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STATISTICS OF $\partial u/\partial y$ IN A TURBULENT SHEAR FLOW

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ABSTRACT

The small scale structure of turbulence is studied by measuring the statistics associated with $\partial u/\partial y$ in the wake behind a circular cylinder. The results indicate that the pdf of $\partial u/\partial y$ has long exponential tails which are Reynolds number dependent. The rate of increase with R_λ of the flatness factor of $\partial u/\partial y$ is higher than that of $\partial u/\partial x$. Important differences are observed in the shapes of spectra of $(\partial u/\partial y)^2$ and $(\partial u/\partial x)^2$. In the inertial range, the autocorrelation of $(\partial u/\partial y)^2$ behaves similarly to that of $(\partial u/\partial x)^2$, each yielding approximately the same value for the intermittency exponent μ.

INTRODUCTION

In the study of small-scale turbulence, a significant amount of effort has been devoted to the investigation of the statistics of $\partial u/\partial x$, the streamwise derivative of the longitudinal velocity fluctuation (Townsend, 1947; Kuo and Corrsin, 1974; Frenkiel and Klebanoff, 1971; Van Atta and Antonia, 1980). Statistical characteristics of ϵ, the fluctuating rate of turbulent kinetic energy dissipation, has been inferred from $\tilde{\epsilon}$ [$\sim (\partial u/\partial x)^2$], which is assumed to be a useful surrogate for ϵ. Recently, with the aid of direct numerical simulation data, attention has also been given to the statistics of other spatial derivatives of the longitudinal velocity fluctuation (She et al., 1988; Kida and Murakami, 1989; Vincent and Meneguzzi, 1991; Jimenez et al., 1993), in particular $\partial u/\partial y$ (here y is in the transverse direction). Results from the simulations indicate that the probability density functions (pdf) of $\partial u/\partial y$ (or $p_{\partial u/\partial y}$ in short) are distinctly non-Gaussian with long exponential tails. This result has not yet been confirmed experimentally. This is probably due to the fact that most of the studies (Antonia et al., 1993; Zhu et al., 1993) associated with the measurement of $\partial u/\partial y$ concentrated primarily on obtaining accurate measurements of $<(\partial u/\partial y)^2>$ (note $<(\partial u/\partial y)^2>$ appears in the expressions for $<\epsilon>$, the mean energy dissipation rate and $<\omega^2>$, the mean square of vorticity or enstrophy; the angular brackets denote time averaging). It is clearly desirable to investigate the statistics of $\partial u/\partial y$, comparing them with those of $\partial u/\partial x$, and ascertain their possible dependence on R_λ ($\equiv <u^2>^{1/2} \lambda/\nu$, λ is the longitudinal Taylor microscale and ν is the kinematic viscosity).

In this paper, we investigate the properties of the small scale structure of turbulence by focusing on the statistical characteristics of $\partial u/\partial y$. We report the measured $p_{\partial u/\partial y}$ and its evolution over a modest range of R_λ. We examine the R_λ dependence of the flatness factor of $\partial u/\partial y$, comparing it with that of $\partial u/\partial x$. In addition, the spectra of $(\partial u/\partial y)^2$ and the power-law behaviour of the autocorrelation of $(\partial u/\partial y)^2$, in the inertial range, are also considered. Such information should provide important insight into the nature of small-scale turbulence.

EXPERIMENTAL DETAILS

Measurements have been made at the centreline in the wake of a circular cylinder at $x_1/d = 70$ (where x_1 is the streamwise distance measured from the cylinder axis, d is the diameter of the cylinder). The present choice ($x_1/d = 70$) was a compromise since relatively large diameter cylinders were needed to obtain relatively large values of R_λ. A detailed description of the test facility and instrumentation can be found in Antonia et al. (1995). A modest range of R_λ (60–260) was achieved for different free stream velocities U_∞ (3.6–14.7 m/s). Two parallel single hot-wires were used to measure $\partial u/\partial y$ using the finite difference approximation. $\partial u/\partial x$ was inferred from the temporal derivative using Taylor's hypothesis. In view of the relatively low turbulence level (typically less than

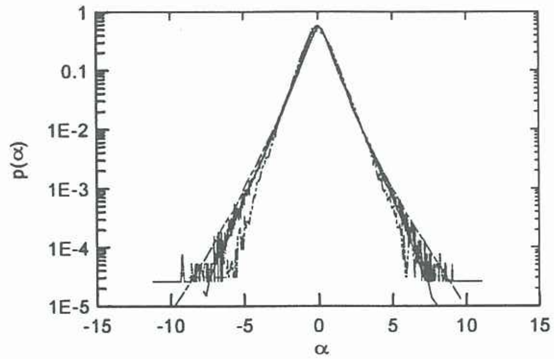

Figure 1: PDF of α ($= \partial u/\partial y / <(\partial u/\partial y)^2>^{1/2}$). Present data, —‐‐‐ —: $R_\lambda = 60$. ·····: $R_\lambda = 190$. — — — : data of Vincent and Meneguzzi, $R_\lambda \simeq 150$. ——: data of Kida and Murakami, $R_\lambda \simeq 128$.

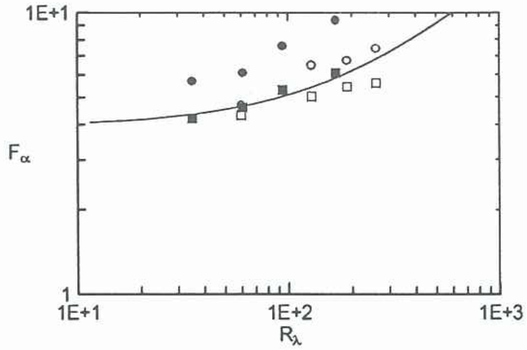

Figure 2: R_λ dependence of F_α. Present data, □ : $\alpha = \partial u/\partial x$; ○ : $\alpha = \partial u/\partial y$. Data of Jimenez et al. (1993), ■ : $\alpha = \partial u/\partial x$; ● : $\alpha = \partial u/\partial y$. ——: $\alpha = \partial u/\partial x$; data compiled by Van Atta and Antonia (1980).

$6 \sim 7\%$), the use of this hypothesis seems justified. The wire length ℓ, and the separation Δy varied between $1.5\eta - 3.5\eta$ and $4\eta - 8.5\eta$ respectively, here η ($\equiv \nu^{3/4} / <\epsilon>^{1/4}$) is the Kolmogorov length scale. All the wires consisted of 2.5 μm Pt-10% Rh wires and were operated with constant temperature anemometers at an overheat ratio of 1.5. Prior to digitization, the signals were amplified and low-pass filtered at a cut-off frequency f_c close to the Kolmogorov frequency f_k ($\equiv \overline{U}_1/2\pi\eta$, $\overline{U}_1$ is the local mean velocity). The signals were digitized at a sampling frequency of $2f_c$ into a NEC386 personal computer using a 12 bit A/D converter.

PDF OF $\partial u/\partial y$

The measured $p_{\partial u/\partial y}$, normalized by its standard deviation, is plotted in Figure 1 for two values of R_λ (60 and 190). Also included in the figure are the $p_{\partial u/\partial y}$ data from the direct numerical simulations of isotropic turbulence of Kida and Murakami ($R_\lambda \simeq 128$) and Vincent and Meneguzzi ($R_\lambda \simeq 150$). The tails of the measured $p_{\partial u/\partial y}$, confirm the exponential behaviour obtained from simulations. The tails of the present $p_{\partial u/\partial y}$ for $R_\lambda = 190$, lie slightly below those of Vincent and Meneguzzi, but close to the $p_{\partial u/\partial y}$ of Kida and Murakami. This can be attributed partly to the effect of spatial resolution of the probe on $p_{\partial u/\partial y}$. Kraichnan (1990) and She (1991) suggested that the self-stretching of the small-scale coherent structures is the main source of non-Gaussianity. The tails of both the measured $p_{\partial u/\partial y}$ and those of direct numerical simulations become flatter with increasing R_λ, reflecting the evolution of $p_{\partial u/\partial y}$ with R_λ.

The measured $p_{\partial u/\partial y}$ is close to being symmetrical with respect to the origin compared to $p_{\partial u/\partial x}$ (not shown) which is asymmetrical. While the non-zero value of the skewness of $\partial u/\partial x$ is closely connected to the stretching of vorticity (e.g. Champagne, 1978), the skewness of $\partial u/\partial y$ is zero for isotropic turbulence. The latter may also be zero by virtue of symmetry on the wake centreline. Measurements of the skewness of $\partial u/\partial y$ on the wake centreline cannot therefore provide unambiguous support for isotropy.

R_λ DEPENDENCE OF $F_{\partial u/\partial y}$

Figure 2 shows the variations of the flatness factors, $F_{\partial u/\partial y}$ and $F_{\partial u/\partial x}$ ($F_\alpha = <\alpha^4> / <\alpha^2>^2$, α is either $\partial u/\partial y$ or $\partial u/\partial x$) as functions of R_λ. Also plotted in the figure are the data for $F_{\partial u/\partial y}$ and $F_{\partial u/\partial x}$ of Jimenez et al. A best fit curve to the $F_{\partial u/\partial x}$ data compiled by Van Atta and Antonia (1980) are also included in the figure. While all three data sets for $F_{\partial u/\partial x}$ are in relatively good agreement with each other, the present $F_{\partial u/\partial y}$ data are consistently smaller than those of Jimenez et al. This is probably partly due to the effect of wire separation on $F_{\partial u/\partial y}$ and also on the choice of f_c. The results indicate that the rate of increase of $F_{\partial u/\partial y}$ is higher than that of $F_{\partial u/\partial x}$ over the limited present range of R_λ. This rate of increase in $F_{\partial u/\partial y}$ is consistent with the evolution of the tail of $p_{\partial u/\partial y}$ and may reflect that $\partial u/\partial y$ is more

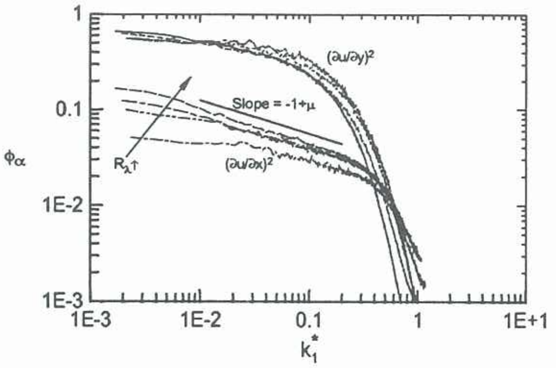

Figure 3: Spectra of α. $\alpha = (\partial u/\partial x)^2$, — - —: $R_\lambda = 60$; —- - - —: 110; — — — : 190; — —: 260. $\alpha = (\partial u/\partial y)^2$, ·····: $R_\lambda = 60$; ----: 110; - - -: 190; ——: 260. The line has slope of $-1+\mu$ ($\mu = 0.65$).

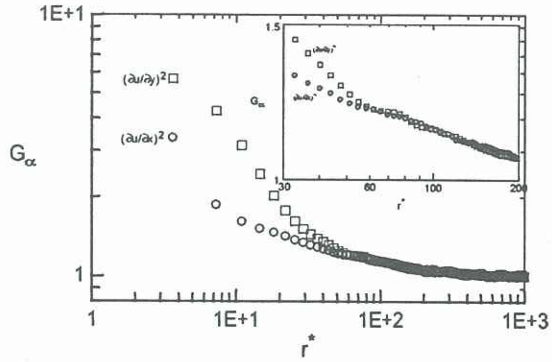

Figure 4: Distribution of G_α as a function of r^*. ○ : $\alpha = (\partial u/\partial x)^2$; □ : $(\partial u/\partial y)^2$, $R_\lambda = 260$. The inset focuses on the inertial range.

intermittent in nature than $\partial u/\partial x$. Kraichnan's probabilistic model suggests that $F_{\partial u/\partial y}$ is asymptotically independent of the Reynolds number. This is certainly not the case with our present $F_{\partial u/\partial y}$ data (and also those of simulation). It remains to be seen if $F_{\partial u/\partial y}$ will asymptote to a constant at large R_λ.

SPECTRA OF $(\partial u/\partial y)^2$

Figure 3 shows $\phi_{(\partial u/\partial y)^2}$ and $\phi_{(\partial u/\partial x)^2}$ for different values of R_λ (60–260). The spectra are normalized such that $\int \phi_\alpha dk_1^* = (\overline{\alpha^{*2}} - \overline{\alpha^*}^2)$. The shapes of $\phi_{(\partial u/\partial y)^2}$ are significantly different from those of $\phi_{(\partial u/\partial x)^2}$. While $\phi_{(\partial u/\partial y)^2}$ is richer in low wavenumber energy, $\phi_{(\partial u/\partial x)^2}$ is richer in high wavenumber energy. $\phi_{(\partial u/\partial x)^2}$ indicates a systematic R_λ dependence at low wavenumbers. Apart from the data for $R_\lambda = 60$, there is a reasonable collapse of the high wavenumber end of $\phi_{(\partial u/\partial x)^2}$ for different R_λ, suggesting approximate R_λ independence. In contrast, no systematic trend can be observed for $\phi_{(\partial u/\partial y)^2}$ at low wavenumbers. Although the high wavenumber region of $\phi_{(\partial u/\partial y)^2}$ appears to be R_λ dependent, this is more likely to be an effect of the spatial resolution of the probe. This effect is expected to be more pronounced for $\phi_{(\partial u/\partial y)^2}$ than for $\phi_{(\partial u/\partial x)^2}$. Different estimates of the intermittency exponent μ (0.3–0.8) have been obtained from the inertial range behaviour of the spectra of the pseudo dissipation $\phi_{\tilde{\epsilon}} \sim k_1^{-1+\mu}$ (Gibson et al., 1970; Wyngaard and Tennekes, 1970; Nelkin, 1981; Antonia et al., 1982). The present $\phi_{(\partial u/\partial x)^2}$ for $R_\lambda = 260$ shows a fairly broad range of power-law behaviour in the inertial range, consistent with $\mu = 0.65$. However, a distinct power-law behaviour cannot be clearly observed in $\phi_{(\partial u/\partial y)^2}$.

AUTOCORRELATION OF $(\partial u/\partial y)^2$

Figure 4 shows distributions of G_α ($\equiv <\alpha(x)\alpha(x+r)> / <\alpha>^2$), where $\alpha = (\partial u/\partial x)^2$ and $(\partial u/\partial y)^2$ for $R_\lambda = 260$. This data set is used here since there exists an inertial range $30 \leq r^* \leq 200$, the extent of which is inferred from the behaviour of the second order velocity structure function of u (not shown). The present value of the Kolmogorov constant C ($\simeq 2.0$) is in reasonable agreement with other laboratory and atmospheric data (Yaglom, 1981). In the tail end of the inertial range, the distributions of both G_α are almost identical. For small separations, the magnitudes of $G_{(\partial u/\partial y)^2}$ are significantly larger than those of $G_{(\partial u/\partial x)^2}$, consistent with the high wavenumber behaviour of $\phi_{(\partial u/\partial y)^2}$ and $\phi_{(\partial u/\partial x)^2}$ (Figure 3). As $r^* \to 0$, the magnitude of G_α approaches that of F_α. Using $G_{\tilde{\epsilon}} \sim (L/r)^\mu$ for $\eta \ll r \ll L$, where L is a macroscale of the flow, a value of about 0.2 for μ has been obtained for laboratory and atmospheric flows for high Reynolds numbers (e.g. Antonia et al., 1982; Sreenivasan and Kailasnath, 1993; Praskovsky and Oncley, 1994). The present estimate of μ_α, obtained from the range of r where $\mu_\alpha = -d(\log G_\alpha)/d(\log r^*)$ is approximately constant, is 0.11 ±0.01, and 0.12 ±0.01 for $\alpha = (\partial u/\partial x)^2$ and $(\partial u/\partial y)^2$ respectively. The relatively smaller value of present $\mu_{(\partial u/\partial x)^2}$ can be partly attributed to the relatively small value of R_λ.

CONCLUSION

Measurements of the statistics of $\partial u/\partial y$ in the wake of a circular cylinder over a moderate range of R_λ (60–260) indicate important differences from those of $\partial u/\partial x$ as well as some similarities. In particular, unlike $p_{\partial u/\partial x}$, $p_{\partial u/\partial y}$ is nearly symmetrical and has long exponential tails, which evolve with R_λ. The rate of increase of $F_{\partial u/\partial y}$ is higher than that of $F_{\partial u/\partial x}$. Moreover, the shapes of $\phi_{(\partial u/\partial y)^2}$ and $\phi_{(\partial u/\partial x)^2}$ are significantly different from each other. While $\phi_{(\partial u/\partial x)^2}$ exhibit a power-law dependence in the inertial range ($R_\lambda = 260$), no such behaviour can be clearly observed for $\phi_{(\partial u/\partial y)^2}$. The estimates for μ_α for $\alpha = (\partial u/\partial y)^2$ and $(\partial u/\partial x)^2$ obtained from the distributions of G_α, are 0.11 $\pm$0.01 and 0.12 $\pm$0.01 respectively.

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