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AN INVESTIGATION OF RENORMALIZATION GROUP BASED ALGEBRAIC TURBULENCE MODEL

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ABSTRACT

An algebraic eddy viscosity model based on Renormalization Group (RNG) formulation with modifications for a flow with separation and wake regions is considered. This model is tested for a flat plate, and then employed for an Axisymmetric Transonic flow around a Secant Ogive Projectile. First the length scale is properly modified for the inner and outer regions of the boundary layer of flat plate flow. Then two new length scales more appropriate for separation and afterbody wake regions are proposed. Finally a more accurate procedure for determination of boundary layer thickness is presented

INTRODUCTION:

To compute turbulent flows there has always been a need to find an appropriate model of turbulence with a desirable degree of accuracy and efficiency. From this point of view it is common to use an algebraic model, particularly the Baldwin-Lomax which is based on Prandtl's mixing theory. However, this model is empirical and has many constants. Recently with the advent of Renormalization Group (RNG) theory, Yakhot and Orzac(1986) developed a new algebraic turbulence model. This formulation is based on the assumption of homogenous turbulent flow and validity of Kolomogrov law. It may be postulated that in the inertial range, this model is completely general. However it needs an appropriate length scale to bring out properly the effects of geometry and boundary conditions. This model has been used by Martinelli and Yakhot (1989) to compute the two dimensional flow around an airfoil at the angle of attack with good results. Recently Kirtley (1992) employed this model successfully for three dimensional turbomachinary flow in a low speed axial compressor rotor. In the present work RNG model is first tested for a flat plate and some modifications for the near wall and outer regions of boundary layer are proposed. Then a new method for determination of boundary layer thickness, which is more accurate for complex flows, is derived. Finally it is applied to a complex transonic turbulent flow around an Axisymmetric Secant Projectile and some modifications in the model for separation and afterbody wake regions are also presented.

RENORMALIZATION GROUP BASED ALGEBRAIC TURBULENCE MODEL:

The eddy viscosity formulations based on RNG theory is rewritten as:

$$\nu=\nu_l\,(\,1+H(\mathrm{X})\,)^{\frac{1}{3}}\quad,\qquad \mathrm{X}=\frac{\epsilon}{\nu_l^3}L^4-C \tag{1}$$

where H(X) is the Heaviside function, υ_l is physical viscosity, L is the common mixing length ,ϵ is the turbulence dissipation rate in equilibrium state and C is a constant about 100. We propose an expression for the mixing length L as below:

$$L=c_e c_\mu \delta.\tanh\left(\frac{kyV_d}{c_\mu\delta}\right)\ ,\qquad c_e=\left[\,1+\left(\frac{\partial u}{\partial y}\right)\Big/\left(\frac{\partial^2 u}{\partial y^2}\right)\,\right]^{0.25} \tag{2}$$

The values of k and c_μ which are obtained analytically and experimentally, are assumed to be 0.4 and 0.09, respectively. and V_d is

van Driest function. Kirtley (1992) also used a similar length scale equation without van Driest factor and c_e, and showed that it works well and leads to good results. However the present modification seems improve the results through V_d in the inner layer and c_e in the outer layer. It takes into account not only the thickness of boundary layer, but also its shape.

MODIFICATION FOR ADVERSE PRESSURE GRADIENT AND AFTERBODY WAKE

Simpson et al (1981) showed that in the regions upstream and downstream from separation point energy comes from diffusion and is not generated through the local mean strain. Driver (1985) also demonstrated that in separation regions convection and diffusion contributions of kinetic energy are substantially increased and the dissipation part of energy is considerably less than production. In addition Delery (1983) showed that near the shock foot there is a very large increase in turbulence kinetic energy. This shows that in the mentioned regions the equilibrium state and isotropic assumptions will not lead us to good results. Therefore this model should be modified. Simpson et al with the extensive experiments have provided sufficient data about length scale, eddy viscosity and F, the ratio of normal stress production to shear stress production for the points upstream and downstream of separation. For the regions downstream of separation Simpson et al indicate that F approaches 0.6 while Delery's results show a value of about 1 for the points after the maximum kinetic energy location. Therefore in the present work ϵ in Eqn 1 is weighted by the factor (1+F). Furthermore, based on using a combined viscosity equation for inner and outer layers and also by assuming the proportionality of viscosity with squared mixing length L, we can derive a length scale as below:

$$L=c_\mu\delta\left[\tanh\left(\frac{ky}{c_\mu\delta}\right)^2\right]^{1/2} \tag{3}$$

It seems that this length scale equation is more appropriate for separated flows, because it is less dependant on the distance from the walls. In the wake region downstream of the body the length scale is measured from the centre line and assumed to be proportional to the local wake width. For the points closer to the base wall with longitudinal reversed velocity the length scale is calculated by

$$L=c_l(\delta_d+|\delta_u-\delta_d|)\tanh\frac{y}{y_s} \tag{4}$$

where c_l is set to 0.27, y is the distance from the center line, y_s is the distance of minimum absolute value of longitudinal velocity component in every cross section from the center line (point S), and also δ_u and δ_d are the absolute distances of upper and lower wake boundaries measured from the point S.

DETERMINATION OF BOUNDARY LAYER THICKNESS

One of the main problems in using algebraic eddy viscosity models is the determination of boundary layer thickness δ. The Baldwin Lomax model (BLM) introduces the function F as $F(y)=y|\omega|.D$, where ω is vorticity and D is van Driest damping factor. In the original BLM model, δ is assumed to be the point with maximum F. However in some complex shear layers occurring in some three dimensional flows, with shocks or with sufficiently strong viscous-inviscid interactions, there is more than one peak in the distribution of F. Thus it is not clear which is the proper peak point. For example we may have a spurious maxima of F at the sublayer edge. In the sublayer we have $u^+=y^+$, so $F=y^+(\partial u^+/\partial y^+)D=y^+D$ and in the log law layer we have $u^+=1/k\ \mathrm{Ln}\ y^++C$, so $F^+=1/k=2.5$. So it is clearly shown that F^+_{max} of the viscous sublayer, which continues until about $y^+D=3.79$, is greater than F^+_{max} of the log law layer. Even for the wake like region, for which the velocity profile equation will be given later, we can find that F^+_{max} for a zero pressure gradient flow is about 4.84 Furthermore when there is a shock with a sufficient favourable pressure gradient, F^+_{max} will be less than 3.79. Therefore at some cross flow sections with the original BLM model we may be misled to a very small boundary layer thickness, which may be less than 10 times that of the adjacent boundary layers. To solve this problem Kirtley (1992) used the point with maximum negative slope of function F as the boundary layer edge. Although this technique is very good for flat plate flows, for complicated shear layers it may lead us to very high values of δ. Furthermore, to find δ, Degani and Schiff (1983) recommend to use the first well defined peak away from the wall, where F drops to less than 90 percent of its local maximum value. It seems that although this technique is better than the original and Kirtley's one, another procedure, explained below seems quite promising.

PROCEDURE FOR DETERMINATION OF BOUNDARY LAYER THICKNESS

Granville (1987) for flows with pressure gradients, in equilibrium or nonequilibrium states, on smooth or rough surfaces gave the velocity profile as below:

$$\frac{U-u}{u_w}=-A\ Ln(\eta)+B[1-3\eta^2+2\eta^3]-A[\eta^2-\eta^3] \tag{5}$$

where A and B are constants, $\eta=y/\delta$, $u_w=(\tau_w/\rho)^{1/2}$, and τ_w = wall shear stress. Furthermore Granville showed that y_{max} occurs at the point $y_{max}=C_{klch}.\delta$ where

$$C_{kleb} = \frac{4(1+6\Pi)}{9(1+4\Pi)}, \quad \Pi = \frac{B}{2A}$$

Π is a constant, which is zero for large favourable pressure gradients, and 0.48 for zero pressure gradient, and greater than 0.5 for adverse pressure gradients. Hence for a zero pressure gradient flow, the maximum of function F occurs at $y_{max}=0.59\ \delta$ and for a strong adverse pressure flow it occurs at $y_{max} = 0.64\ \delta$. This means depending on the specific conditions there is a specific relation between y_{max} and boundary layer thickness δ. Now we intrduce the following expression:

$$y_m = \left(\int_0^{y_m} F.dy\right) / \left(\int_0^{\delta} F.dy\right) \tag{6}$$

If we assume $F= y\,|\partial u/\partial y|$, from eqn (5) we can derive y_m

$$y_m = \frac{C_{kleb}[12-3C^3_{kleb}+(2+12\Pi)(4C^2_{kleb}-3C^3_{kleb})]}{(11+12\Pi)} \tag{7}$$

From this equation we find that for a flat plate $y_m = 0.62$ and for a flow with a strong adverse pressure ($\Pi=2$) $y_m = 0.61$ and in the separation region where $\Pi\to\infty$.and $C_{kleb}\to 2/3$ we have $y_m \to 0.592$. Accordingly, it seems that the factor y_m is not so dependant on the pressure gradient, so that we can assume $y_m = 0.6$ as a general factor in boundary layers. This means that the area under the curve F(y) until the point with maximum F is approximately a fraction 0.6 of its total area. With the aid of this concept we can easily evaluate δ in every cross section of flow. In order to calculate F instead of $\partial u/\partial y$ we have used vorticity ω.

SOLUTION PROCEDURE AND RESULTS

Eqn 1 and energy dissipation rate equation ϵ should be solved iteratively to find the value of ν at every iteration. The Navier-Stokes equation is solved using Runge-Kutta time stepping with Residual-Averaging acceleration technique.

Typical results for a flat plate at zero incidence are shown in Figs 1 - 3 for longitudinal velocity , C_f the skin friction coefficient and boundary layer development along the surface. The second case as reported by Lin et al (1993) is that of a Secant Ogive Projectile in a flow with M=0.97 and Re=13 x 10^6. Pressure coefficients , pressure and mach number contours around the body are given in Figs 4-6. The velocity profiles for the sections just before expansion, and upstream from separation point are shown in Fig 7. The velocity profiles in various afterbody cross sections are illustrated by Fig 8. They indicate that, although this eddy viscosity model is very simple and derived analytically, it is possible to get quite good results even for complex flows.

CONCLUSION

The work presents a modified, algebraic, RNG model of turbulence. Kirtley's length scale is amended in near wall and outer layer regions. Further a length scale which is more appropriate for the region in the vicinity of separation is presented. The effect of normal stresses which are more important in this region are taken into account, guided by past experimental results. Further a new procedure for determination of boundary layer thickness which is more suitable for complex flows, is derived. Through these modifications it is possible to achieve good results for complex geometries including shocks, separation regions, and afterbody wake flows.

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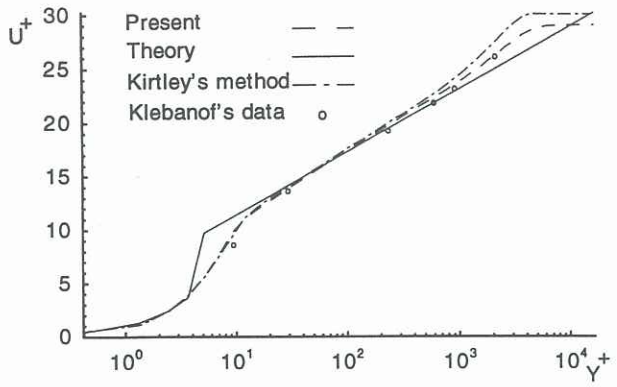

Fig-1 VELOCITY PROFILE IN WALL COORDINATES.

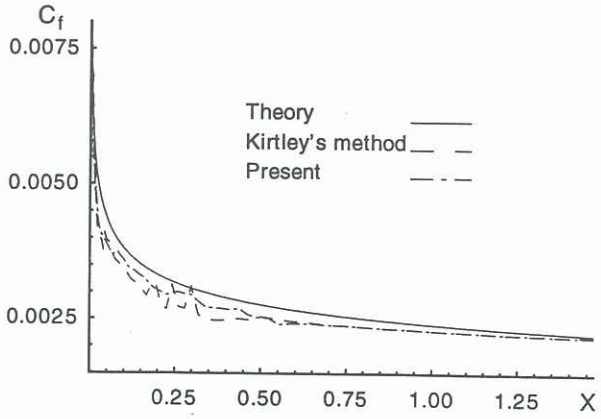

Fig-2 SKIN FRICTION FOR FLAT PLATE, Re=10^6

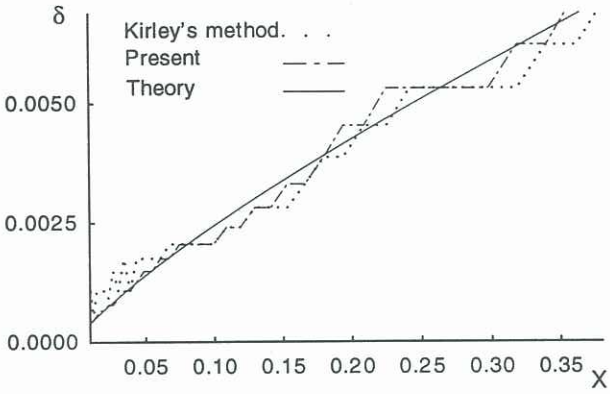

Fig-3 BOUNDARY LAYER THICKNESS DEVELOPMENT ALONG THE FLAT PLATE.

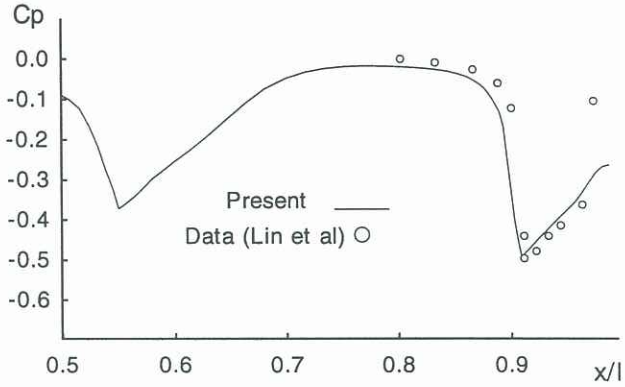

Fig-4 PRESSURE COEFFICIENTS ALONG THE PROJECTILE.

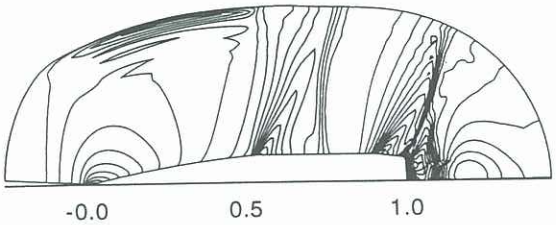

Fig-5 PRESSURE CONTOURS.

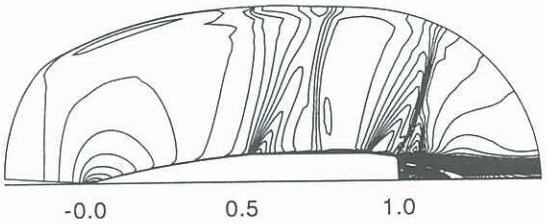

Fig-6 MACH NUMBER CONTOURS.

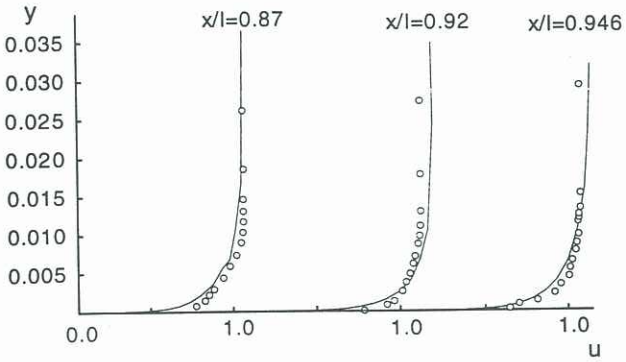

Fig-7 VELOCITY PROFILES NEAR SEPARATION POINT — PRESENT, ∘ Data (LIN AND CHIENG).

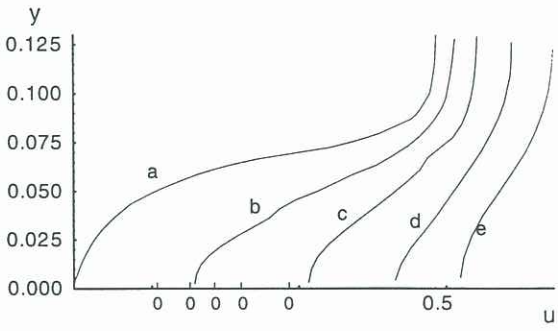

Fig-8 VELOCITY PROFILES AT VARIOUS AFTER -BODY POSITIONS., X/L= ***a***, 1.0015; ***b***, 1.0453; ***c***, 1.1174; ***d***, 1.215; ***e***, 1.349