmetric bodies. There are certain features specific to this problem [8,9].

1. Statement of the problem and numerical method

The problem of transonic flow past a prolate solid of revolution can be examined within the framework of the non-linear transonic theory of small perturbations. Here, we obtain steady-state solution by proceeding on the basis of the equation for unsteady conditions. This unsteady transonic equation can be written as follows in dimensionless form, as it has been done in [10]

$$M_\infty^2\varphi_{tt} + 2M_\infty^2\varphi_{xt} = [(C_1 + C_2\varphi_x)\varphi_x]'_x + \frac{1}{r}(r\varphi_r)'_r \qquad (1.1)$$

where

$$C_1 = 1 - M_\infty^2, \quad C_2 = -\frac{\gamma+1}{2}M_\infty^2, \qquad (1.2)$$

φ is the potential of velocity perturbations, the x and r axes of a cylindrical system of coordinates with origin at the middle of the body are parallel and perpendicular to the unperturbed flow, respectively, M_∞ is the Mach number of the incident flow, and γ is the adiabatic index.

All the quantities in (1.1) are dimensionless. The perturbation potential is relative to the velocity of the incoming flow U_∞ and the length of the body L, the independent variables x and r are relative to L and the time t is relative to L/U_∞.

The initial and boundary-value problem is obtained by adding initial and boundary conditions to Eq. (1.1). The initial conditions are obtained by assigning the potential and its derivative with respect to time. The boundary conditions are more complicated. Since (1.1) is non-linear and is mixed elliptic-hyperbolic with respect to the space variables, depending on whether the incident flow is subsonic or supersonic, the conditions on the outer boundaries of the calculation domain must be of two forms.

However, as we are studying unsteady flow, it could happen that the incident flow changes from subsonic to supersonic and vice versa. Moreover, with a finite calculation domain, the usual subsonic condition expressing decay of the perturbation potential far from the body means that perturbations, which have reached the boundary, are reflected. Since transonic flows are sensitive to small changes in the flow parameters, we must eliminate any possible influence of the boundary conditions on the flow field near the body in the form of perturbations reflected from external boundaries. We do this by using special non-reflecting boundary conditions, which are appropriate for both the subsonic and supersonic domains. Conditions of this kind were obtained for the plane case in [10] and for axisymmetric case in [11] by an analysis of the asymptotic behaviour of a relation that was satisfied on the characteristic surface of the original equation.

We use here the results of [11]:

$$C\varphi_x - M_\infty(M_\infty + \sqrt{C+M_\infty^2})\varphi_t = 0, \qquad x \to -\infty$$

$$C\varphi_x - M_\infty(M_\infty - \sqrt{C+M_\infty^2}\varphi_t = 0, \qquad x \to \infty$$

$$\sqrt{C\varphi_r} + M_\infty\sqrt{C+M_\infty^2}\varphi_t = 0, \qquad r \to \infty$$

The formulation of the condition on the lower boundary requires special consideration. Unlike the plane case, Eq. (1.1) has a singularity at $r=0$ and the impermeability condition on the body cannot be taken down to the axis of symmetry. Thus, the boundary condition is determined from the solution of Eq. (1.1) as $r \to 0$ allowing for the impermeability condition

$$\lim_{r\to 0}(r\varphi_r) = RR_x' = S_x'/(2\pi), \tag{1.3}$$

where $R(x)$ is the coordinate of the body and $S(x)$ is its cross-sectional area. This relation will be used as a boundary condition on a cylindrical surface with small cross-sectional radius r_*, which we will also take as the lower boundary of the outer domain:

$$\varphi_r = 0, \quad |x| > 0.5, \quad r\varphi_r = S_x'/(2\pi), \quad |x| \le 0.5, \quad r = r_*$$

The numerical calculations of transonic flows for the outer domain were derived by using the Alternating Direction Method, developed in [8].

2. Method of calculating wave drag

We use the integral momentum theorem to determine the wave drag of bodies. Certain transformations must be performed before the integral theorem of moment can be used to calculate the wave drag of a solid of revolution by integrating over a shock wave. With the assumption that the perturbations are small, we write the expression for the pressure coefficient in the form

$$c_p = -2u - v^2$$

The transverse component of velocity on the body is known from the boundary conditions. To determine the longitudinal component, again integrating Eq. (1.3):

$$\varphi = \frac{S_x'}{2\pi}\ln r + g(x)$$

Here, $g(x)$ is a function determined by joining the result with the solution of the external problem. For the longitudinal component of velocity we obtain

$$u = \varphi_x = \frac{S_{xx}''}{2\pi}\ln r + g_x' \tag{2.1}$$

Since the boundary-value problem has been solved, i.e. since the field of perturbed velocity is known, then we also know the distribution of the longitudinal component u_* on the surface of the imaginary cylinder at $r = r_*$. Combining the internal and external solutions at $r = r_*$, we find the value of g' in (2.1) and, thus, the sought distribution of the longitudinal component of velocity on the surface of the body:

$$u(x,R) = \frac{S''}{2\pi}\ln\frac{R}{r_*} + u_*$$

Now we can determine the wave-drag coefficient of the body as the integral of pressure on the body:

$$c_x = -\int_{-o.5}^{0.5} 2\pi RR_x'\left(2u_* + \frac{S_{xx}''}{\pi}\ln r\frac{R}{r_*} + (R_x')^2\right)dx.$$

We take condition (1.3) into account and write this expression in the form of the sum of two integrals:

$$c_x = -4\pi\int_{-0.5}^{0..5} r_*u_*v_*dx - 2\pi\int_{-0.5}^{0.5}\left(S_{xx}''\ln\frac{R}{r_*} + (R_x')^2\right)RR'dx \tag{2.2}$$

The second term can be integrated:

$$2\pi\int_{-0.5}^{0.5}\left(\frac{S_{xx}''}{\pi}\ln\frac{R}{r_*} + (R_x')^2\right)RR_x'dx = \frac{\left(S_x'\right)^2}{2\pi}\ln\frac{R}{r_*}\Big|_{x=-0.5}^{x=0.5} \tag{2.3}$$

Since only thin and (as a rule) closed bodies are examined within the framework of the transonic theory of small perturbations, the terms of (2.3) vanish. Specifically, this occurs because the limit $R^2\ln R \to 0$ as $R \to 0$ in the nose and tail portion of the body. In the case of a body with a nontrivial radius in the bottom part, the complement of resistance – the term with $x = 0.5$ -, may not be zero. In this case, it can be calculated in accordance with (2.3). Thus, the first term remains unknown in the expression for drag (2.2) and must be found.

To examine a system consisting of the stationary analogy of Eq. (1.1) and the equation expressing the condition for the absence of vorticity in the flow

$$r(1-M_\infty)u_x - r(\gamma+1)M_\infty^2 uu_x^2 + (rv)_r = 0$$
$$u_r - v_x = 0.$$

Multiplying the first equation of the system by u and the second by rv and adding the result, we obtain a relation having a divergent form:

$$\left(r\left(\frac{1-M_\infty^2}{2}u^2 - \frac{\gamma+1}{3}M_\infty^2u^3 - \frac{v^2}{2}\right)\right)_x + (ruv)_r = 0.$$

Taking the double integral of this expression over the entire theoretical flow region and using Green's formula to reduce it to a curvilinear integral over a closed contour as in figure 1 we find that

$$\oint r\left(\frac{1-M_\infty^2}{2}u^2 - \frac{\gamma+1}{3}M_\infty^2u^3 - \frac{v^2}{2}\right)dr - ruvdx = 0 \tag{2.4}$$

At $M_\infty \le 1$, the asymptote of the long-range flow field is such [4] that the integrals over the external boundaries approach zero as the boundaries of the theoretical region approach infinity.

Thus, we can write (3.4) in the form

$$\int_s r\left[\frac{1-M_\infty^2}{2}u^2 - \frac{\gamma+1}{3}M_\infty^2u^3 - \frac{v^2}{2}\right]dr - \int_s r[uv]dx - \int_{-0.5}^{0.5} u_*v_*r_*dx = 0,$$

where the index s denotes integration along a shock wave or several shock waves. The brackets denote that the enclosed quantity undergoes a discontinuity in the transition through the shock wave.

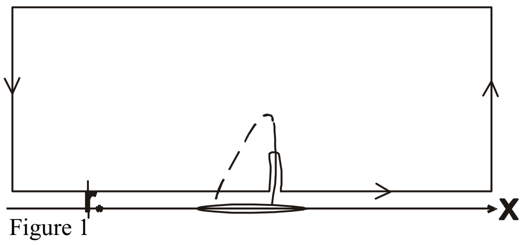

Figure 1

The resulting expression means that, to within the multiplier 4π, the first term in Eq. (2.2) can be expressed in terms of integrals over a shock wave. Here, the integrands differ from the corresponding expressions for the plane case [4] only in the presence of independent coordinate r, which is continuous in the transition through the shock. Omitting the intermediate calculations (where are analogous to the calculations performed in [4]), we present the final result in the form

$$\int_{-0.5}^{0.5} u_* v_* r_* dx = -M_\infty^2 \frac{\gamma+1}{12} \int_s r[u]^3 dr,$$

so that

$$c_x = 4\pi \frac{\gamma+1}{12} M_\infty^2 \int_s r[u]^3 dr \qquad (2.5)$$

In contrast to the first integral in Eq. (2.2) integral (2.5) is always positive, since the integrand is positive due to the physical nature of the problem.

3. Results of calculations

On the base of the implicit numerical method of Variable Directions a computer program was developed. This program calculates both stationary and nonstationary transonic flows and aerodynamic characteristics of bodies different forms. Figure 2 compares the distribution of c_p over body

$$R(x) = 2\tau(0.25 - x^2), \quad |x| \le 0.5, \quad \tau = 0.167, \qquad (3.1)$$

obtained by solving the steady problem with $M_\infty = 0.98$ and experimental results (separate points) of [12]. In [12] and also in [13] a different approximation schemes for the steady analogue of Eq. (1.1) were used; the results are not shown because they are in good correlation with the curve. The efficiency of the method is demonstrated by the good agreement between the theoretical and experimental curves.

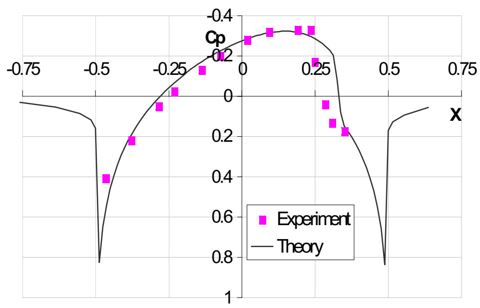

Figure 2 Experimental and theoretical data, arc body, Mach number 0.98

We will now consider the results for the same body but different Mach numbers of transonic flow as well as some other body.

Pressure distributions over the arc-body of revolution (3.1) for the transonic Mach Numbers 0.92 to 0.99 are shown at the figure 3. It can be seen that the shock moves along the body surface with the change of Mach number, when approaching Mach Number 1. The horizontal lines show the critical values of pressure when the local velocity of the flow is sonic.

Figure 4 demonstrates pressure distribution over the same body but the velocities of transonic flow are slightly supersonic, i.e. Mach number equals 1.01 to 1.05. There are two shocks at the leading and the rear edge of the body of revolution.

There are horizontal lines suggested the levels of sonic pressure in supersonic flow. The flow on the surface of the body has supersonic velocity, excluding the vicinity of leading and rear points. The horizontal lines show the levels of critical pressure c_p^* when local Mach number equals 1.

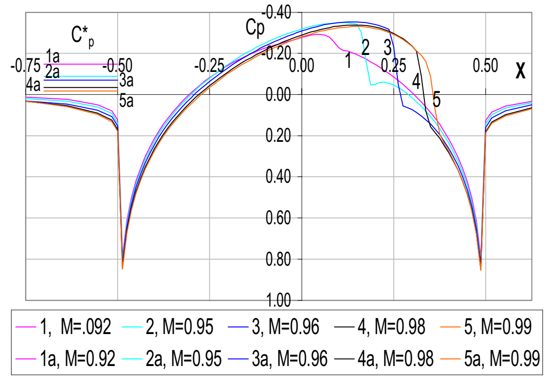

Figure 3 Pressure distribution on arc body of revolution, $\tau = 0.167$, M=0.92 – 0.99

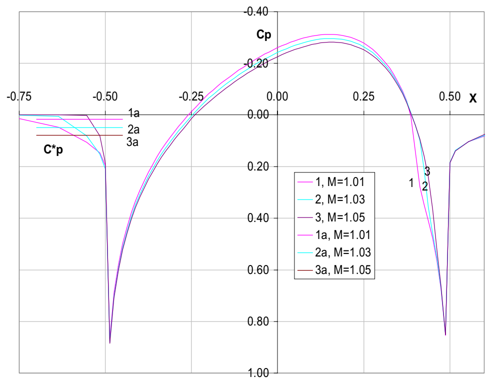

Figure 4 Pressure distribution on arc body of revolution, $\tau = 0.167$, M=1.01 – 1.05

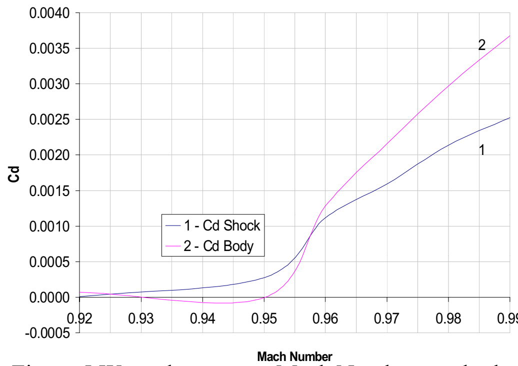

Figure 5 Wave drag versus Mach Number, arc body of revolution

As it is mentioned above there are some difficulties to obtain integral characteristics – wave drag of the body. The usual method of using pressure integration over the body can have large errors. It can be seen at the Figure 5. There are a zone of Mach Numbers with slightly negative values of wave drag, when the drag is calculated by integrating pressure over the body. The discrepancy can be especially big if the bodies are non-symmetric, as it was shown in [8].

Using the formula (2.5) to calculate wave drag of bodies in slightly supersonic flow demands special analysis, because the local supersonic domain on the body is infinite. The calculations show the quality true character of the curve decreasing in supersonic flow as $1\big/\sqrt{M_\infty^2 - 1}$ in correspondence with the linear supersonic flow theory.

In conclusion, aerodynamic characteristics of complicated 3-D elongate body representing modern hypersonic vehicle can be obtained if the transonic equivalence rule is used. It is shown that the results obtained in [8], this method was used together with the transonic equivalence rule to calculate the wave drag of a schematised hypersonic aircraft.

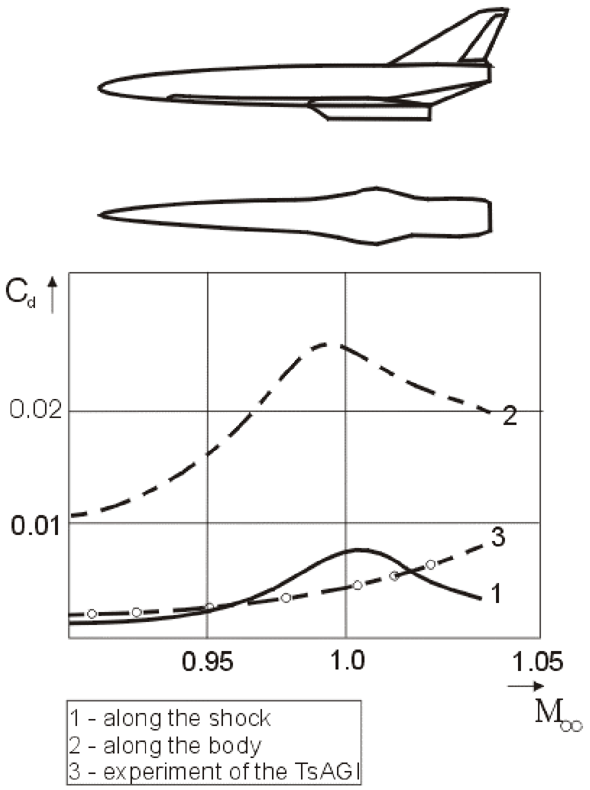

Figure 6 Wave drag of hypersonic aircraft versus Mach Number

Figure 6 shows the schematised aircraft and the corresponding equivalent solid of revolution. This Figure 6 shows the dependence of wave drag on the Mach number of the aircraft and the equivalent body of revolution. The dashed experimental curve was obtained at TsAGI (Central Aerohydrodynamics Institute, Moscow) [8] and corresponds to the wave drag of the aircraft model calculated as the difference between the total drag for the specified Mach number and the drag at $M_\infty = 0.7$ for subcritical flow. The dashed curve corresponds to the drag calculated by the method of integration over the body. It is apparent that the values of wave drag calculated by the method of integration over a shock wave (solid curve) are considerably closer to the experimental data than the values obtained by integrating over the body. It should be pointed out that the aircraft model being discussed is fairly complex, including an engine nacelle and corresponding to a thick equivalent body of revolution (thickness 14%). This is probably the source of error in the theoretical curve. In particular, the complexity of the model is probably responsible for the location of the drag maximum in the supersonic region.

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