14th Australasian Fluid Mechanics Conference
Adelaide University, Adelaide, Australia
10-14 December 2001

Physical Mechanism of Incipient Separation in Shock Wave / Boundary layer Interactions Induced by a Sharp Fin

Hua-Shu Dou[1] and Nhan-Phan-Thien[2]

[1] School of Aeronautical, Mechanical and Mechatronic Engineering
The University of Sydney, NSW 2006, AUSTRALIA

[2]Department of Mechanical Engineering
The National University of Singapore, SINGAPORE 119260

Abstract

In this paper, the incipient separation induced by shock wave/turbulent boundary layer interactions at sharp fin is predicted with Dou and Deng's theory, and is compared with Lu and Settles' experimental data. The physical mechanism of the incipient separation induced by shock wave/turbulent boundary layer interactions at sharp fin is explained through the surface flow pattern analysis. The reason for the discrepancy in the predicted and experimental incipient separation conditions is clarified. In addition, a correlation for the correction of incipient separation angle predicted by theory is also given.

Introduction

In the past 40 years, the incipient separation in shock wave/turbulent boundary layer interactions (SW/TBLI) induced by a sharp fin on a flat plate [2,13,14] has received considerable attention, Fig. 1. Because the occurrence of incipient separation changed the topology of the flow field, one problem encountered in research is how to judge the incipient separation. The typical topology of the surface flow pattern is shown in Fig.2.

Stanbrook[15] first defined that incipient separation takes place when the wall limiting streamlines align with the inviscid shock wave. Based on Stanbrook's criterion, McCabe[11] proposed a simple inviscid theory to predict the incipient separation by calculating the deflection of the vortex tubes caused by the lateral pressure gradient when the boundary-layer passes through the shock. For engineering purposes, Korkegi[6] carried out approximations to McCabe's theory and also corrections with test data and obtained one semi-empirical formula for incipient separation: $M_\infty \alpha_i$ =0.30, for k=1.4 and $M_\infty \geq 1.60$. Later, Lu[8] took into account the stretch of the vortex when the boundary-layer passes through the shock, and improved McCabe's theory. Based on 3D compressible boundary layer theory, Dou and Deng[3] proposed a method for analyzing the secondary flow within the boundary layer and predicted incipient separation conditions. This analysis appears better physically founded than those by McCabe and Korkegi as well as Lu, for it is easily understood, and the results also show the tendency of α_i decreasing with increasing Re_θ. This issue has been discussed by Settles and Dolling [14], Lu[9], and Leung and Squire [7]. Both McCabe [11], and Dou and Deng [3] predicted the incipient separation angle through skin-friction line (also called "limiting streamlines" or "surface streamlines"[14]) calculation and also correctly predicted the wall limiting streamline direction before separation. However, they overpredict the incipient separation condition. This problem has perplexed people for many years [14].

In this paper, Dou and Deng's theory [3] is used to predict the surface streamline direction and is compared with Lu and Settles's experimental data [10]. Then, the variation of the surface flow pattern with increasing deflection angle is analyzed, and the mechanism of the incipient separation induced by SW/TBLI at sharp fin is explained.

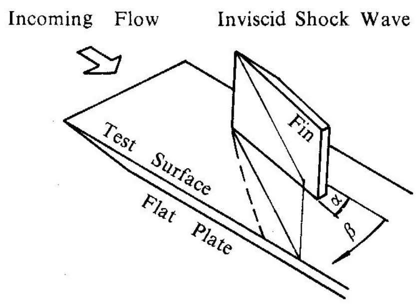

Fig.1 Sketch of sharp fin on the flat plate

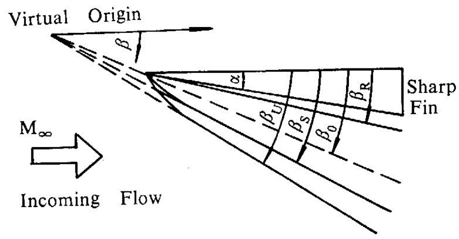

Fig.2 Main features of the surface flow pattern generated by sharp fin on flat plate. All the angles are measured from incoming freestream direction, centered at the virtual origin or the fin apex. β_U : angle of upstream influence line; β_S : angle of the separation line; β_0 : angle of the shock wave trace; β_R : angle of the reattachment line.

Theory

The theory developed by Dou and Deng [3] is used to the prediction of surface flow behaviour, and it is concisely introduced as follows. For the flow in SW/TBL interaction generated by a sharp fin in supersonic flow, the boundary layer on the walls is skewed owing to the pressure gradient and the streamline curvature. The fluid particles near the walls flow along the path with larger curvature than the outer inviscid flows. The direction of wall limiting streamline deviates from the primary streamline by an angle γ_W. The angle γ_W increases along the flow toward downstream. When the direction of the wall limiting streamline coincides with a conical polar line direction, a three-dimensional separation occurs. The wall limiting streamline becomes perpendicular to the direction of local pressure gradient at the separation line. Therefore, it is possible to predict the separation line by calculating the variation of the direction of wall limiting streamline.

For this type of pressure-driven three-dimensional turbulent boundary-layer, if the wall shear angle γ_W is not very large and the lateral flow is not bi-directional, Johnston's triangular model gives the best approximation(Olcmen and Simpson [12]). This model has been widely used in many engineering problems (Swafford and Whitfield [16]). Johnston [5] divided the turbulent boundary layer into two regions in the direction of boundary layer thickness. He assumed that a collateral region (inner region) near the wall exists and the direction of the velocity vector at this region is coincident with the shear stress vector. In the outer region, the behavior of flow is primarily dominated by the outer inviscid flow. According to this model, the crossflow velocity profile of the boundary layer can be expressed as follows.

$$\left.\begin{aligned} \frac{w}{u_e} &= \tan\gamma_W \frac{u}{u_e} \quad for \quad \frac{u}{u_e} \le \left(\frac{u}{u_e}\right)_p \\ \frac{w}{u_e} &= A\left(1-\frac{u}{u_e}\right) \quad for \quad \frac{u}{u_e} \ge \left(\frac{u}{u_e}\right)_p \end{aligned}\right\} \qquad (1)$$

where, γ_W is the angle between the wall limiting streamline and the external streamline, $(u/u_e)_p$ is the streamwise velocity ratio at the apex of the triangle. If the variation of the direction of external flow is known, the direction of the wall limiting streamline can be calculated by evaluating the angle γ_W.

From Eq.(1), the following expression is obtained

$$\tan\gamma_w = A\left[(u/u_e)_p^{-1} - 1\right] \qquad (2)$$

It can be seen that the parameters A and $(u/u_e)_p$ must be determined for the calculation of γ_W. Johnston[5] expressed the parameter A as a function of the parameters of the main flow even for the cases of a pressure gradient existence,

$$A = 2u_e^2 \int_0^\alpha \frac{d\alpha}{u_e^2} \qquad (3)$$

where, u_e is the velocity at the outer edge of the boundary-layer, α is the turning angle of main flow and is measured relative to the main flow direction at the beginning of the turn of main flow streamline.

According to the conservation of energy, the velocity ratio across the shock wave is expressed as follows.

$$\frac{u_{e2}}{u_{e1}} = \frac{M_2}{M_1}\sqrt{\frac{1+[(k-1/2)]M_1^2}{1+[(k-1/2)]M_2^2}} \qquad (4)$$

Introducing Eq.(8) into Eq.(3), and using the Prandtl-Meyer relation, then integrating and simplifying, the Eq.(12) can be solved

$$A = \frac{M_2^2}{1+\frac{k-1}{2}M_2^2}\left(\arccos\frac{1}{M_1} - \arccos\frac{1}{M_1} - \frac{\sqrt{M_1^2-1}}{M^2{}_1} - \frac{\sqrt{M_2^2-1}}{M^2{}_2}\right)$$

(5)

Prandtl-Meyer relation is expressed as

$$v(M) = \sqrt{\frac{\gamma+1}{\gamma-1}}\arctan\sqrt{\frac{\gamma+1}{\gamma-1}\left(M^2-1\right)} - \arctan\sqrt{M_1^2-1} \qquad (6)$$

The deflection angle is related to Prandtl-Meyer by

$$\alpha = v(M_2) - (M_1)$$

(7)

The parameter $(u/u_e)_p$ is expressed as [5,15]

$$\left(\frac{u}{u_e}\right)_p = \bar{y}_p\sqrt{\frac{\rho_e}{\rho_W}\frac{c_{fx}\cos\gamma_w}{2}} \qquad (8)$$

where, $\bar{y}_p = y_p / v_w\sqrt{\tau_w/\rho_w}$, and c_{fx} is the component of the skin friction coefficient in the direction of main flow. The density ratio ρ_W/ρ_e can be calculated from the energy equation for compressible flow. By the physical relationship for the three-dimensional boundary-layer, the value of $\bar{y}_p$ is related to the conditions of boundary layers. It is assumed that $\bar{y}_p$ =11.0 from the experimental data in supersonic flows (Dou and Deng [3]). The local skin friction coefficient toward the streamline direction was obtained by Dou and Deng [3],

$$c_{fx}/c_{fxi} = \left(1+0.13M^2\right)^{-0.73} \qquad (9)$$

where c_{fx} is the coefficient of skin friction for incompressible turbulent boundary layer on flat plate. In this paper, the Karman-Schoenherr's equation recommended by Hopkins and Inouye[4] is employed. This equation is applicable to the whole range of the Reynolds number of turbulent boundary layer.

$$1/c_{fxi} = 17.08(\log \mathrm{Re})^2 + 25.11\log \mathrm{Re} + 6.012 \qquad (10)$$

Substituting Eqs(12) and (13) into Eq.(2), the following equation can be derived

$$\tan\gamma_W = A\left[0.13\left(\frac{\rho_e}{\rho_W}c_{fx}\cos\gamma_W\right)^{-0.5} - 1\right]$$

(11)

The shock wave angle can be calculated by the implicit oblique shock wave theory, or by the approximate equation given by Dou and Deng[1]. Using Eqs(6) to (15), the variation α-γ_W along the streamwise direction can be calculated for given Mach number and α-γ_W number of incoming flow with the increasing α-γ_W. Then, the turning angle α-γ_W of surface streamline on the wall by the action of shock disturbance can be evaluated. When the turning angle α-γ_W at the wall equals to the shock angle α-γ_W, the separation of the three-dimensional boundary layer is considered to occur as was shown by Stanbrook[15]. Similar calculations can be carried out for various incoming flow conditions.

Results and Discussion

Comparison of the Theories with Experiments

The data of incipient separation were generally obtained by an oil film visualization technique in tunnel experiments[2,14]. The incipient separation in experiments was mostly decided in terms of the formation of convergent line from the upstream, according to Lighthill's criterion [14]. Figure 6 shows the comparison of the experimental data reported by Lu and Settles [10] and the predictions by using Dou and Deng's method for four Mach numbers. The intersection point of the turning angle σ of surface streamlines with the shock wave angle β_0 corresponds to the condition set by Stanbrook's criterion (A-A line). The agreement of σ value between the theory and the experiments is very good before incipient separation (even for $\sigma > \sigma_i$, i.e. about A-A line), which successfully confirms the theory. The arrows at the abscissa indicate the incipient separation judged with Korkegi's equation (B-B line), as reported by Lu and Settles [10]. They are lower than those obtained by using Stanbrook's criterion. The incipient separation reported by experiments is even lower a little than Korkegi's values[6].

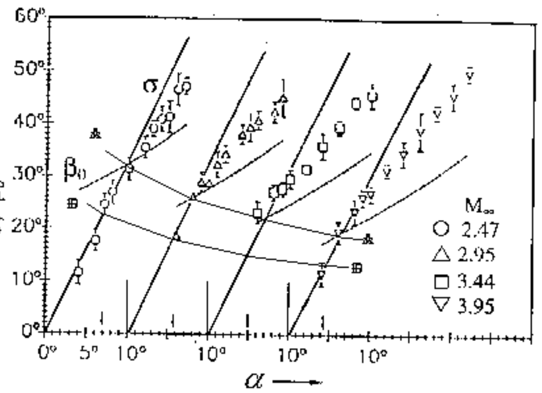

Fig.3 Comparison of predictions by Dou and Deng's theory with the experimental data of Lu and Settles[21] for the surface streamline direction.

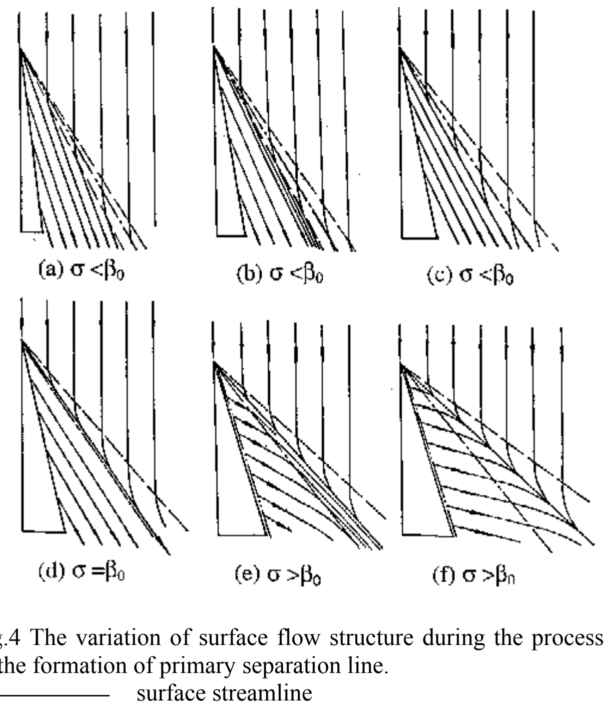

Fig.4 The variation of surface flow structure during the process of the formation of primary separation line.

——— surface streamline
— – — shock wave
.... reattachment line
— — — upstream influence; and downstream boundary of interaction region.

(a) $\sigma < \beta_0$ attached flow. (b) $\sigma < \beta_0$ attached flow. (c) $\sigma < \beta_0$ incipient separation line formed. (d) $\sigma < \beta_0$ wall limiting streamlines parallel to shock. (e) $\sigma < \beta_0$ "incipient separation line" turning gradually. (f) $\sigma < \beta_0$ primary separation line formed.

Analysis of Surface Flow Pattern

The process of formation of the incipient separation could be described by analyzing the evolution of the surface flow pattern. The variation of surface flow pattern with the increase of the strength of shock wave for given incoming Mach number could be divided into the following six stages (Fig. 4).

(a) The deflection angle is small and the shock wave is weak, and the effect of secondary flow is negligible. (b) On increasing the deflection angle, the wall limiting streamlines behind the shock turn to the shock wave gradually. The main feature of this stage is the gathering of surface streamlines from the upstream behind the shock. (c) Further increasing the deflection angle, the wall limiting streamlines converge and coalesce onto a single line from the upstream, when the turning of the surface streamlines behind the shock are still not large enough to deflect the surface streamlines parallel to the shock. The single line formed from the upstream is just the "incipient separation line" exhibited by oil streak pattern technique in experiments, which symbolizes the beginning of the separation process. (d) Upon further increasing the deflection angle, this single line formed from the upstream rotates (shifts) continuously with the increasing shock wave angle, and the surface streamlines behind the shock are parallel to the shock. This is the condition defined by the Stanbrook's criterion. Of course, this condition arrives later than the appearance of "incipient separation line" indicated by experiments. This is the reason why the theories overpredict the occurrence of the incipient separation compared with the experiments shown in Fig. 6 and Fig. 7. (e) On further increase of the deflection angle, this single line formed from the upstream rotates (shifts) continuously with the increasing shock wave angle, and the wall limiting streamlines converge to this line from both sides. (f) When the deflection angle is increased to a certain value, the "primary separation line" is formed.

Physical Mechanism for Incipient Separation

The three-dimensional separations induced by SW/TBLI at sharp fin can be described by the model of Maskell or Lighthill [14]. According to Lighthill's criterion, it is considered to be separated when the wall limiting streamlines converge to a single line. In terms of mass conservation, converging only from the upstream to a single line is enough to be considered separated. From the equilibrium of force, the vector of skin-friction force is perpendicular to the direction of local pressure gradient at the incipient separation line. Therefore, when the skin-friction lines of incoming flow becomes perpendicular to the direction of the local pressure gradient, a formation of the "incipient separation line" becomes possible. In fact, Stanbrook's criterion satisfies this condition. However, on the gradual increasing of the shock wave angle, the state marked by Stanbrook's criterion is not the first appearance of this condition. This condition is satisfied indeed somewhat earlier, at the case(c) of Fig.8. This is the real reason for the formation of incipient separation line.

The surface flow pattern formed by a sharp fin is a conical one, and not cylindrical [2,14]. For both the cylindrical and conical interactions to be possible, it is required that the direction of the skin-friction line at incipient separation line is perpendicular to the local pressure gradient. However, for conical interactions, it is not necessary for the "incipient separation line" to align with shock wave, while it is necessary for cylindrical interactions. The interaction region on the flat plate generated by SW/TBLI at sharp fin is a conical zone, which is across the inviscid shock wave. Thus, the maximum turning of the surface streamlines as well as the primarily formed convergence line is behind the shock wave, and this convergence line makes an angle to the inviscid shock line. Therefore, this convergence line is first formed as the increasing of shock wave angle.

From the above discussions we can find that when the wall limiting streamlines after shock align along with one polar line from the fin apex(or a virtual origin) with the increasing strength of shock wave, the incipient separation line is generated. At this polar line, the direction of the skin-friction vector is perpendicular to the local pressure gradient. The wall limiting streamlines of incoming flow converge and coalesce to it. Thus, this polar line could prevent the oil substance from passing through and could be detected in experiments with oil streak pattern technique, and is considered as the incipient separation line by Lighthill's criterion.

If the flow is cylindrical, there is no the stage from the case(c) to the case (d) in Fig.8, and the case (d) coincides with the case (c). The incipient separation line formed from the upstream uniquely corresponds to that defined by Stanbrook's criterion[15]. Therefore, the process of the formation of the primary separation line for conical interactions is very different from that for cylindrical interactions. The Stanbrook's criterion is applicable to cylindrical interactions, but not directly applicable to conical interactions. The discrepancy of incipient separation conditions between the predictions using this criterion and the experimental data is resulted from the intrinsic behavior of conical interactions.

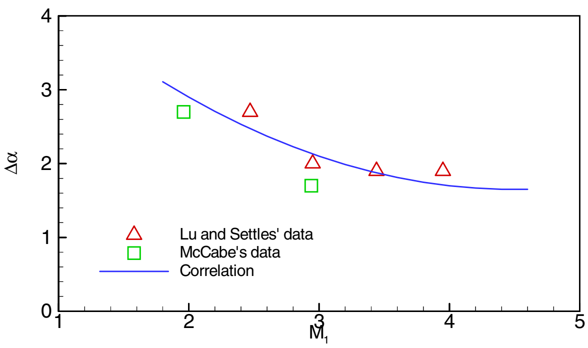

Fig. 5 Correlation for the correction of incipient separation angle predicted by theory.

Correction for Incipient Separation Angle

The difference of the deflection angle between the condition(c) and (d), $\Delta\alpha$ is shown in Fig.5 for two set of data. It is found that the $\Delta\alpha$ decreases with the increasing Mach number. This accords with the physical mechanism of the interaction because the pressure ratio across the shock wave increases with the Mach number, and the pressure ratio at separation line is almost constant for three-dimensional separation of supersonic flows[2,14]. Since it is difficult to find a criterion to define the condition of Fig.8(c), we still take Stanbrook's criterion as the incipient separation criterion, and add a correction to the predicted incipient separation angle α_i using the experimental data. Assume that the correction $\Delta\alpha$ is only related to the Mach number, a correlation for the correction to the theoretical prediction is

$$\Delta\alpha = 0.20M_1^{\ 2} - 1.80M_1 + 5.70 \quad for \quad 1.60 < M_1 < 5 \quad (11)$$

The corrected incipient separation angle α_{ic} is

$$\alpha_{ic} = \alpha_i - \Delta\alpha \quad for \quad 1.60 < M_1 < 5 \qquad (12)$$

Conclusions

The conclusion goes along the following lines.

1. A theoretical model to analyze the three-dimensional turbulent boundary-layer in SW/TBLI is described, which is firstly reported in [9]. The prediction of the wall limiting streamline direction by this model yielded good agreement with Lu and Settles' experimental data.

2. The "incipient separation line" formed from the upstream is generated by the secondary flow induced by the lateral pressure gradient. The incipient separation line indicated by experiments is the condition of the first appearance of the wall limiting streamline perpendicular to the local pressure gradient.

3. The process of the formation of the primary separation line for conical interactions is very different from those for cylindrical interactions. The disagreement between the prediction by this criterion and the experiments is caused by the intrinsic behavior of conical interaction.

4. The difference of the deflection angle for incipient separation between the prediction and the experiments, $\Delta\alpha$ decreases with the increasing Mach number. A correlation equation for the correction to the theoretical predicted incipient separation angle $\Delta\alpha$ is given.

References

[1] Dou,H.-S.,and Deng X.-Y., "Approximate Formula of Weak Oblique Shock Wave Angle," AIAA Journal, 30, 1992, 837-839.

[2] Dou, H.-S., and Deng X.-Y., "Experimental Investigations of the Separation Behaviour in 3D Shock Wave/Turbulent Boundary-Layer Interactions," Proceedings of the 18th Congress of ICAS (International Council of the Aeronautical Sciences) /AIAA, Beijing, Sep., 1992, pp.1543-1553.

[3] Dou,H.-S., and Deng X.-Y., "Prediction for the Incipient separation of Fin-induced 3-D Shock Wave /Turbulent Boundary-Layer Interactions," Acta Aerodynamica Sinica, Vol.10,No.1, 1992, pp.45-52(in Chinese with English abstract).

[4] Hopkins, E.J., and Inouye, M., " An Evaluation of Theories for Predicting Turbulent Skin Friction and Heat Transfer on Flat Plates at Supersonic and Hypersonic Mach Numbers," AIAA Journal, Vol.9, No.6, 1971, 993-1003.

[5] Johnston,J.P., " On the Three- Dimensional Turbulent Boundary Layer Generated by Secondary Flow," ASME *Journal of Basic Engineering*, Vol.82, No.1, 1960, pp.233-248.

[6] Korkegi, R.H., "A Simple Correlation for Incipient Turbulent Boundary Layer Separation due to a Skewed Shock Wave," AIAA Journal, Vol.11, Nov.1973, pp.1578-1579.

[7] Leung, A.W.C., and Squire, L.C., "Reynolds Number Effects in Swept-Shock-Wave/ Turbulent-Boundary-Layer Interaction," AIAA Journal, Vol.33, No.5, 1995, pp.798-804.

[8] Lu, F. K., "Semi Empirical Extension of McCabe's Vorticity Model for Fin-Generated Shock Wave Boundary-Layer Interactions," Proc. of the 4th Asian Congress of Fluid Mechanics, The Hong Kong Univ. Press, Hong Kong, Aug. 21-25,1989, pp.A170-173.

[9] Lu, F. K., "Quasiconical Free Interaction between a Swept Shock and a Turbulent Boundary Layer," AIAA Journal, Vol.31, No.4, 1993, pp.686-692.

[10] Lu, F.K., and Settles, G.S., "Color surface-flow Visualization of Fin-generated shock Wave Boundary-Layer Interactions," Experiments in Fluids, Vol.8, No.6, 1990, 352-354.

[11] McCabe, A., "The Three- Dimensional Interaction of a Shock Wave with a Turbulent Boundary Layer," Aero. Quart. 17, 1966, 231-252.

[12] Olcmen, M. S., and Simpson, R. L., "Perspective: On the Near Wall Similarity of Three-Dimensional Turbulent Boundary Layers," ASME *J. of Fluid Engineering*, 114, 1992, 487-495.

[13] Panaras, A.G., "Review of The Physics of Swept-Shock/Boundary Layer Interactions," Progress in Aerospace Science, 32, 1996, 173-244.

[14] Settles, G.S., and Dolling, D. S., "Swept shock /Boundary-Layer Interactions--Tutorial and Update," AIAA Paper 90-0375, 1990.

[15] Stanbrook, A.,"An Experimental Study of the Glancing Inter-action between a Shock Wave and a Boundary Layer," British ARC CP-555, July 1960.

[16] Swafford, T. W., and Whitfield, D.L., "Time-Dependent Solution of Three-Dimensional Compressible Turbulent Integral Boundary-Layer Equations," *AIAA Journal*, 23, 1985, 1005-1013.