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A Comparison of Forced and Freely Oscillating Cylinders

J. Carberry[1], J. Sheridan[1] and D. Rockwell[2]

[1]Department of Mechanical Engineering
Monash University, Clayton, Victoria, 3800 AUSTRALIA

[2]Department of Mechanical Engineering and Mechanics
Lehigh University, Bethlehem, PA, 18015, USA

Abstract

The fluid-structure interactions between an oscillating cylinder and the natural instability of the wake have been studied by simultaneously measuring the structure of the near wake and the forces on the cylinder. In this paper we examine the results of externally forced sinusoidal oscillations and compare them with the vortex induced oscillations of an elastically mounted cylinder. While wake structure and the variation of the vortex lift phase show some remarkable similarities, the results of these, and previous investigations, show that the forced oscillations can result in negative energy transfer for purely sinusoidal motion at frequencies and amplitudes at which free vibrations are known to occur.

Introduction and Preliminary Discussion

Vortex induced vibration of natural and engineering structures is of significant practical interest as it can result in structural failure, an infamous example being the Tacoma Narrows bridge.

Vortex induced vibrations typically occur when f_o, the natural frequency of the wake generated by flow over the stationary body, approaches the natural structural frequency of the body f_{struc}. The relationship between the vortex induced motion and the wake of the oscillating body is complicated by the fact that these parameters are intrinsically inter-dependent. A common method of simplifying this problem is to control or force the oscillation of the body and examine the response of the wake to a defined motion. The transverse motion of a freely oscillating body is typically very sinusoidal and when the oscillations are forced, this motion is replicated by a purely sinusoidal motion transverse to the free-stream.

As the frequency of the forced oscillations, f_e passes through f_o significant changes are observed in both the forces on the cylinder and the structure of the wake. A jump in the phase and amplitude of lift force around $f_e/f_o \approx 1$ was first identified by Bishop & Hassan [1], and subsequently observed by a number of investigators, including [4, 12, 13, 11, 2]. A change in the mode of vortex shedding from 2P to 2S, [15] and a switch in the sign of the phase-referenced initially formed vortex, [9, 6] was also observed at $f_e/f_o \approx 1$. A link between the jump in the lift force and the change in the phase and mode of vortex shedding was established experimentally by Carberry *et al* [2], who describe these events as a transition between the low- and high frequency wake states.

The analogous case of an elastically mounted cylinder which is constrained to oscillate transversely has also received considerable attention. The response of the elastically mounted cylinder depends primarily on the mass, damping and natural response frequency of the cylindrical structure as well as the reduced velocity, U* of the flow past the cylinder. The early investigation of Feng [3] showed that as U* varied the cylinder displayed two response branches, subsequently defined as the initial and lower branches. The transition between the initial and lower branches was characterised by a jump in the phase of the lift force and occurred at $f_e/f_o \approx 1$. Further investigations [8, 5] found that at very low values of mass damping, $m^*\zeta$ there is an additional upper response branch. Generally the amplitude response is plotted against U*, where $U^* = U/(f_{struc} D)$. However, for lower values of $m^*\zeta$ when the response is plotted against U^*/f^* or $(U^*/f^*)St_o$, there is a general collapse of the data, [8, 5], where $f^* = f_e/f_{struc}$, $St_o = f_o D/U$ and therefore $(U^*/f^*)St_o = f_o/f_e$. Thus, the collapse occurs when the free-stream velocity is normalised by f_e, the frequency at which the system responds to the wake instability rather than f_{struc}, the resonance frequency of the structure.

The total fluid force, F_{total} on a moving body can be broken into two components: the force due to the vorticity field F_{vortex} and F_{fi} the force due to the inertia of the fluid displaced by the motion of the body, [16, 5].

$$\begin{aligned}\bar{F}_{total} &= -\rho \frac{d}{dt} \int_{V_c} \bar{r} \times \bar{\omega}\, dV + \rho \frac{d}{dt} \int_{V_b} \bar{u}\, dV \\ &= \bar{F}_{vortex} + \bar{F}_{fi}\end{aligned} \qquad (1)$$

where u is the velocity of the body, V_c is a distant external boundary containing all the shed vorticity and V_b is the volume bounding the solid body, in our case the cylinder. For transverse oscillations the cylinder's acceleration is parallel to the lift force and has no component in the direction of the drag. Thus $C_{L\,fi}$ is equivalent to F_{fi} to and is in-phase with the displacement of the cylinder. The vectorial relationship between the total lift force $C_L(t)$ and the vortex lift force $C_{L\,vortex}(t)$ is shown in Figure 1, where $C_{L\,vortex}(t)$, rather than $C_L(t)$ represents the lift force due to the vorticity field. Therefore, as demonstrated by Govardhan & Williamson [5], the vortex force should be used to provide insight into the changes in the structure of the wake to the forces experienced by the body.

The forced sinusoidal motion of the cylinder is given by

$$y(t) = A \sin(2\pi f_e t) \qquad (2)$$

where A is the amplitude of oscillation. When the cylinder's wake is locked-on to the oscillations the approximately sinusoidal total lift force can be represented by:

$$C_L(t) \approx C_L \sin(2\pi f_e t + \phi_{lift}) \qquad (3)$$

where C_L is the amplitude of the total lift force coefficient and ϕ_{lift} is the phase of the total lift with respect to the cylinder's displacement y(t). Similarly, the vortex lift force can be expressed as a sinusoidal function with phase $\phi_{lift\,vortex}$ and amplitude $C_{L\,vortex}$:

$$C_{L\,vortex}(t) \approx C_{L\,vortex} \sin(2\pi f_e t + \phi_{lift\,vortex}) \qquad (4)$$

For transverse oscillations, the energy of the cylinder represents the energy transfer from the fluid to the cylinder and is simply the integrated product of the total lift force and the velocity of the cylinder. The energy transfer varies with the component of the lift force which is out-of-phase with the cylinder's displacement, while the in-phase component of the lift force has no effect on the energy transfer. Figure 1 demonstrates an interesting link between the vortex and total lift forces: their out-of-phase components are equal and the energy transfer can be expressed in terms of either the total or vortex forces. If the total and vortex lift forces are accurately represented by equations (3) and (4) then the normalised energy transfer is given by

$$C_E \approx \pi \frac{A}{D} C_L \sin(\phi_{lift}) \approx \pi \frac{A}{D} C_{L\,vortex} \sin(\phi_{lift\,vortex}) \qquad (5)$$

The sustained motion of a free oscillating system requires a net positive energy transfer and will only occur when $0^o<\phi_{lift}<180^o$, or equivalently when $0^o<\phi_{lift\,vortex}<180^o$. The forced motion of a

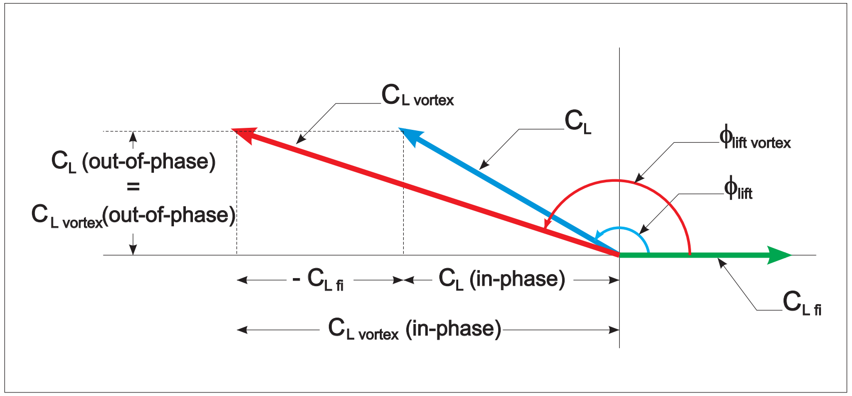

Figure 1 Vector diagram showing the relationship between C_L (t) and $C_{L\ vortex}$ (t).

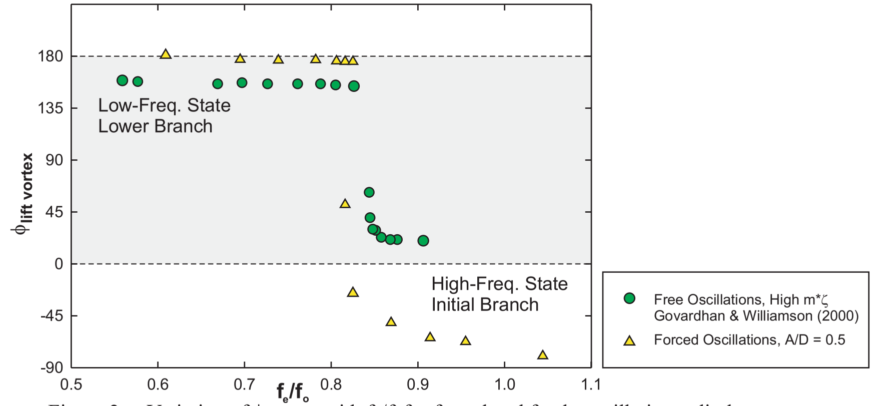

Figure 2 Variation of $\phi_{lift\ vortex}$ with f_e/f_o for forced and freely oscillating cylinders
● Free oscillation, $m^* = 320$, $m^*\zeta = 0.250$, [5], △ Forced oscillation, $A/D = 0.5$, $Re = 2300$.

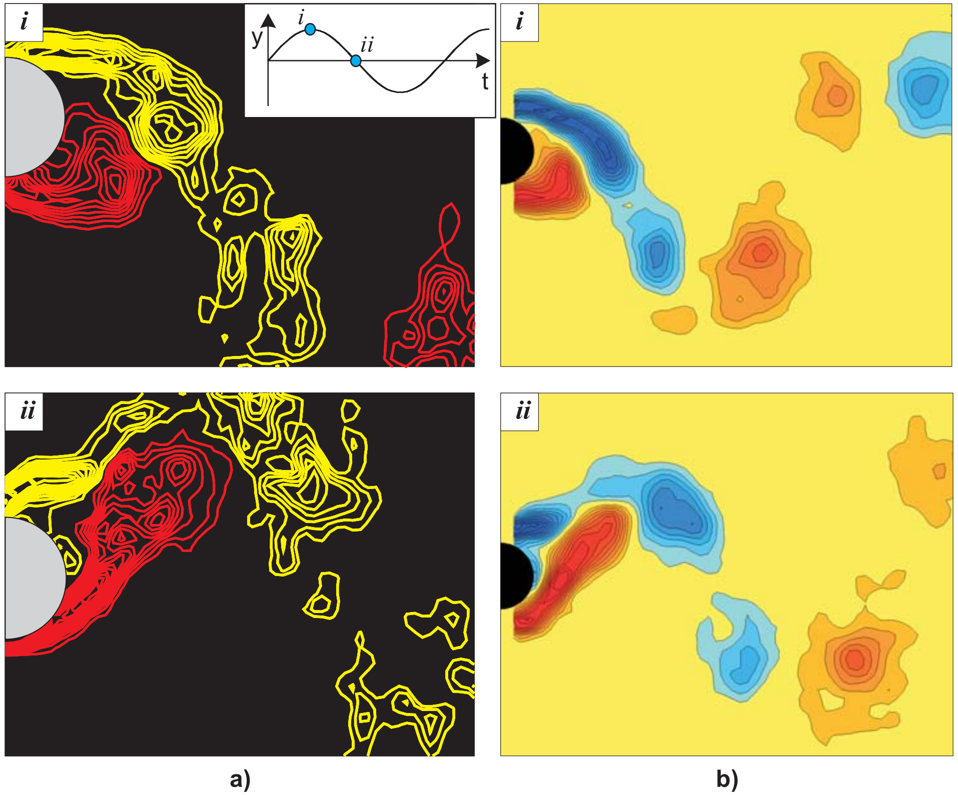

Figure 3 Phase averaged vorticity fields for a) forced oscillations low-frequency state, $A/D = 0.6$, $f_e/f_o = 0.82$, $Re = 2300$ and b) free oscillations lower branch, $A/D = 0.6$, $U^* = 6.40$, $Re \approx 3700$, [5]. The fields are shown at (i) the top and (ii) the mid-point of downwards motion, as indicated by the displacement insert.

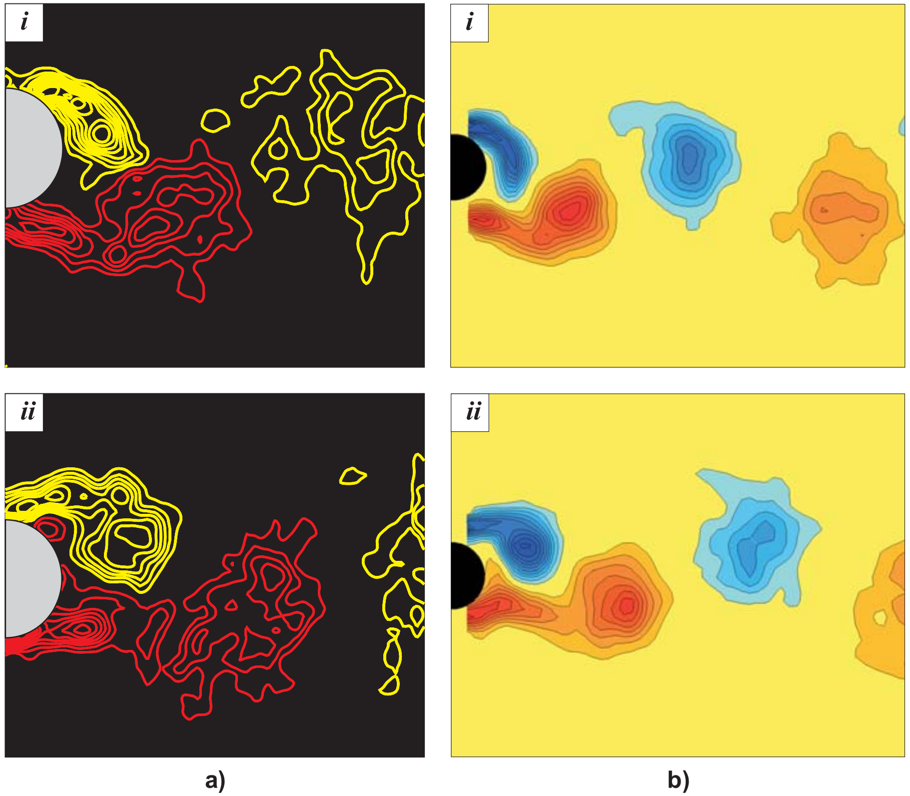

Figure 4 Phase averaged vorticity fields for a) forced oscillations high-frequency state, $A/D = 0.25$, $f_e/f_o = 1.15$, $Re = 4400$ and b) free oscillations initial branch, $A/D = 0.33$, $U^* = 5.18$, $Re \approx 3000$, [5]. The position of the images (i) and (ii) is the same as in Figure 3.

body is not subject to this constraint and all values of ϕ_{lift} and $\phi_{lift\ vortex}$ are theoretically possible.

One of the important questions arising from the investigations of forced and freely oscillating cylinders is whether the results from the forced oscillations be used to predict and interpret the oscillations of a freely vibrating system. A number of investigations, [13, 7] have considered this problem, however they were not able to predict the response of a freely oscillating cylinder over a wide range of parameters, and to a large extent this question remains unresolved. In this paper we present forces and vorticity fields for the forced oscillation of a cylinder and compare these results with those of an elastically mounted cylinder, [5]. The similarities and differences between these two sets of results will be discussed, in particular the ability of the forced oscillations to predict, or provide insight into, the freely oscillating case will be considered.

Experimental Method

The experiments were performed in a free-surface water channel at Lehigh University. The cylinder was mounted horizontally and oscillated transverse to the free-stream, such that its vertical motion was given by equation (2). During each experiment the amplitude of oscillation was held constant, while the frequency varied between $0.74 < f_e/f_o < 1.27$. Experiments were conducted at a number of amplitudes including A/D = 0.25, 0.4, 0.5 & 0.6, where D is the diameter of the cylinder. Over the range of A/D and f_e/f_o studied the wake was locked-on to the cylinder's oscillations. Two different cylinders, 25.4 mm and 50.8 mm in diameter, were used, giving aspect ratios of 12.5 and 7.6 respectively. The larger cylinder was used at A/D = 0.25 giving a Reynolds number, $Re = U_\infty D/\nu$, of 4400, while for the higher amplitudes the smaller cylinder was used and Re = 2300. The near wake velocity field was measured using a laser scanning version of high-image density PIV [10]. The images were recorded on high resolution 35 mm film and digitised at 106 pix/mm resulting in velocity fields with approximately 3500 vectors. The phase averaged vorticity fields reported here were calculated from 4-9 instantaneous velocity fields. The time varying lift force on the cylinder was measured by strain gauges mounted on a support sting. The inertia force due to the acceleration of the cylinder's mass was subtracted from the lift force.

Results and Discussion

Both the wake states for the forced oscillations, and the response branches of the elastically mounted cylinder, are characterised in terms of the wake structure and the forces on the cylinder [2, 5]. The variation of C_L and $C_{L\ vortex}$ with f_e/f_o has a distinctive shape for both the forced and freely oscillating cases. However, it is difficult to compare these shapes as for the free cylinder the amplitude of oscillation varies with f_e/f_o, while for the forced case A/D is held constant. The variable which best allows us to compare the lift forces on the forced and freely oscillating cylinder is the phase of the vortex lift force. The phase of the vortex lift force relates directly to changes in the distribution of vorticity and does not include the fluid inertia force, which varies directly with A/D. Additionally, the value of $\phi_{lift\ vortex}$ indicates the direction of energy transfer and has been used previously by Govardhan & Williamson [5] to characterise response branches.

Low-frequency state ↔ lower response branch

The structure of the near wake for the low-frequency state is shown in Figure 3a at the top and mid-point of the cylinder's oscillation. The low-frequency wake is characterised by the production of long shear layers which extend across the back of the cylinder, resulting in a very wide wake. As discussed by Carberry *et al* [2], the long extended shear layers promote the 2P mode of shedding, where vorticity of both signs is found throughout the vertical extent of the wake. At the top of the cylinder's oscillation a long negative shear layer extends well into the lower half of the wake, separating the attached positive vorticity from a positive vortex structure which was shed in the previous half cycle. As the cylinder moves downwards the positive shear layers swings around the back of the cylinder and at the mid-point of the cylinder's oscillation it has a distinct upwards angle. At the mid-point of the oscillation, Figures 3a(ii) and 3b(ii), a small portion of the negative shear layer has already separated from the rest of the shear layer, forming a counter rotating pair with previously shed positive vorticity and the attached negative vorticity is about to be shed into the near wake.

For the elastically mounted cylinder the wake for the initial branch, Figure 3b is very similar to the low-frequency wake in Figure 3a, where for both cases A/D = 0.6. Not only do they both display the same 2P mode of shedding, but the distribution of vorticity and phase of vortex shedding are remarkably similar. The phase of the vortex lift force for the low-frequency wake, Figure 3a is 184^o, while for the lower branch, Figure 3b $\phi_{lift\ vortex}$ is just under 180^o. Thus, the values of $\phi_{lift\ vortex}$ and the phase of vortex shedding for the low-frequency state and the lower branch are very similar, however, as the values of $\phi_{lift\ vortex}$ lie either side of 180^o the energy transfer is in the opposite direction.

The difference in the values ϕ_{lift} for the two cases in Figure 3 is approximately 25^o. The larger difference in ϕ_{lift} does not relate to additional changes in the wake but is symptomatic of the fact that the phases fall into the 2nd and 3rd quadrants. Examination of Figure 1 shows that when $\phi_{lift\ vortex}$ is near 180^o adding $C_{L\ fi}$ to the vortex force acts to push ϕ_{lift} away from 180^o, and the difference in the values of ϕ_{lift} does not necessarily represent the difference the phase of vortex shedding.

High-frequency state ↔ initial response branch

The mode of vortex shedding for the high-frequency wake, Figure 4a, is 2S and the structure of the wake clearly differs from the low-frequency wake. The attached shear layers are much shorter resulting in a relatively narrow wake. The initial branch wake also sheds in the 2S mode, as shown in Figure 4b, and the general structure of the near wake is similar to the high-frequency wake. Comparison of the wakes for the forced and freely oscillating cylinders at the top and mid-point of the oscillation indicates that shedding is occurring earlier in the high-frequency wake. Compared to the 2P mode of shedding the exact timing of vortex shedding for the 2S mode is often not well defined. As shown in Figure 4a(ii), the shorter negative shear-layer and the fact that the positive vortex is shed from the lower shear-layer as the wake tilts upwards, combine to make the separation of vorticity less distinct. It is important to remember that $\phi_{lift\ vortex}$, and therefore the direction of energy transfer, is linked to the maximum rate of change in the horizontal vortex moment, equation (1), rather than the point in the cycle at which we define the "pinching off" of a new separated vortex structure.

There is a significant difference in the values of $\phi_{lift\ vortex}$ for the high-frequency and initial branch wakes shown in Figures 4(a & b). For the high-frequency state $\phi_{lift\ vortex} = -78^o$, whist for the initial branch $\phi_{lift\ vortex}$ is just greater than 0^o. The large difference in $\phi_{lift\ vortex}$ is consistent with a shift in the phase of vortex shedding, however it is difficult to quantify the difference in phase from the vorticity fields of Figure 4. The energy transfer for the initial branch is small and positive, while for the high-frequency state the energy transfer is negative and relatively large, $C_E = -1.30$.

Vortex lift phase and energy transfer

The variation of $\phi_{lift\ vortex}$ with f_e/f_o for the forced and freely oscillating cylinders is shown in Figure 2. The freely oscillating cylinder has a relatively high mass damping ($m^*\zeta = 0.250$), and only the lower and initial response branches are observed. The amplitude of oscillation varies from close to zero at the bounds of lock-in, to A/D ≈ 0.55 just before the transition from the initial to the lower branch. At the transition between response branches there is typically a jump in A/D and a corresponding jump in $C_{L\,fi}$. The amplitude of the forced oscillation is constant, A/D = 0.5 and therefore $C_{L\,fi}$ varies only with the frequency of oscillation. The vortex lift phase for the freely oscillating cylinder is constrained by the requirement of positive energy transfer and will always fall within the shaded positive energy region in Figure 2, $0^o<\phi_{lift\ vortex}<180^o$. The forced oscillation of the cylinder can result in either positive or negative energy transfer, and $\phi_{lift\ vortex}$ extends either side of the positive energy region. Despite the difference in the range of $\phi_{lift\ vortex}$ the shape of the plots for the forced and free oscillations in Figure 2 is remarkably similar.

For an elastically mounted cylinder the range of $\phi_{lift\ vortex}$ varies with $m^*\zeta$, [5, 3]. At high values of $m^*\zeta$ the values of $\phi_{lift\ vortex}$ associated with the lower branch are well below 180^o while for the initial branch $\phi_{lift\ vortex}$ is well above 0^o. As $m^*\zeta$ decreases the values of $\phi_{lift\ vortex}$ for the lower and initial branch approach their limiting values of 180^o and 0^o respectively. The variation of $\phi_{lift\ vortex}$ with $m^*\zeta$ and f_e/f_o is similar to the variation of the phase of a driven elastically mounted mass with $m^*\zeta$ and f_e/f_{struc}, [14], where for the simpler case of the driven mass the force generating the motion is independent of the systems response. For the forced oscillations at A/D = 0.5, shown in Figure 2, $\phi_{lift\ vortex}$ is approximately equal to 180^o for the low-frequency state, while for the high-frequency state $\phi_{lift\ vortex}$ appears to asymptote towards -90^o. Experiments over a range of A/D and Re showed a general collapse of the data towards these two values. This suggests that when the wake is not restricted by the requirement of positive energy transfer there is a preferred phase of vortex shedding for each wake state which does not necessarily fall within $0^o<\phi_{lift\ vortex}<180^o$. The high-frequency state is often associated with negative energy transfer, particularly at high values of f_e/f_o, while for the low-frequency state the direction of energy transfer depends on whether $\phi_{lift\ vortex}$ is just above, or just below 180^o. For values of f_e/f_o above the transition to the high-frequency state the vortex lift force is typically in-phase with the cylinder's oscillation and $\phi_{lift\ vortex}$ is close to zero. Therefore, as shown in Figure 2, close to transition there may be a narrow region of positive energy transfer. At A/D = 0.4 and 0.5 these narrow regions of positive C_E appear to coincide with the values of f_e/f_o at which these amplitudes of oscillation are observed to occur for a freely oscillating cylinder, as for higher f_e/f_o the freely oscillating cylinder does not oscillate at these amplitudes. The motion of the elastically mounted cylinder at smaller amplitudes is observed over a wider range of f_e/f_o. However, for a forced amplitude of A/D = 0.25 we did not observe a wide band of positive energy transfer, and in fact C_E was negative for the entire high-frequency state.

Conclusions

Despite the remarkable similarities in the wake structures and modes of vortex shedding for the forced wake states and free response branches, differences in the phase of the lift force mean that in many cases the forced wakes have negative energy transfer and hence do not predict free cylinder oscillations. The fundamental difference between the two systems is emphasised by the results for identical values of A/D in Figure 3, where despite very similar wakes and values of $\phi_{lift\ vortex}$, the negative energy transfer for the forced oscillation incorrectly suggests that the elastically mounted cylinder will not oscillate at this amplitude. If the forced oscillation accurately represents the vortex induced motion of the elastically mounted cylinder then the wakes for the two cases should be the same. Although the motion of the elastically mounted cylinder, particularly at higher A/D, differs only very slightly from a pure sinusoid, it appears that the subtle differences in the displacement profile can result in significant changes in the energy transfer. An understanding of how small changes in the motion of the cylinder can alter the phase of vortex shedding and the energy transfer is required to determine the relationship between the forced and freely oscillating cylinders. Therefore, despite encouraging similarities in the wake states of the forced and free oscillations, further work is required to determine how the forced case can be used to predict the vortex induced motion.

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