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REDUCTION OF FREE STREAM TURBULENCE IN THE TEST SECTION OF A CRYOGENIC WIND TUNNEL

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ABSTRACT

Observed unexpectedly high turbulence intensities in the test section at cold runs have indicated a drawback of the cryogenic technology. The present theoretical study suggests two options of lowering the turbulence intensity in the test section at a cold run so that measurements performed on a model are applicable to the full-scale structure.

INTRODUCTION

The turbulence intensity is usually defined as the ratio of the r.m.s. velocity fluctuation of the streamwise component to the magnitude of the mean velocity. The turbulence intensity of the oncoming stream in the test section, Tu, is a very important parameter in measurements in the KKK(Kryo-Kanal Köln) and in the ETW(European Transonic Wind Tunnel). The atmospheric pressure cryogenic wind tunnel KKK- designed for low speed testing -is a conversion of a conventional closed-circuit fan-driven atmospheric pressure tunnel built at DLR Köln. The cryogenic pressure tunnel ETW is designed to reach close to full-scale flight Reynolds numbers on transonic transport aircraft models. Figure 1 shows an outline of a section of the ETW. The turbulence intensity Tu determines the degree to which measurements performed on a model can be applied to the full-scale structure. It is necessary to design wind tunnels of low turbulence intensity if model measurements are to be applicable to the design of full-scale aircraft/6/. Figure 2 shows the turbulence intensity Tu as a function of the magnitude of the mean velocity of the oncoming stream for two values of the temperature of the working fluid /9/. Results like those shown in figure 2 indicate a drawback of the cryogenic technology. The control of the turbulence level at the start of the settling chamber by means of fine-mesh screens seems to be sufficiently well understood /6,9/. The present study focuses solely on the following open issues concerning the ETW operating at subsonic test-section Mach numbers:

A. Consider the ratio of the r.m.s. velocity fluctuation of the streamwise component at the start of the test section to the r.m.s. velocity fluctuation of the streamwise component at the start of the settling chamber. What are the physical reasons for the departure of this ratio at a cold run from the corresponding ratio at a run at normal temperature?

B. Can the turbulence intensity in the test section at a cold run be lowered so that the measurements performed on a model are applicable to the full-scale structure?

SETTLING CHAMBER

The present brief account focuses solely on two sections of the tunnel ahead of the test section: the nozzle and the settling chamber. The flow in the settling chamber is considered a Newtonian fluid flow of constant density ρ and constant kinematic viscosity ν; turbulent fluctuations of an external force appearing in an inertial frame of reference are excluded. The turbulent kinetic energy equation then becomes in Cartesian tensor notation /4/

$$\partial e/\partial t + V_k \partial e/\partial x_k = S_{ik}(\Sigma_{ik}/\rho) - \partial(J_k/\rho)/\partial x_k - \varepsilon , \qquad (1)$$

where $t, x_k, e, \varepsilon, V_k, J_k, S_{ik}$ and Σ_{ik} are, respectively, time, Cartesian coordinate, turbulent kinetic energy, dissipation of turbulent energy, Cartesian component of mean velocity, of turbulent energy flux, of Reynolds stress, and of mean strain rate. Except for a wall zone, the mean-velocity field is a steady homogeneous parallel stream. The turbulence in this stream can be considered isotropic, and homogeneous in planes perpendicular to the tunnel axis. e is independent of time. A rigorous argument that the diffusion term is negligible is given following eq. (13). The x_1-axis collapses with the tunnel axis, $x_1=0$ at the start of the settling chamber, $x_1=x_{1d}$ at the start of the nozzle. The balance equation (1) in the homogeneous parallel stream then simplifies to

$$de/dx_1 = -ae \ , \ a = (10\nu)/(\lambda^2 V_1(0)) \tag{2}$$

where λ is the Taylor microscale. If a in eq.(2) were known the relationship (2) would be come an ordinary differential equation for e. Its general solution can be written

$$e = k \exp(-\int_0^{x_1} a(z)dz) \quad , \quad 0 \leq x_1 \leq x_{1d} \tag{3}$$

where z is a dummy variable for x_1. The arbitrary constant is specified by requiring k = e(0).

NOZZLE

A modelled form of the balance equation for the turbulent kinetic energy in Favre-averaged form in steady compressible flow at high turbulence Reynolds numbers and without external-force effects is in Cartesian tensor notation /1/

$$V_i \partial e/\partial x_i = S_{ik}(\Sigma_{ik}/\rho) - (1/\rho)\partial J_i/\partial x_i - \varepsilon - (C_t e^2/\rho^2 \sigma_t \varepsilon)(\partial \rho/\partial x_i)(\partial p/\partial x_i), \ -J_i = (C_t \rho e^2/\sigma_e \varepsilon)\frac{\partial e}{\partial x_i} \tag{4}$$

where e, V_i, S_{ik} are, respectively ,-in Favre-averaged form- turbulent kinetic energy, x_i-component of fluid velocity, Cartesian component of Reynolds stress. Σ_{ik} and ε denote Cartesian component of the strain rate of the Favre-averaged-velocity field, the (not negative) viscous dissipation function(cf. /3/). ρ and p are, respectively , (unweighted) ensemble-averaged density and static pressure. The coefficients C_t ,σ_e and σ_t are expected to be of physical order unity. In the following we put $x_1=x$, $V_1=V$, $-S_{11}/\rho=s$, $J_1=J$. The quasi-one-dimensional equivalent of eq.(4),

$$(V/2)ds/dx = (1/\rho)d\left[((C_t \rho e^2)/(\sigma_e \varepsilon))(de/dx)\right]/dx - sdV/dx + Q(x) \tag{5}$$

with

$$Q(x) = -(V/2)d(2e-s)/dx - \varepsilon - ((C_t e^2)/(\rho^2 \sigma_t \varepsilon))(d\rho/dx)(dp/dx) \ , \tag{6}$$

is adopted in a streamtube along the tunnel axis. A rigorous argument that the diffusion term in eq.(5) is negligible is given following eq.(13). The streamtube relation (5) then becomes

$$(V/2)ds/dx = -sdV/dx + Q(x) \ . \tag{7}$$

If Q(x) and V(x) were known eq.(7) would be ordinary differential equation for s. A formal representation of its general solution is

$$s = (V(x_d)/V(x))^2 \ (C + \int_{x_d}^{x} 2Q(z) \ (V(z)/(V(x_d))^2)dz), \ x_d \leq x \leq x_t \tag{8}$$

where $x=x_t$ at the start of the test section. The arbitrary constant C is determined by requiring $s(x_d)=C$. It follows from eqs. (3),(8) and

$$\int_{x_d}^{x} Q(z)V(z)dz < 0 \quad \text{for} \quad x_d < x \leq x_t \tag{9}$$

that

$$s(x)/s(0) < (V(x_d)/V(x))^2 \quad , \quad x_d \leq x \leq x_t \ . \tag{10}$$

A rigorous argument that the inequality (9) holds is given following the inequality (12). The velocity ratio in the inequality (10) can be replaced by a product of an area and density ratio with the aid of the streamtube approximation of the continuity equation for the nozzle flow,

$$\rho(x_d)V(x_d)A(x_d) = \rho(x)V(x)A(x) \quad , \tag{11}$$

where A(x) is the nozzle-area distribution. We obtain

$$s(x)/s(0) < ((\rho(x)/\rho(x_d))(A(x)/A(x_d)))^2 \quad , \quad x_d \leq x \leq x_t \quad . \tag{12}$$

The well-known streamtube relation between the local fluid density and the local Mach number for steady, continuous, nonviscous, nonconducting, nondiffusing flow of an ideal gas of constant specific heats in the absence of external forces yields: $\rho(x_t)/\rho(x_d)$ is practically determined by the test-section Mach number at $x=x_t$ if the settling-chamber Mach number is less than 0.1. Well-known streamtube relations for steady, continuous, nonviscous nonconducting, nondiffusing flow of an ideal gas of constant specific heats in the absence of external forces, and eq.(11) lead to dp/dx, $d\rho/dx \nless 0$ in the contraction. Experiments indicate $d(2e-s)/dx \nless 0$ in the contraction , at least if density fluctuations are insignificant(cf. /6/).

Introduce a one-parameter family of mean flows through the settling chamber and the nozzle , feasible in the ETW. The parameter is the Reynolds number

$$Re = (V(x_t)\rho(x_t)W)/\mu(x_t) \tag{13}$$

with the reference length W, $p(x_t)$, and the Mach number

$$M = V(x_t)/(\gamma(R/m)T(x_t))^{0.5} \qquad \text{where } T = (pm)/(\rho R)$$

fixed. γ ,m,R, μ ,and T denote, respectively , ratio of specific heats, mean molecular weight, universal gas constant, ensemble-averaged viscosity and temperature. The mean flow of a cold run and of a run at normal temperature are included in the family. Appropriate outer expansions - in the sense of the method of matched asymptotic expansions -(cf. /8/) are of the form

$$G = G_0 + o[1] \quad , \quad J/(\rho(x_d)V(x_d)s(x_d)) = o[1] \tag{14}$$

in the limit process

$$Re \to \infty \text{ with position, } M,\ p(x_t),\ W \text{ fixed.} \tag{15}$$

The subscript 0 denotes outer limit -different from zero and finite. The symbol G stands for $s/s(x_d)$, $e/e(0)$, $q = (2QV)/(s(x_d)V^2(x_d))$, $V/V(x_t)$, $s(x_d)/e(0)$; $a = \varepsilon/(V(0)e)$ for $0 \leq x \leq x_d$. Statement (14) relies on the assumption that $V_1 \partial e/\partial x_1$ and ε in eq.(4) are of the same mathematical order, and the Reynolds stress in the Favre-averaged momentum equation tends to zero in the limit process (15).

In view of statement (14),(15): All terms of eq.(4) - in the homogeneous parallel stream in the settling chamber - multiplied by $1/(e(0)V(0))$, and all terms in eq.(5) multiplied by $1/(s(x_d)V(x_d))$ are O[1](i.e. different from zero and finite) in the limit pro- (15) except for the diffusion terms which are o[1] in the limit process (15).

The outer expansion of a(x) and of q(x) can be formally written:

$$a(x)=a_0(x)+a_1(x)g(Re)+o[g(Re)]\ ,a_0>0 \quad (16)\ ,\quad q(x)=q_0(x)+q_1(x)h(Re)+o[h(Re)] \tag{17}$$

in the limit process (15). The set of coefficients in eqs. (16),(17) for a cold run and for a run at normal temperature agree. There is no need to specify the gauge functions g(Re),h(Re) in the present study.

An immediate consequence of eqs. (3) (8),and (11) for the nozzle flow is

$$\ln(s(x)/s(0)) = -\int_0^{x_d} a(z)dz + 2\ln((\rho(x)A(x))/(\rho(x_d)A(x_d))) + \ln H(x) + \ln(1+B(x)) \tag{18}$$

for $x_d \leq x \leq x_t$, where

$$B(x) = \int_{x_d}^{x} (q(z) - q_0(z))dz(H(x))^{-1}\ ,\quad H(x) = 1 + \int_{x_d}^{x} q_0(z)dz. \tag{19}$$

At sufficiently high Reynolds number Re, we can write approximately $\ln(1+B(x))=B(x)$. (20)
In view of eqs. (18) to (20) and (16),(17) we find

$$\ln\frac{s(x)}{s(0)} = -\int_0^{x_d} a_0(z)dz + 2\ln\frac{\rho(x)A(x)}{\rho(x_d)A(x_d)} + \ln H(x) + \frac{h(Re)}{H(x)}\int_{x_d}^{x} q_1(z)dz - g(Re)\int_0^{x_d} a_1(z)dz\ , \tag{21}$$

$x_d \leq x \leq x_t$, if higher order terms o[g(Re)] and o[h(Re)] are disregarded. An immediate consequence of eq.(21) is

$$\ln((s(x)/s(0))_C/(s(x)/s(0))_N) = \frac{1}{H(x)}\int_{x_d}^{x} q_1(z)dz(h(Re_C)-h(Re_N))-(g(Re_C)-g(Re_N))\int_0^{x} a_1(z)dz\ , \tag{22}$$

$x_d \leq x \leq x_t$, where the subscripts C and N denote conditions evaluated at a cold run and at a run at normal temperature, respectively.

Let the superscript ' refer to a slightly modified ETW. The modification consists in the installation of n additional screens between the honeycomb and the start of the settling chamber(at x=0). All screens installed are of the same type, tunnel/model geometry unaltered. Experience suggests(cf. /10/) that λ^2/ν is - for given (single) screen geometry - a function solely of x and V(0) but not of n. To the streamtube approximation , the flow speed V(0) can be can be expressed in terms of $V(x_t)$ and the ratio

$$(\rho(x_t)A(x_t))/(\rho(x_d)A(x_d)) \text{ where } \rho(x_t)/\rho(x_d) \text{ is determined by M.}$$

Hence $a' \approx a$ for for $M=M'$, $T(x_t)=T'(x_t)$. In view of eq.(18) and the corresponding relationship for the slightly modified ETW with $M = M', T(x_t) = T'(x_t)$ we find

$$(s(x_t)/s(0))'/(s(x_t)/s(0)) = 1 + (1+\int_{x_d}^{x_t} q(z)dz)^{-1}\int_{x_d}^{x_t}(q'(z) - q(z))dz \quad . \tag{23}$$

Consider the case where the numerator and the denominator on the left-hand side of eq. (23) refer to a cold run in the slightly modified ETW and in the original ETW , respectively , $M=M'$, $p(x_t) = p'(x_t)$, $T(x_t) = T'(x_t)$. In this case the departure of $q'(z)$ from $q(z)$ in eq.(23) is solely due to the installation of n additional screens. The empirical formula (cf. /6/)

$$s'(0) = (\ 1 + c(Re)\ )^{-n} s(0)\ , \tag{24}$$

where the resistance coefficient of a single screen,c,is a function solely of Re for a given W,M ,screen and tunnel geometry, does not hold for arbitrarily large n. Note that the fine-mesh screens themselves generate fine-grained turbulence. The resistance coefficient c tends to zero as $Re \to \infty$ with W and M fixed(cf. /9/). This statement and eq.(24) suggest that

$$s'(x_t)/s(x_t)\ ,\ s'(0)/s(0) \to 1 \text{ as } Re \to \infty \text{ with } W\ ,\ M\ ,\ p(x_t) \text{ fixed} \tag{25}$$

in the case under consideration. In view of eqs. (23) and (25) we find

$$(\int_{x_d}^{x_t}(q'(z) - q(z))dz)/(1 + \int_{x_d}^{x_t} q(z)dz) \to 0 \text{ in the limit process (15)}\ , \tag{26}$$

in the case under consideration. Hence, for sufficiently high Re, the right-hand side of eq.(23) is close to 1 in the case under consideration. Combination of eq.(23) and eq.(24) then leads to

$$Tu'(x_t) = (\ 1 + c(Re)\ )^{-n/2}\ Tu(x_t)\ . \tag{27}$$

Tu refers to a cold run at pressure $p(x_t)$, Mach number M,temperature $T(x_t)$ in the original ETW, Tu' refers to a cold run at $p'(x_t)=p(x_t)$, $M'=M$, $T'(x_t)=T(x_t)$ in the slightly modified ETW. Formula (27) formally holds for the KKK(cf. /5/). "Mean" in the sense of Favre-averaged in the definition of Tu applies to (27) for the ETW, in contrast , "mean" in the sense of unweighted ensemble-averaged in the definition of Tu applies to the corresponding formula for the KKK.

CONCLUDING REMARKS

The present study suggests the following answers to the questions put in the in introduction:

A. In view of the result (22): The departure of this ratio at a cold run at $M,p(x_t)$ from the corresponding ratio at a run at normal temperature (and at the same $M,p(x_t)$) is solely due to the dependence of the product viscosity times sound speed upon the absolute temperature.

B. The result (12) suggests two options of lowering the turbulence intensity in the test section at a cold run below an adequate upper bound:

1. Lower s(0) accordingly by installing an adequate device of screens.

2. Reduce the nozzle-area ratio $A(x_t)/A(x_d)$ accordingly.

Table 1 shows the variation of $r = Tu'(x_t)/Tu(x_t)$ for various n after eq.(27) for c(Re) = 0.68.

Wind tunnel test data, in the form of dimensionless force and moment coefficients, are functions of a lengthy list of tunnel/model related parameters of various levels of importance in terms of their influence on aerodynamic behaviour. A recent review /2/ of the cryogenic wind tunnel focuses solely on the Reynolds number issue. The present study illustrates the concern over the Tu issue and then moves towards its solution for the ETW. Finally, it is worth calling attention to the issue of model support influence /7/.

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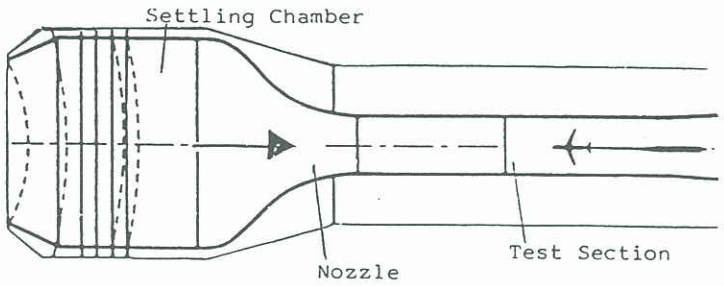

FIG. 1 OUTLINE OF A SECTION OF THE ETW .

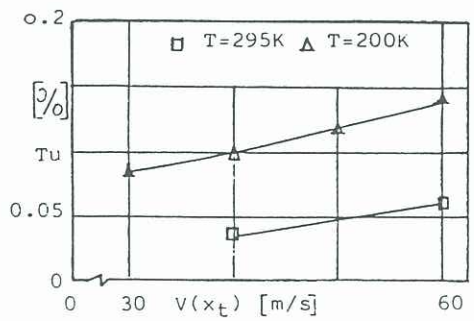

FIG. 2 ILLUSTRATION OF THE SENSITIVITY OF THE TURBULENCE INTENSITY IN THE TEST SECTION OF THE KKK TO THE FLUID TEMPERATURE T. $V(x_t)$,MAGNITUDE OF THE MEAN VELOCITY OF THE ONCOMING STREAM.

TABLE 1 REDUCTION OF THE TURBULENCE INTENSITY IN THE KKK BY INSTALLATION OF n ADDITIONAL SCREENS.

n	1	2	3	4	5
r	0.771	0.595	0.459	0.354	0.273
$V(x_t)$=80m/s , T=100K					