13th Australasian Fluid Mechanics Conference
Monash University, Melbourne, Australia
13-18 December 1998

THREE DIMENSIONAL VORTICAL STRUCTURES IN FLOWS AROUND PLATES

Boon T. TAN, Mark C. THOMPSON and K. HOURIGAN

Department of Mechanical Engineering
Monash University, Clayton, Victoria, AUSTRALIA

ABSTRACT

Numerical simulations of the flow around plates with blunt or elliptical leading edges and square trailing edges have been performed. Plates with elliptical leading edges display three-dimensional flow structures in the wake similar to those observed for a circular cylinder (Williamson 1988, Thompson *et al.* 1996). Modes corresponding to *Mode A* and *Mode B* can be observed depending on the Reynolds number and the plate length. For a short plate with $c/t = 2.5$ the wavelengths of these modes are approximately 6 and 0.7 plate thicknesses (t) respectively. For a longer plate with $c/t = 7.5$, only *Mode B* structures are present with a wavelength of approximately $1t$. On the surface of the plate with a blunt leading edge, staggered hairpin vortex structures with a wavelength of approximately 3 diameters are observed at $c/t = 10$ and 13 similar to the *Pattern B* three-dimensional shedding mode found in the experiments performed by Sasaki and Kiya (1991) at $Re = 350$.

NOMENCLATURE

c/t	Chord to Thickness Ratio (or Aspect Ratio)
Re	Reynolds Number
l_x	Streamwise Wavelength
l_z	Spanwise Wavelength
K	Number of Spectral Elements

INTRODUCTION

For flow past two-dimensional circular cylinders, the wake becomes three dimensional above a critical Reynolds number. The flow past elongated bluff bodies (such as long plates) also exhibit three-dimensional wakes. For plates with elliptical (or streamlined) leading edges, shedding is suppressed from the leading edge and only trailing-edge shedding can occur. The wake in this case shows similar characteristics to that from a circular cylinder. For plates with sharp leading edges, shedding can occur from both the leading and the trailing edges leading to a complex interaction between both the broadly two-dimensional vortex rollers and also the three-dimensional vortex structures which develop in both the leading- and trailing-edge shear layers.

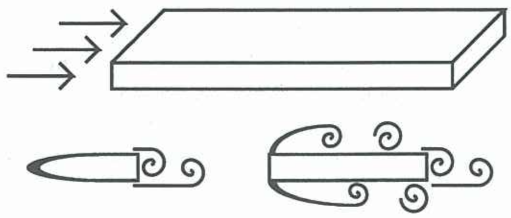


Figure 1: Diagram of the type of flow examined (top). The cases examined are: (1) trailing-edge shedding from a plate with an elliptical leading edge (bottom left) and (2) both leading and trailing edge shedding from a rectangular cross-sectioned plate.

Figure 1 shows a sketch of the two cases examined in this paper, which both consist of two-dimensional bluff bodies placed in a uniform flow. The geometries investigated are: (1) a rectangular plate and; (2) a similar plate with an elliptical leading edge. The diagram also shows sketches of the two-dimensional flow structures observed for these geometries. The elliptical leading edge is sufficiently streamlined to prevent vortex shedding from the leading edge of the plate. For rectangular plates, the aspect ratios considered in this study are large enough for the leading-edge shear layers to form large vortical structures which convect downstream next to the top and bottom plate surfaces, and also for the trailing-edge shear layers to roll up and form large vortical structures.

The three-dimensional modes of shedding from short bluff bodies (especially circular cylinders and vertical plates) have been extensively studied experimentally (e.g. Williamson, 1988, Norberg, 1994), theoretically (Henderson and Barkley, 1996) and numerically (Thompson *et al.*, 1996, Mittal and Balchandar, 1996, Karnidakis and Triantafyllou, 1992, Naijar and Vanka, 1995). It has been well documented that the transition to three dimensionality in the wake involves the growth and saturation of two separate instabilities. The first of these, *Mode A*, occurs above a critical Reynolds number of approximately 190, and between $Re = 230$—260 there is a gradual shift in energy to the second instability, *Mode B*. At higher Reynolds numbers the existence of *Mode A* (which

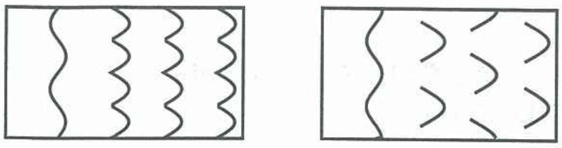


Figure 2: Plan view of the flow over a rectangular plate showing sketches of the observed dye patterns for the two instability modes found by Sasaki and Kiya (1991). The first mode to occur ($Re < 380$) called *Pattern A* is shown on the left and the second mode *Pattern B* is shown on the right.

has a longer wavelength) becomes less apparent but still appears to be important to the overall wake dynamics (Thompson *et al.* 1996). *Mode B* persists well into the turbulent regime although the regularity decreases as the flow becomes more turbulent and it is more difficult to visualise. Spanwise correlations of the flow at Reynolds numbers above 1000 clearly indicate the present of *Mode B* streamwise vortical structures (Wu *et al.*, 1994).

The spanwise wavelength of *Mode A* is approximately 3–4 diameters and that of *Mode B* is about 0.8–1 diameters (Henderson and Barkley, 1996, Williamson, 1988). An analysis by Williamson (1996) showed that these wavelengths scale with different physical structures. *Mode A* appears to be an elliptical instability of the two-dimensional vortex cores, while *Mode B* appears to be an instability of the braid regions between the rollers. Importantly, the different instabilities lead to two different topologies for the modes. For *Mode A*, the streamwise vortical structures connecting the largely two-dimensional vortex rollers are aligned in the downstream direction so that they are of opposite sign on opposite sides of the wake. For *Mode B*, the reverse is true, here the structures are of the same sign on each side of the wake.

The two-dimensional flow over long rectangular cross-sectioned plates has been the subject of investigation in many previous studies (e.g., Mills *et al.* 1995, Nakamura *et al.* 1991, Thompson *et al.* 1997, Bearman *et al.* 1982). The vortices shed from the leading edge, convect downstream and interact with the trailing edge. As vortices pass the trailing edge, a pressure pulse results which travels upstream to lock further leading-edge shedding. This causes regular shedding and leads to discrete changes in the Strouhal number based on plate length as the shedding changes from one possible shedding mode to the next (as the plate length is increased). These shedding modes correspond to different integer numbers (n) of vortices between the leading and trailing edges and is classified as the impinging leading-edge vortex (ILEV) instability (Rockwell and Naudascher, 1979).

Only a limited amount of work has been carried out on the three dimensional instability of the vortices shed from the leading edge of a plate. Sasaki and Kiya (1991) obtained clear flow visualisation of two different modes. The measurements were taken between 0.6 to 2 times the re-attachment length from the leading edge for plates with a c/t ratio ranging between 10 to 40.

The flow remained two-dimensional up to a Reynolds number of approximately 320. At higher Reynolds numbers the mean re-attachment length was about 5 plate thicknesses. Between $320 < Re < 380$, the flow above the plate becomes three-dimensional. Secondary streamwise vortex structures appear forming the first pattern shown in figure 2. This is referred to by Sasaki and Kiya (1991) as *Pattern A*. For this mode the streamwise vortex structures which occur between the two-dimensional spanwise vortices are in phase with subsequent streamwise vortices. The streamwise and spanwise wavelengths are both approximately 2–2.5 plate thicknesses. For Reynolds numbers in excess of 380, a different mode becomes dominant which has been called *Pattern B*. This mode has a wavelength of 3–4 thicknesses in both the spanwise and streamwise directions. In this case, the streamwise vortices structures still form in rows but each row is staggered with respect to the next one. A sketch of these two instability modes is shown in Figure 2.

The remainder of the paper is organised as follows. The next section contains an overview of the numerical method used for the simulations. Following this, results are presented: firstly for plates with elliptical leading edges (to suppress the leading-edge shedding); then for plates with square leading-edges allowing a complex interaction between leading and trailing edge vortex structures.

NUMERICAL METHOD

The time-dependent incompressible Navier-Stokes equations in primitive variables form are the governing equations for these simulations. The equations are discretised using a time-split spectral/spectral-element method as described by Karniadakis and Triantafyllou (1992) and Thompson *et al.* (1996). Only a brief overview will be presented here.

The momentum equations are integrated forward in time using a classical three-step method. The steps account for the convection, pressure and diffusion terms respectively. The convection step is treated implicitly with a 3rd order Adams-Bashforth method and the diffusion and pressure/mass-conservation steps lead to Helmholtz and Poisson equations which are handled implicitly. Second-order temporal accuracy is achieved by using a first-order pressure boundary condition (Karniadakis *et al.* 1991). The time step used ranged from 0.0065 to 0.008 dimensionless time units. The natural shedding period was approximately 5 units.

The spectral-element technique (Karniadakis and

Triantafyllou, 1992) was employed for the two-dimensional (stream-wise) flow, *i.e.*, the x-y planes. A full Fourier spectral approach was taken in the spanwise direction (z). Typically of the order of 350 macro (spectral) element were used to discretise the domain. In each element a tensor product of 6th order polynomial interpolants is used. In the spanwise direction, between 24 and 48 Fourier terms were used. Typically, the spanwise domain size was set at $2\pi t$, sufficient to capture all of the observed three-dimensional modes.

The velocity at all external boundaries was set to the upstream value except for the outflow boundary which had zero normal gradients. A no-slip condition was applied on the surface of the plates.

This code has been used for simulating the three-dimensional flow past a circular cylinder (Thompson *et. al*, 1996) where it successfully predicted the two experimentally observed three-dimensional shedding modes (*Mode A,B*) (Williamson, 1988). The resolution and domain size for the current simulations are broadly based on that work. The resolution in each planes has been established from two-dimensional simulations. Several simulations with increased resolution in the spanwise direction has been performed to verify the accuracy of the predictions (i.e. varying resolution between 32 and 48 planes).

RESULTS

Three-dimensional simulations were started with results from two-dimensional simulations which had reached an asymptotic state. The simulations were integrated for typically 200 time units ($\approx$ 40 cycles) until the mean value of the base pressure became constant in time. For the visualisation results presented later the spanwise domain has been doubled to better show the streamwise vortical structures.

Plates with Elliptical Leading Edges

c/t=2.5

Simulations were performed at a $Re = 300$, 350 and 380. An iso-surface plot of the streamwise vorticity together with mean base pressure is shown in Figure 4. The spanwise wavelength is $2\pi t$ at $Re = 300$ (which is the maximum size allowed on the computational domain.) The alignment of streamwise vortex cores from the braid region on one side of the wake to the other, shows that the topology of this three-dimensional mode is similar to *Mode A* shedding for the circular cylinder wake. (That is, streamwise vorticity on one side of the wake is aligned with streamwise vorticity of opposite sign on the other side of the wake.) The pressure iso-surfaces highlight the significant distortion of the predominantly two-dimensional spanwise vortex cores.

If the Reynolds number is increased to $Re = 350$, there is evidence of competition between *Mode A* and *Mode B* shedding. At any particular time, both the *Mode A* and *Mode B* structures may be visible and the amplitude of each changes with time. The *Mode B* structures have a wavelength of approximately one nineth of the *Mode A* wavelength. The spanwise vortex cores are less distorted than for the lower Reynolds number case resulting in a recovery of base pressure.

Only *Mode B* is present at $Re = 380$ with a spanwise wavelength of 0.7 diameters and the streamwise vortex filaments are of the same sign as matching filaments of the opposite side of the wake. The distortion of the spanwise vortex cores is negligible and this results in a strong recovery in base pressure relative to the *Mode A* regime.

c/t=7.5

Simulations were performed at Reynolds numbers between 250 and 500. Below $Re = 300$, the wake remained two-dimensional. Above $Re = 300$ there is a gradual growth in three-dimensionality corresponding to the *Mode B* instability with increasing Reynolds number. The spanwise wavelength remained constant at approximately 1 plate thickness. Figure 5 shows an iso-surface plot at $Re = 500$. For these simulations the *Mode A* instability has not been observed.

Rectangular Plate

Results of the simulation of flows over rectangular plates with $c/t = 10$ and 13 at $Re = 350$ are shown in Figure 6. The *Pattern B* structures are visible above the surface of the plate with a spanwise wavelength (l_z) of πt. The streamwise vortex loops (hairpins) are stretched in the streamwise direction and display the staggered arrangement found by Sasaki and Kiya (1992). The 'legs' of the vortices for the simulations with $c/t = 13$ are stretched quite significantly. The streamwise wavelength (l_y) is 4. The interaction of the (spanwise and streamwise) vortices from opposite sides of the plate and from the trailing edge shedding leads to a very complex wake interaction. Simulations at $Re = 340$ and below have not shown any three dimensional instability and *Pattern A* has not been simulated as yet.

DISCUSSION

The simulations were performed with a fixed domain size and periodic boundary conditions in the spanwise direction. This may restrict some modes from developing, especially those with a larger wavelength. The *Mode A* structures for the elliptical leading edge with $c/t = 2.5$ had a wavelength equivalent to the domain width. This wavelength may not be the most unstable mode. At $c/t = 7.5$, *Mode A* could be unstable for wavelengths larger than the spanwise domain size and therefore cannot be captured.

It is possible that the spanwise domain size may prevent the occurrence of *Pattern A*, although the experimentally observed *Pattern A* wavelength suggests that it should be captured with the current domain size. Simulations with larger spanwise domains are underway to investigate this possibility. *Pattern A* was only observed in a small Reynolds number range (320-380) by Sasaki and Kiya (1991). They did not use end plates and it may be possible the differences in spanwise boundary conditions could account for the discrepancies between those experiments and these simulations.

CONCLUSIONS

The wake for the shorter plate ($c/t = 2.5$) with an elliptical leading edge showed a similar transition scenario to the wake of a circular cylinder. With increasing Reynolds number first *Mode A* develops at around $Re = 300$. For slightly higher *Re* *Mode B* appears but competes with *Mode A*. At $Re = 380$ the streamwise vortex structures in the wake are predominantly *Mode B*.

For the longer plate ($c/t = 7.5$) the wake remained two-dimensional up to $Re = 300$. For higher *Re*, only *Mode B* structures were observed. In this case, the shear layers separating from the trailing edge are considerably thicker than for a circular cylinder. It is possible that this leads to a much longer spanwise wavelength for the *Mode A* structures which cannot be captured on the computational domain.

Pattern B has been simulated for a rectangular plate with $c/t = 10$ and 13 at $Re = 350$. Simulations at lower Reynolds numbers have not captured the experimentally observed *Pattern A* structures.

ACKNOWLEDGEMENTS

The first author would like to acknowledge the financial support of the Monash Postgraduate Scholarship and Overseas Postgraduate Research Scholarship. The Monash High Performance Computing facility made the simulations in this paper possible.

REFERENCES

BEARMAN, P.W. and OBASAJU, E.D., "An Experimental Study of Pressure Fluctuations on Fixed and Oscillating Square-Sectioned Cylinders", J. Fluid Mech., 119, 297–321, 1982.

HENDERSON, R.D. and BARKLEY, D., "Secondary instability in the wake of a circular cylinder", Phys. Fluids, 8, 1683, 1996.

KARNIADAKIS, G.E., ISRAELI, M. and ORSZAG S.A., "High-Order Splitting Methods for the Incompressible Navier-Stokes Equations", J. Comp. Phys., 97, 414–443. 1991.

KARNIADAKIS, G. E. & TRIANTAFYLLOU, G. S. "Three-dimensional Dynamics and Transition to Turbulence in the Wake of Bluff Objects", J. Fluid Mech., 238, 1–30, 1992.

MILLS, R., SHERIDAN, J., HOURIGAN, K. and WELSH, M.C., "The Mechanism Controlling Vortex Shedding from Rectangular Bluff Bodies", Proceedings of the 12th Australasian Fluid Mechanics Conference, 227–230, Sydney, December 1995.

MITTAL, R. and BALACHANDAR, S. "Direct Numerical Simulation of Flow Past Elliptic Cylinders", J. Comp. Phys., 124, 351, 1996.

NAIJAR, F.M. and VANKA, S.P., "Effects of Intrinsic three-dimensionality on the drag characteristics of a normal flat plate", Phys. Fluids, 10 2516–2518, 1995.

NAUDASCHER, E. and WANG Y., "Flow induced vibrations of prismatic bodies and grids and prisms", J. Fluids Struct., 7, 341–373, 1993.

NAKAMURA, Y., OHYA, Y., and TSURUTA, H., "Experiments on Vortex Shedding from Flat Plates with Square Leading Edges", J. Fluid Mech, 222, 437–447, 1991.

NORBERG, C., "An experimental investigation of the flow around a circular cylinder: influence of aspect ratio", J. Fluid Mech., 258, 287–316, 1994

OHYA, Y., NAKAMURA, Y., OZONO, S., TSURUTA, H. and NAKAYAMA, R., "A Numerical Study of Vortex Shedding from Flat Plates with Square Leading and Trailing Edges", J. Fluid Mech., 236, 445–460, 1992.

ROCKWELL, D. and NAUDASCHER, E., "Self-Sustained Oscillations of Impinging Free Shear Layers", Ann. Rev. Fluid Mech., 11, 67–94, 1979.

SASAKI, K. and KIYA, M., "Three-dimensional Vortex Structure in a Leading Edge Separation Bubble at Moderate Reynolds Number", J. Fluids Eng., 113, 405, 1991.

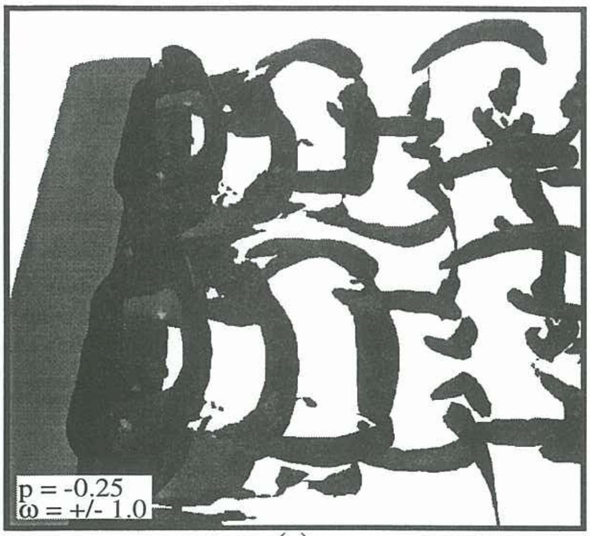
THOMPSON, M.C., HOURIGAN, K. and SHERIDAN, J., "Three-Dimensional Instabilities in the Wake of a Circular Cylinder", International Journal of Experimental Heat Transfer, Thermodynamics, and Fluid Mechanics, 12, 190–196, 1996.

THOMPSON M.C., TAN B.T. and HOURIGAN K., "Simulation of Perturbed Flows Past Long Blunt Bodies", FEDSM97-3675 published on CDROM, ASME Fluids Engineering Division Summer Meeting, Vancover, 1997.

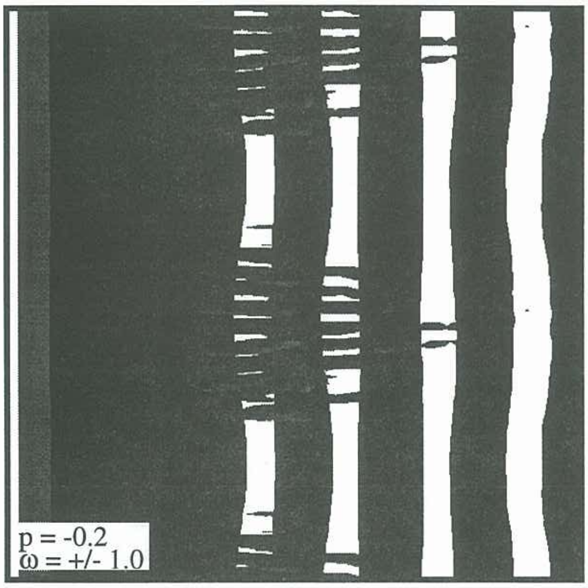
WILLIAMSON, C. H. K., "The Existence of Two Stages in the Transition to Three-Dimensionality of a Cylinder Wake", Phys. Fluids, 31, 3165, 1988.

WILLIAMSON, C. H. K., "Three-dimensional Wake Transition", J. Fluid Mech., 328, 345, 1996.

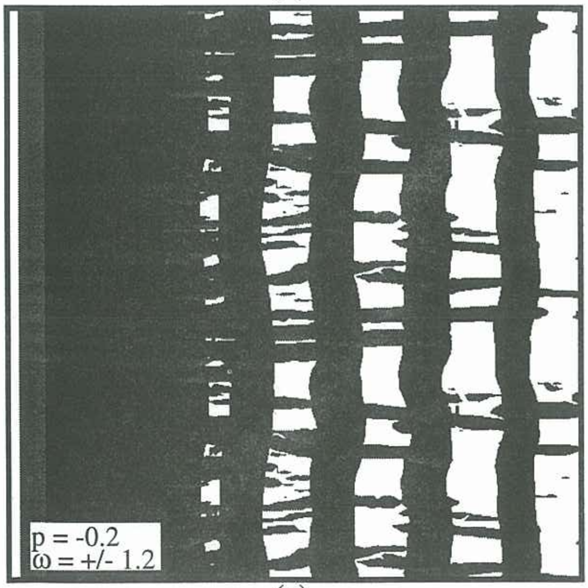
WU, J., SHERIDAN, J., SORIA, J. and WELSH, M.C., "An Experimental Investigation of Streamwise Vortices in the Wake of a Bluff Body", J. Fluids Structures, 8, 621–625, 1994.



(a)



(b)



(c)

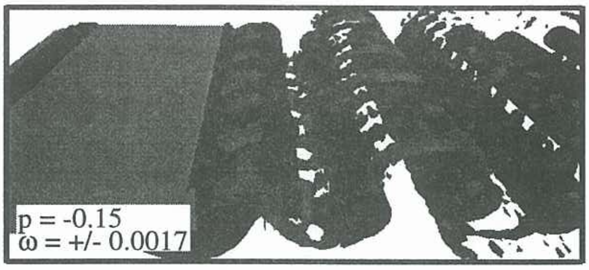
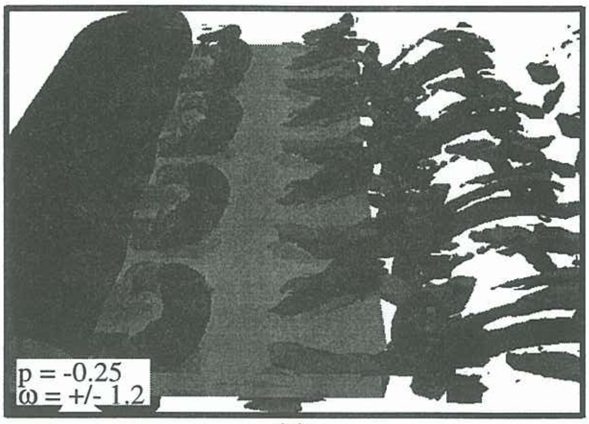
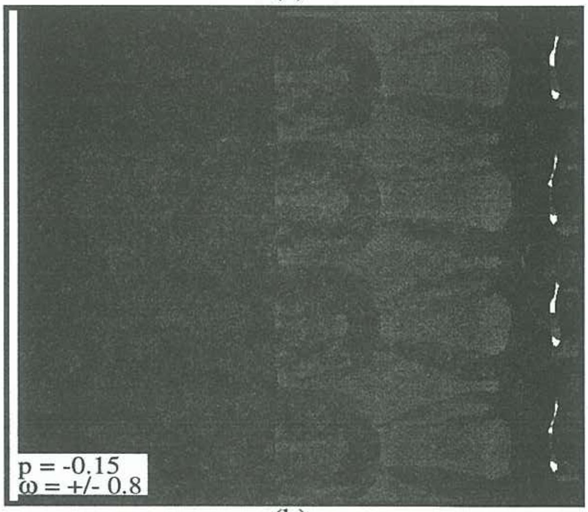


Figure 5: Iso-surface plot for an elliptical leading edge plate with a $c/t = 7.5$ at $Re = 500$ viewed from the side.



(a)



(b)

Figure 6 (Above): Iso-surface plot for a rectangular plate with a (a) $c/t = 10$ and (b) $c/t = 13$ at $Re = 350$ viewed from the side and top respectively.

Figure 4 (Adjacent): Iso-surface plot for an elliptical leading edge plate with a $c/t = 2.5$ at (a) $Re = 300$, (b) $Re = 350$ and (c) $Re = 380$ viewed from the top.

Note : Light and dark grey iso-surfaces are constant positive and negative streamwise vorticity respectively, and black iso-surfaces are surfaces of constant pressure with levels labeled in each diagram. The bluff body is shown as the lightest grey surface.