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Transient flow in a ventilated enclosure containing a vertically distributed source of buoyancy

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Abstract

An experimental and numerical investigation of the fluid mechanics within an enclosure containing a plane, vertically distributed source of buoyancy is described. Two separate situations are considered in which the enclosure is either sealed or naturally ventilated. In the sealed enclosure the stratification develops in qualitatively the same way as for a 'filling' box containing a single, point source of buoyancy on the floor. In the case of the ventilated box it is found that for some values of non-dimensional vent area a complex stratification develops with a number of distinct layers fed by horizontal intrusions from the vertical source. We present results from saline solution experiments and compare them with a theoretical model developed using plume theory. Numerical solutions to the theoretical equations which provide detailed information on the transient development of the density profile within the enclosure are also presented.

Introduction

The "filling box" is a fluid mechanics problem that has applications in many areas of geophysical fluid mechanics and engineering. The most fundamental configuration is the situation with a point source of buoyancy located on the floor of a sealed enclosure. The buoyancy source gives rise to an axi-symmetric, turbulent plume, which lays down a stably stratified buoyant layer of fluid at the top of the enclosure. The depth of this layer increases with time, as does its buoyancy at any given height. In this paper we are concerned with a filling box with a buoyancy source distributed over a vertical surface within the enclosure (as shown schematically in figure 1).

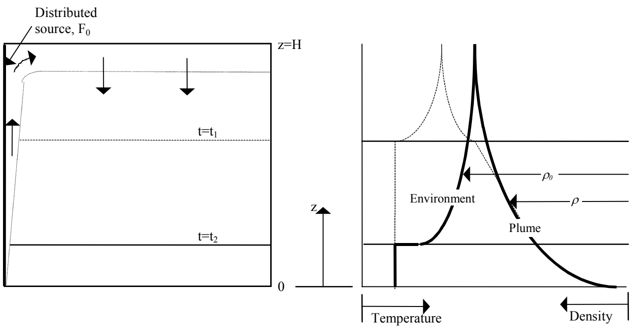

Figure 1. Schematic of: a) development of a plume from a plane, vertical, distributed source of buoyancy on one wall of a sealed box of height H showing position of the first density front at times t_1 and t_2 $(t_2 > t_1)$; b) corresponding density profiles in the plume and the ambient fluid (not to scale).

The first comprehensive analysis of the filling box problem with an axi-symmetric plume was carried out by Baines and Turner [1], hereinafter referred to as B&T, who predicted the depth of the buoyant layer as a function of time and the ambient stratification in the *asymptotic* limit (as $t \to \infty$). Worster and Huppert [13], hereinafter referred to as W&H, subsequently refined B&T's analysis of this situation to provide a method of predicting the stratification in the buoyant layer at any time. To do this they invoked an approximation whereby the buoyancy flux in the plume rising through the buoyant layer varied linearly with height above the lower limit, or "first front", of the layer. Germeles [6] carried out a numerical analysis of the same situation, which W&H used to validate their approximate analytical solution.

The filling box problem is of considerable interest to building services and air conditioning designers and a logical extension of the work of B&T was made by Linden, Lane-Serff & Smeed [8] who investigated the behaviour of naturally ventilated filling boxes with various configurations of opening to the external ambient. Other researchers who have also studied the ventilated box include Sandberg and Lindström [10,11] who were concerned with forced ventilation of enclosures containing point and vertical line sources of buoyancy.

The motivation for the present research comes, in part, from the need to develop a model of the air flows and thermal stratification that arise in enclosures such as buildings that have distributed heat sources on one or more walls. Solar gains to the walls of large atria, for example, can lead to intense thermal stratification of tens of degrees Celsius [4]. This can significantly affect thermal comfort conditions within the atria and increases the air conditioning heat loads of adjacent rooms.

Natural convection boundary layers are generated on heated vertical surfaces in such enclosures and surprisingly little research has been reported where the thermal boundary condition is one of uniform heat flux and high Rayleigh number [12]. Given that the effect of molecular viscosity is relatively small in these turbulent boundary layers, they may be treated as plumes with buoyancy added uniformly with height. We have used this assumption and the approach of B&T and W&H to determine the characteristics of the transient development of stratification in the filling box.

Plane plume in a uniform environment

The plane, vertically distributed source of buoyancy shown in figure 1 is taken to emit a buoyancy flux, F_0, per unit area with zero momentum and volume fluxes at the surface. The source has horizontal width, L, and we represent the plume (or boundary layer) as having "top-hat" vertical velocity and density profiles for computational convenience. The plume properties are vertical velocity, w, and buoyancy, $\Delta = g(\rho_e - \rho)/\rho_l$, where $\rho(z)$ is the density of the plume, $\rho_e(z)$ is the density of the ambient fluid and ρ_l is a reference density. The equations governing the development of the plume through an ambient of arbitrary stratification are as follows (full details given in [3]):

$$\frac{d(bw)}{dz} = \alpha w \tag{1a}$$

$$\frac{d(bw^2)}{dz} = b\Delta \tag{1b}$$

$$\frac{d(bw\Delta)}{dz} = (bw)\frac{\partial \Delta_e}{\partial z} + F_0, \tag{1c}$$

where b is the width of the plume and $\Delta_e = g(\rho_e - \rho_l)/\rho_l$ is the buoyancy of the ambient fluid. For convenience we use the following nomenclature for the volume flux, $Q=(bw)$, momentum flux, $M = (bw^2)$, and buoyancy flux, $F = (bw\Delta)$, per unit horizontal length of the plume, and α denotes the entrainment coefficient. For a plume in a uniform environment a similarity solution to (1) may be found such that:

$$Q = \frac{3}{4}\left(\frac{4}{5}\right)^{1/3} \alpha^{2/3} F_0^{1/3} z^{4/3} \tag{2a}$$

$$M = \frac{3}{4}\left(\frac{4}{5}\right)^{2/3} \alpha^{1/3} F_0^{2/3} z^{5/3} \tag{2b}$$

$$\Delta = \frac{4}{3}\left(\frac{5}{4}\right)^{1/3} \alpha^{-2/3} F_0^{2/3} z^{-1/3} \tag{2c}$$

$$w = \left(\frac{4}{5}\right)^{1/3} \alpha^{-1/3} F_0^{1/3} z^{1/3}. \tag{2d}$$

Plane plume in a linearly stratified environment

When a plane plume develops in a linearly stratified environment equation (1) may be conveniently rewritten using the following non-dimensional variables:

$$\zeta = \alpha^{1/2} F_0^{-1/2} N^{3/2} z \tag{3a}$$

$$f = \alpha^{1/2} F_0^{-3/2} N^{3/2} F \tag{3b}$$

$$q = F_0^{-1} N^2 Q \tag{3c}$$

$$m = \alpha^{1/2} F_0^{-3/2} N^{5/2} M \tag{3d}$$

to yield

$$\frac{dq}{d\zeta} = \frac{m}{q}; \qquad \frac{dm}{d\zeta} = \frac{qf}{m}; \qquad \frac{df}{d\zeta} = -q + 1, \tag{4}$$

where N is the buoyancy frequency of the ambient fluid. These equations have been solved numerically and the results are presented in figure 2.

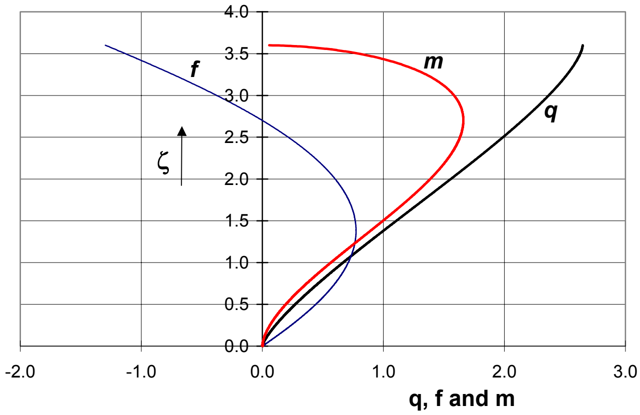

Figure 2. Variation in the non-dimensional volume flux, q, buoyancy flux, f, and momentum flux, m, of a plume due to a plane, vertically distributed source of buoyancy in a linearly and stably stratified environment of buoyancy frequency N.

It can be seen that the local buoyancy flux, f, is predicted to reach a maximum at $\zeta \sim 1.4$ and then zero at $\zeta = 2.7$. Beyond this point the plume is negatively buoyant with respect to the ambient and momentum decreases. In practice the plume will therefore overshoot the neutrally-buoyant height and eventually fall back to form a horizontal intrusion into the ambient in the region $2.7 < \zeta < 3.6$ (cf. equivalent analysis by Morton, Taylor and Turner [9]). A new plume will therefore develop adjacent to the distributed buoyancy source immediately above the horizontal intrusion.

Plane plume in a sealed filling box

In modelling the situation shown in figure 1 we have adopted a similar approach to that of B&T and W&H. In addition to the plume equations (1) the development of the ambient stratification within the box (neglecting molecular diffusion) is:

$$\frac{\partial \Delta_e}{\partial t} = Q\left(\frac{L}{A}\right)\frac{\partial \Delta_e}{\partial z}, \tag{5}$$

where A is the horizontal cross-sectional area of the box. The governing differential equations are then arranged in their most convenient form by defining the following variables

$$\zeta = zH^{-1} \tag{6a}$$

$$\tau = \alpha^{2/3} H^{1/3}\left(\frac{L}{A}\right) F_0^{1/3} t \tag{6b}$$

$$\delta = \alpha^{2/3} H^{1/3} F_0^{-2/3} \Delta_e \tag{6c}$$

$$f = H^{-1} F_0^{-1} F \tag{6d}$$

$$q = \alpha^{-2/3} H^{-4/3} F_0^{-1/3} Q \tag{6e}$$

$$m = \alpha^{-1/3} H^{-5/3} F_0^{-2/3} M. \tag{6f}$$

The governing equations for the filling box are then:

$$\frac{dq}{d\zeta} = \frac{m}{q}, \qquad \frac{dm}{d\zeta} = \frac{qf}{m}, \qquad \frac{df}{d\zeta} = q\frac{\partial \delta}{\partial \zeta} + 1 \tag{7}$$

$$\frac{\partial \delta}{\partial \tau} = q\frac{\partial \delta}{\partial \zeta}. \tag{8}$$

The position, ζ_0, and density, δ_0, of the first front may then be determined using (2) so that:

$$-\delta_0 = \left.\frac{f}{q}\right|_{\zeta=1} = \frac{4}{3}\left(\frac{5}{4}\right)^{1/3} \approx 1.436 \tag{9}$$

$$\zeta_0 = \left[1 + \frac{1}{4}\left(\frac{4}{5}\right)^{1/3}\tau\right]^{-3}, \tag{10}$$

and the time scale associated with the flow is $t^* \sim AH^{-1/3}F_0^{-1/3}/L$, where $\tau = t/t^*$. The density within the buoyant layer that develops at the top of a box containing a *point source* of buoyancy was found analytically by W&H by making a linear approximation to the buoyancy flux in the rising plume. This type of approximation is not possible in our case and we have therefore carried out a numerical solution of the governing equations (7) and (8) adopting a procedure similar to that described by Germeles [6]. Results of this numerical procedure are given in figures 3 and 4.

The ventilated filling box

When openings of respective areas a_t and a_b are provided in the top and bottom of the box that connect to an external environment of constant density then a ventilation flow, Q_{vent}, moves through these vents such that:

$$Q_{vent} = A^*\left(\int_0^H \Delta_e dz\right)^{1/2} = A^*\left(\int_{z_0}^H \Delta_e dz\right)^{1/2}, \tag{11}$$

where H is the total height of the enclosure. The "effective vent area", A^*, is defined as [5]:

$$A^* = \frac{c_d a_t a_b}{\left(\left(\frac{c_d^2 a_t^2}{c} + a_b^2\right)/2\right)^{1/2}}, \quad (12)$$

where c is the pressure loss coefficient associated with the inflow through a sharp-edged opening and c_d is a discharge coefficient that accounts for the vena contracta arising at the downstream side of the sharp-edged upper vents. In terms of the dimensionless variables (6) the ventilation flow rate (per unit length of source) is therefore:

$$q_{vent} = \frac{A^*}{\alpha HL}\left(\int_0^1 \delta \, d\zeta\right)^{1/2}. \quad (13)$$

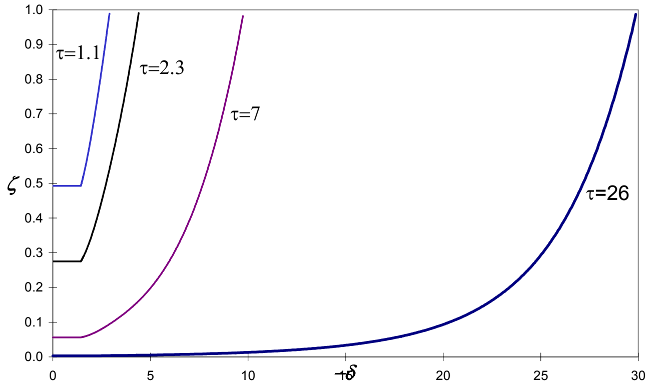

Figure3. Development of the non-dimensional density field in a filling box containing a vertical, plane distributed source of buoyancy with increasing time, τ.

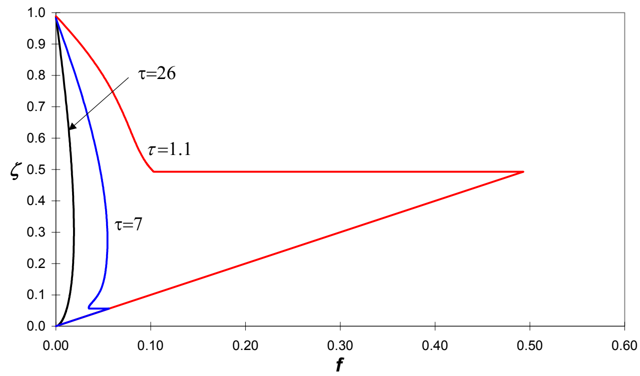

Figure 4. Non-dimensional buoyancy flux, f, of a plane plume relative to the local ambient fluid in a filling box containing a vertical, plane distributed source of buoyancy.

In a ventilated box it is possible for the stratification at a given height to be sufficiently intense that the plume becomes neutrally buoyant and forms a horizontal intrusion as shown schematically in figure 5. It is postulated that a new plume will then start immediately above this point.

Numerical predictions of the stratification in a ventilated box with a non-dimensional vent area of $A^*/(\alpha HL)$= 0.042 are shown in figure 6 by way of example. The buoyant layer initially grows in depth in a manner similar to that for the sealed box. However, the first front does not continue to descend indefinitely due to the upward ventilation flow of the ambient fluid through the box. After a long period, $\tau > 40$, the plume detrains a total of three times over the height of the box in this particular case.

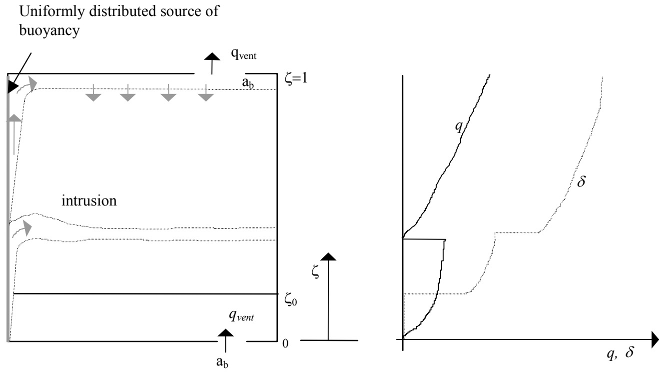

Figure 5. Schematic of a plume from a distributed source of buoyancy on one wall of a ventilated box. The plume rises to a point where it is neutrally buoyant with respect to the local ambient and subsequently forms a horizontal intrusion.

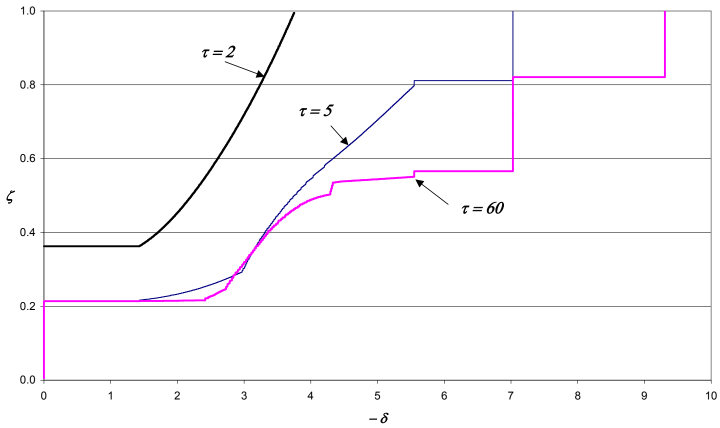

Figure 6. Transient development of the ambient stratification within a filling box containing a plane, vertically distributed source of buoyancy with a non-dimensional vent area of $A^*/(\alpha HL)$= 0.042.

Experiments

Experiments to validate some of the results above were undertaken using an enclosure of internal dimensions 25 x 25 x 25cm. Water was the working fluid and the vertically distributed source of buoyancy was implemented by replacing one of the vertical acrylic walls of the enclosure with a porous membrane through which salt solution was injected (see figure 7). The experiments were visualised by means of a shadowgraph and recorded on videotape via a CCD camera.

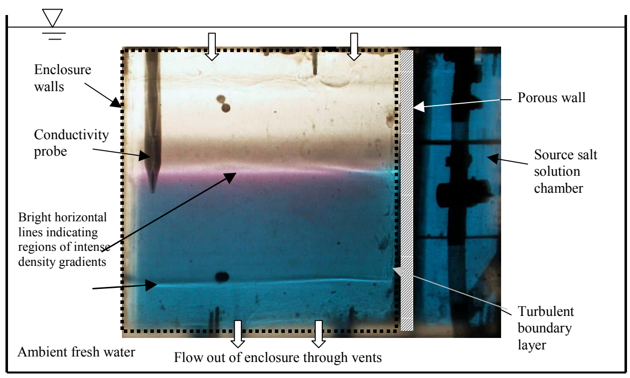

Figure 7. Annotated digitised image showing a typical ventilated experiment at $t = 30$ minutes. Dye has been added to the source salt solution to aid in the interpretation of the density field. The porous wall on the right provides a vertically distributed plane source of buoyancy. Experimental conditions were $A^* = 0.56\text{cm}^2$, $F_0 = 1.5\ \text{cm}^2/\text{s}^3$.

A major assumption in our model is that the turbulent, high Rayleigh number natural convection boundary layer may be modelled as a plume and that the entrainment assumption is valid for this flow. This has been experimentally validated by comparing $z_0(t)$ with that predicted by (10). The first front height was determined by processing the video images with the Dig*Image* software package [2]. Results from six experiments are shown in figure 8.

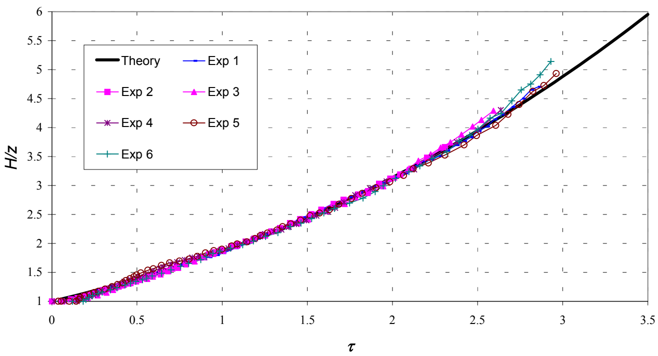

Figure 8. Position of the first front as a function of non-dimensional time in a "sealed" enclosure containing a plane, vertically distributed source of buoyancy. The entrainment constant was determined from a least-squares best fit to the theory given by (10).

It can be seen that the entrainment assumption is valid for non-dimensional time $0 < \tau < 2.5$. Beyond this time the entrainment assumption did not appear to hold, most probably because the boundary layer on the upper part of the source (for small ζ) was laminar and thus entrainment in this region was less than that in an idealised turbulent plume. As a result, when the first front reached the laminar portion of the boundary layer so its ascent rate decreased relative to that predicted by (10). The mean of the entrainment constants determined from the six experiments shown in figure 8 was $\alpha = 0.020$.

The transient stratification profile in a ventilated filling box was determined using a conductivity probe similar to that described by Leppinen [7]. Results from an experiment with a non-dimensional vent area of $A^*/\alpha HL = 0.094$ are presented in figure 9 and compared with the results from the numerical solution of (7), (8) and (13).

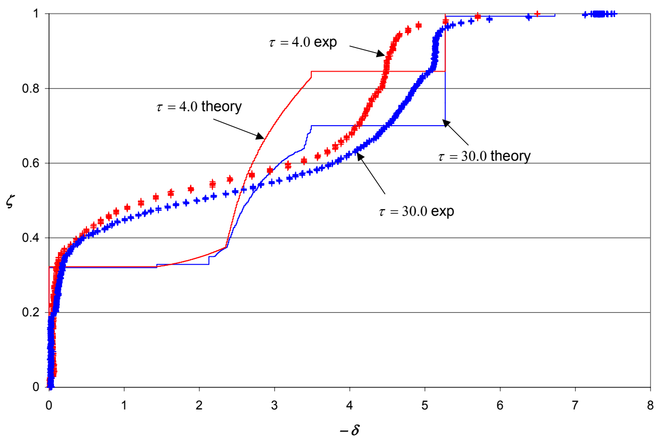

Figure 9. Ambient fluid non-dimensional density, δ, as a function of non-dimensional height, ζ, at two values of non-dimensional time, τ, for a non-dimensional area of $A^*/\alpha HL = 0.094$. Symbols represent experimental data and solid lines are numerical results.

Although the numerical results do not replicate exactly the experimental measurements, qualitative agreement is evident. Moreover, the plume detrainment process was observed experimentally at a number of locations for particular non-dimensional vent areas. This detrainment was usually accompanied by the appearance of bright horizontal lines on the shadowgraph indicating regions of intense stratification (figure 7).

A number of significant difficulties are faced when attempting to represent the idealised flows of figures 1 and 5 at the laboratory scale. In particular, any natural convection boundary layer arising from a real vertical distributed source will be laminar for small ζ.

Conclusions

This numerical and experimental study of the flows arising from a vertically distributed source of buoyancy has led to the following outcomes. Analytical models have been developed for prediction of the volumetric flow rate, momentum, velocity and buoyancy of the plume in uniform and linearly stratified environments. An analytical model of the stratification that develops in a sealed and a ventilated box as a result of a vertically distributed buoyancy input on one wall has been developed and solved numerically. The assumption that the turbulent natural convection boundary layer on a wall of uniform buoyancy flux obeys the entrainment assumption has been experimentally validated and the entrainment constant for this flow was found to be of order 0.02. The flow field in the ventilated case is found to be complex and involves detrainment of the plume from the vertical source at particular non-dimensional vent areas.

References

[1] Baines, W.D. and Turner, J.S., Turbulent buoyant convection from a source in a confined region, *J. Fluid Mech.*, **37**, 1968, 51-80.

[2] Cenedese, C. and Dalziel, S.B., Concentration and depth fields determined by the light transmitted through a dyed solution, *Proc. 8th Int. Symp. Flow Visualization*, ed. Carlomagno & Grant. ISBN0953399109, 1998, paper 061.

[3] Cooper, P., Hunt, G.R. and Linden, P.F., The filling box containing a vertically distributed source of buoyancy, submitted to *J. Fluid Mech*, 2001.

[4] Cooper, P. and Kulkarni, R., Overheating and thermal stratification in multi-storey glazed spaces, *Proc. ANZSES Conf.*, Sydney, 1994, 333-338.

[5] Cooper, P. and Linden, P.F., Two sources of buoyancy in a naturally ventilated enclosure, *J. Fluid Mech.*, **311**, 1996, 153-176.

[6] Germeles, A.E., Forced plumes and mixing of liquids in tanks. J. Fluid Mech. 71, 601-623.

[7] Leppinen, D. M., *Aspects of Convection*, PhD Thesis, 1997 University of Cambridge.

[8] Linden, P. F., Lane-Serff, G. F. and Smeed, D. A., Emptying filling boxes: the fluid mechanics of natural ventilation, *J. Fluid Mech.*, **212**, 1990, 309-335.

[9] Morton, B., Taylor, G.I. and Turner, J.S., Turbulent gravitational convection from maintained and instantaneous sources, *Proc. Royal Soc. London,* A, **234A**, 1956, 1-22.

[10] Sandberg, M. & Lindstrom, S., A model for ventilation by displacement, Proc of Roomvent '87, Session 3, Stockholm, 10-12 June, 1987, 1-14.

[11] Sandberg, M. & Lindstrom, S., Stratified flow in ventilated rooms - a model study, Proc Roomvent '90, Session D1-6, 1990, 1-15.

[12] Vliet, G. C. and Liu, C. K., An experimental study of turbulent natural convection boundary layers, *J. Heat Transfer*, **91**, 1969, 517-531.

[13] Worster, M.G. and Huppert, H. E., Time-dependent density profiles in a filling box, *J. Fluid Mech.*, **132**, 1983, 457-466.