Simulation and Scaling of Unstable Natural Convection Flow in Stratified Open Cavities

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Abstract
Cavity flow with a heated vertical wall facing an open boundary with top and bottom impervious and adiabatic and background stratification exhibits a characteristic bifurcation in the downstream region of the thermal boundary layer. The bifurcation has been shown to be associated with the free jet produced by the downstream turning boundary layer flow. The open cavity bifurcation is similar to that observed in closed cavities with heated and cooled opposing walls and in that context it has been suggested that the bifurcation instability is shear driven and corresponds to the well known instability of a Bickley jet. Results for a range of Rayleigh number have been obtained allowing the thermal boundary layer exit jet to be scaled. It is also shown that the exit jet may be shifted from the upper boundary by changing the ambient stratification, while retaining the general form and bifurcation behaviour, showing that the bifurcation is not dependent on the presence of the upper boundary.

Introduction
Natural convection flow in open cavities with a heated vertical wall, insulated top and bottom and open vertical boundary has application to refrigerators and ovens in open door conditions, solar thermal receiver systems, thermosyphons, fire spread in buildings, natural convection cooling of electronic devices, brake-housing systems in aircraft, and pipes joining reservoirs and oceans. In many of these cases the ambient fluid is stratified [3, 4, 9, 13].

The heated wall forms a natural convection boundary layer that is discharged as a jet beneath the upper boundary. This fluid travels across the upper part of the cavity and exits through the upper part of the open boundary. Ambient fluid is entrained across the remainder of the open boundary, travels across the cavity and is entrained by the boundary layer.

It has been shown that the open cavity, with stable linearly stratified ambient and Prandtl number $Pr = 0.7$, undergoes a bifurcation from steady to oscillatory unsteady flow at a Rayleigh number of approximately $8.25 \times 10^4$ [6]. A similar bifurcation is observed in the equivalent closed cavity flow where the open boundary is replaced by a closed cooled boundary [8, 10]. The bifurcation in the closed cavity has been shown to have characteristics similar to those of a Bickley jet, and it has therefore been suggested that the cavity bifurcation is a result of the free jet formed by the boundary layer at the downstream corner [5]. The jet initially exits the corner as an attached wall jet with buoyancy forces causing it to subsequently separate from the upper boundary resulting in a free jet, in both the closed and open cavities [6, 14].

Results obtained using a time-accurate finite volume scheme for open cavity natural convection flow with both homogeneous and stratified ambient at Prandtl number $Pr = 0.7$ have been obtained and show that a bifurcation occurs for the flow with stratified ambient only. Solutions obtained for a range of Rayleigh numbers show that the boundary layer exit jet has the same scaling with Rayleigh number as the thermal boundary layer itself, indicating that the jet is formed directly from the boundary layer without any significant modification. Additionally results have been obtained with varying background stratification allowing the position of neutral buoyancy to be moved below the upper boundary. In this case the flow has the same features as those obtained when the position of neutral buoyancy is located at the upper boundary, with the same $S$ shaped jet flow and bifurcation behaviour.

Numerical Method
The two-dimensional governing equations, using the Boussinesq approximation, are,

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + Ra \frac{T}{Pr}$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = 1 \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} - \nu \frac{\partial u}{\partial y} = 0,$$

where

$$Pr = \frac{\nu}{\kappa}, \quad Ra = \frac{g \beta \Delta TH^3}{\nu \kappa},$$

and where $u$ and $v$ are the velocity components in the $x$ and $y$ directions respectively, $t$ is the time, $T$ and $T_b$ are the fluctuating and background temperature respectively, with $T_b$ a function of $y$ only, with the total temperature $T + T_b$. As $T_b$ is steady, independent of $x$ and a linear function of $y$ it is zero in all but the vertical advection term in the temperature transport equation (3), and the background component of this term is then separated and included as the last term on the right hand side. The terms $\beta$, $\kappa$ and $\nu$ are the coefficients of thermal expansion, thermal conductivity and kinematic viscosity respectively, $g$ is the acceleration due to gravity and $H$ is the height of the cavity. Lengths are non-dimensionalised by the height of the cavity $H$, the temperature relative to the mean temperature by $\Delta T$, and time by $H^2/\nu$. $\Delta T$ is the dimensional difference between the total temperature on the heated wall and the background temperature at the height at which the non-dimensional background
The computational domain, together with boundary conditions for the fixed boundaries, are shown in Figure 1. With a finite-volume scheme an additional node is included outside the domain and for Dirichlet boundary conditions the average of the dependent variable at the exterior and immediate interior nodes is set to the required value. For Neumann boundary conditions the gradient in finite difference form using the dependent variable at the exterior and immediate interior nodes is set to the required value.

On the open boundary the normal gradient of vertical and horizontal velocity is set to zero. The horizontal gradient of fluctuating temperature is set to zero in the outflow region, that is where the horizontal velocity is positive on the boundary, while in the inflow region where the horizontal velocity on the open boundary is negative, the fluctuating temperature is set to zero.

A non-uniform mesh is used allowing grid nodes to be concentrated in regions of rapid solution variation, adjacent to the heated plate and upper and lower boundaries. The origin lies at the bottom left corner of the domain with $y$ increasing up the heated plate and $x$ increasing horizontally into the domain. The horizontal mesh size adjacent to the heated wall is $\Delta x = 0.002$ with a grid stretching factor of 1.08 per cell until $x = 0.2$, after which the stretching factor is gradually reduced until the grid becomes uniform in the interior. The vertical mesh size adjacent to the upper and lower boundaries is $\Delta y = 0.002$ with again a grid stretching factor of 1.08 per cell until $y = 0.2$ for the lower domain and $y = 0.8$ for the upper domain, after which the stretching factor is gradually reduced until the grid becomes uniform in the interior, resulting in a mesh of $59 \times 97$ nodes. The time step used was $\Delta t = 5 \times 10^{-7}$.

Time-step dependency tests were carried out by obtaining additional results at half the time step given above. The variation in the amplitude of the bifurcated modes was found to be less than 2%, while no discernible difference was observed in the frequencies. Mesh dependency tests have also been carried out by halving the mesh size at the heated wall and upper and lower boundaries, halving the stretching factor and halving the time step. Again the solutions on the two meshes showed a less than 2% variation with no discernible difference in the frequencies and other features of the bifurcation.

To determine any effect of the cavity aspect ratio and open boundary condition simulations were also carried out in a cavity with twice the horizontal length and the same grid spacing's given above, resulting in an $86 \times 97$ grid. This led to a less than 3% change in the lateral temperature profile in the boundary layer, the most sensitive quantity in the solution. The occurrence of the bifurcations and other quantitative features of the two flows were identical.

Results

Results have been obtained for $Ra$ ranging from $5 \times 10^6$ to $1 \times 10^{10}$, with $Pr = 0.7$, in a square open cavity. The cavity is initially filled with a quiescent fluid. At time $t = 0.0$ the left wall is instantaneously heated to a non-dimensional temperature of 1.0 and the flow allowed to develop. Flows with both homogeneous and stably stratified ambient conditions have been considered. In the stratified case, the background stratification gradient has been set to $dT_b/dy = 2.0$ with $T_{top} = 1.0, 1.25, 1.75, 2.00$. The flow with homogeneous ambient has $T_b = 0$. For all flows considered by time $t = 0.05$ a quasi-steady flow has developed.

Figure 2 contains the streamfunction and temperature contours for the flow with zero stratification at time $t = 0.06$ and $Ra = 0.5 \times 10^7$, with the heated wall on the left facing the open boundary on the right. A thermal boundary layer has developed travelling up the heated wall, entraining ambient fluid from the cavity over the lower 90% of its height. This fluid is discharged as a heated wall jet that travels across the cavity immediately beneath the upper boundary and exits through the open boundary without any significant change in character. By this stage of development the flow is steady and no bifurcation is observed.

Figure 3 contains the streamfunction and temperature contours for the stratified flow at time $t = 0.06$ and $Ra = 0.5 \times 10^7$. Again a thermal boundary layer has developed travelling up the heated wall, with ambient fluid now entrained over only the lower half of its height, and discharged over the upper half. A component of the discharge forms a boundary layer exit jet at the down-

Figure 1: Schematic presentation of the computational domain.
stream corner that almost immediately turns down into
the cavity, and then rises again to from a tight reverse
S shaped structure. The jet then spreads out to form a
gravity intrusion with vertical extent several times that
of the initial jet. At this Rayleigh number the stratified
flow is also steady, however the character of the exit jet
and intrusion is completely different to that of the non-
stratified flow.

At $Ra = 0.825 \times 10^8$ it is observed that the stratified flow
is no longer steady at large time but exhibits wave-like
behaviour. Figure 4 shows temperature time series at $x =
1/120$ and a range of vertical locations inside the thermal
boundary layer, adjacent to the upper part of the heated
wall. The sinusoidal behaviour of the time series clearly
demonstrates the unsteadiness of the flow. Spectra of the
time series at $y = 0.91$ are shown in Figure 5, where it is
seen that the dominant mode has a frequency $f = 1100$.

The boundary layer exit jet may be characterised by the
maximum velocity and jet width. In table 1 these quan-
tities are presented for a range of Rayleigh numbers with
d$T_b/\, dy = 2$ and $T_{top} = 1.0$ both in raw form and scaled
with Rayleigh number. The scalings for the jet width and
velocity are those given in [12] for the thermal boundary
layer thickness and velocity for natural convection bound-
dary layers on semi-infinite heated plates. It is clear that
these scalings provide an accurate description of the jet
structure, indicating that the corner region acts to simply
turn the jet without significantly affecting its structure.

Figure 6 shows streamline contours for the $Pr = 0.7$
flow with background stratification $dT_b/\, dy = 2$ and
$T_{top} = 1.25, 1.50, 1.75, 2.00$. As can be seen the loca-
tion of the boundary layer exit jet is dependent on the
local ambient temperature. Increasing $T_{top}$ shifts the lo-
cation of neutral buoyancy, that is the height at which
the temperature on the boundary ($T = 1.0$) is equal to
the background temperature. This correspondingly shifts
the position at which the boundary layer detaches from
the wall forming a horizontal jet. However it is seen that
the character of the exit jet is essentially unchanged, in-
dicating that it is primarily generated by buoyancy forces
rather than the presence of the upper boundary.

**Discussion and Conclusions**

Natural convection flow in an open cavity with stratified
ambient has been investigated. The flow generates a nat-
ural convection boundary which entrains fluid from the
ambient, discharging it into an exit jet which forms at
the upper end of the boundary layer. For non-stratified
ambient the jet transits the cavity as an attached wall jet
beneath the top surface exiting through the open bound-
dary, with no unsteadiness observed for Rayleigh numbers
up to $Ra = 1 \times 10^{10}$. For stratified ambient the jet turns
immediately downwards into the cavity and expands to
form a broad intrusion which transits the cavity and exits
through the open boundary. In this case the flow is ob-
served to undergo a bifurcation at a Rayleigh number of
$0.825 \times 10^8$ similar to that observed in the closed cavity
flow. The behaviour of the flow is not dependent on the
jet Reynolds number is obtained as $Re$ with no real change to its character. On this basis the scaling that the boundary layer essentially turns the corner to those for the natural convection boundary layer, showing that these scalings conform to those for the natural convection boundary layer, showing that the boundary layer essentially turns the corner with no real change to its character. On this basis the jet Reynolds number at bifurcation is therefore approximately 75.

The jet may be scaled in terms of its width and maximum velocity. It has been shown that these scalings conform to those for the natural convection boundary layer, showing that the boundary layer essentially turns the corner with no real change to its character. On this basis the jet Reynolds number at bifurcation is therefore approximately 75.

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