MULTI-CELL FLOW IN DIFFERENTIALLY HEATED SLOTS

Paul W. Cleary
Division of Mathematics and Statistics
CSIRO
Melbourne, Victoria
AUSTRALIA

ABSTRACT

An important natural convective phenomenon is the break up of a single convective cell into multiple cells as the Grashof number (Gr) is increased beyond a critical value Gr_c. This is a common in cavities with large aspect ratios. This paper reports the results of a numerical study of the transition from a single cell flow (at low Gr) to multicellular flow (at higher Gr) for a long vertical differentially heated cavity filled with air, for an aspect ratio of 20. This problem has previously been studied previously by Lee and Korpela (1983). In general, our higher resolution simulations are in agreement. Important differences exist in the numbers of cells at the upper and lower end of the range of transition Gr and the value of Gr_c.

INTRODUCTION

The simulations were performed using the CSIRO finite element CFD package *Fastflo*. The algorithm involves the segregated solution of the Boussinesq equations for natural convection. The Navier-stokes part of the equations are solved using a very robust, accurate and stable operator-splitting method. The simulations are time accurate and fully transient. This is required to follow the transition from one flow state to another. See Cleary (1995) for details. 12×60 and 10×100 element structured quadrilateral meshes were used with appropriate timesteps. Extensive mesh refinement tests and other diagnostics, such as ratios of the Nu and overall heat conservation indicate that the solutions are highly accurate.

There are three aspects of the flow that are of interest. The structure of the final flows and the numbers of cells involved, for a range of Gr; the time dependent behaviour of the instability that leads to the multi-cell flows and its variation with Gr; and finally the behaviour of the heat transfer characteristics throughout these complex changes in the flow.

MULTI-CELL FLOWS AND THE INSTABILITY

For Gr$< 8,800$ the flow consists of a single elongated recirculation cell. Its formation is very rapid and the cell is stable. For $8,900 <$ Gr $< 50,000$ the single large cell still forms, but is unstable. After a long period of seemingly steady behaviour, there is a transition from the single cell to a multi-cell flow.

Figure 1 shows the transition process for Gr $= 11,000$. The early part of the flow evolution is indistinguishable from those belonging to lower Gr. Figure 1a (the first pair) shows the flow at $t = 45$, (the isotherms are on the left and the streamlines on the right of each pair). Here the left wall is heated and the right wall is cooled. The top and bottom are adiabatic. For this Gr, this flow pattern is unstable to infinitesimal perturbations that grow exponentially. The basic flow appears unchanged until around $t = 75$, (shown in Figure 1b). Here the innermost streamline shows slight indentations at about 10% of the slot height above and below the slot centerline. As the perturbations grow the indentations enlarge and others are formed above and below. This causes the formation of 5 very weak recirculation cells, symmetrically placed along the centerline of the slot. These are clearly visible by $t = 85$ and are shown in Figure 1c. A slight waviness has appeared in the previously straight and vertical isotherms. The flow around the cells strengthens for a short time and soon after becomes steady again. This flow pattern is stable. The steady final flow is shown in Figure 2b.

The shape of the isotherms near the ends of the slots and of most of the streamlines are little changed. The change is restricted to the central area. The region that was previously filled with unstable stratified stationary fluid with straight vertical isotherms now contains 5 stable elongated recirculation cells. The recirculation direction is the same in all the cells. This means that the region directly between the cells ex-

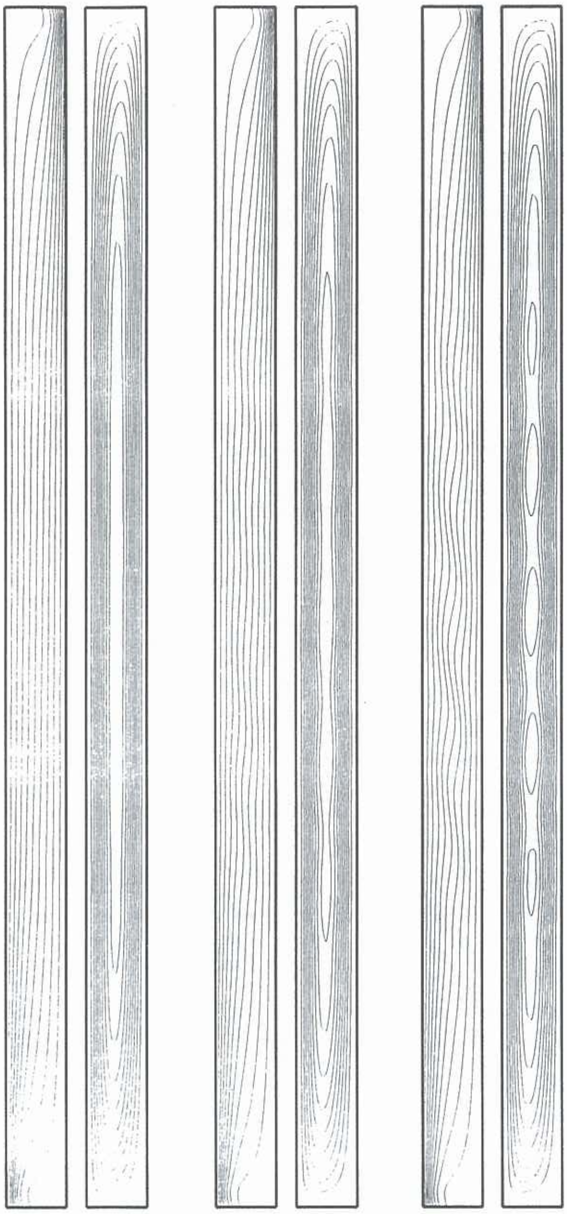

Figure 1: FLOW (TEMPERATURE AND STREAMFUNCTION) at (a) $t = 45$, (b) $t = 75$ and (c) $t = 85$.

periences quite strong shear. This acts to keep the cells some distance apart. These openings between cells are windows through which heat can pass preferentially.

In their earlier study, Lee and Korpela (1983), presented a relatively simple picture: There was a single transition from a single celled flow to a 5 cell flow with a critical Gr between 10,000 and 11,000. Our simulations are generally consistent with Lee and Korpela's but suggest that the situation is more complex.

Figure 2 shows the final steady flow patterns for increasing Gr. The important changes observed in the final flows, are summarised:

- the vertical length of the innermost streamline that encircles all the cells becomes progressively smaller.
- the number of cells begins at 6, and decreases to 5, then 4 and then 3.
- the cells become larger and stronger and move closer together.
- as they become larger they become tilted slightly to the right.
- the isotherms become increasingly wavy. For $\mathrm{Gr} = 25,000$ and above they become inverted near the centers of the cells.
- the isotherm patterns near the top and bottom walls are largely unchanged.
- the main recirculation is modified by the cells in the central area, but is largely unaffected near the boundaries. Even at $\mathrm{Gr} = 50,000$ more than 50% of the fluid still recirculates around the primary outer cell.

The time elapsed before the onset of the transition is comparatively short for $\mathrm{Gr} \geq 11,000$. It is always in the range 30-45. The time of the onset of the transition increases rapidly as the Gr approaches a critical value, of around 8,900. At $\mathrm{Gr} = 10,000$ the onset time doubles to 90 and at $\mathrm{Gr} = 9,000$ has risen to 235. The calculations of Lee and Korpela were very limited in their duration because of cpu restrictions. It is likely that these short calculations failed to detect this increase in onset time. Such a short duration cutoff for the instability to manifest would produce a critical Gr of between 10,000 and 11,000. This is where their simulations and the experiments predicted the transition should occur. Linear stability analysis using a conduction temperature profile suggests $\mathrm{Gr}_c = 8038$. This is a lower bound. The inclusion of a small vertical temperature gradient along the entire slot stabilises the flow so that the linear stability analysis gives a Gr_c in the range 10,000 and 11,000 (Lee and Korpela). The constant temperature gradient actually only appears in 80% of the slot (see Figure slotRa7810a) and this estimate over predicts the stabilisation and therefore the Gr_c. Our value of $\mathrm{Gr}_c = 8900$ lies within this range. Our lowest Gr transitions are to a 6 cell state, but for $\mathrm{Gr} > 9800$ the transition is to a 5 cell state. So our rapid onset transitions (for $\mathrm{Gr} > 11,000$) are therefore consistent with Lee and Korpela.

We define the activity of the flow DV as the norm of the velocity difference between timesteps integrated over the entire flow. The time dependent behaviour of the transition can be studied using this activity. For $\mathrm{Gr} < 8,900$, DV drops quickly from an high initial value and quickly becomes zero as the flow becomes steady. For higher Gr, the early flow activity is indistinguishable. As the multi-cell perturbation begins to grow at the transition onset time

Figure 2: ASYMPTOTIC FLOW PATTERNS (TEMPERATURE AND STREAMFUNCTION) (a) Gr $= 9,600$, 6 CELLS, (b) Gr $= 11,000$, 5 CELLS, (c) Gr $= 12,000$, 5 CELLS, (d) Gr $= 25,000$, 3 CELLS AND (e) Gr $= 50,000$ 3 CELLS

then the activity DV increases smoothly from zero. Between this time and the peak in DV the multi-cell flow in the central area continues to grow. The cells become established near the peak of DV and are then able to transfer heat from one side to the other. The new flow settles down and the activity DV vanishes again. As Gr increases the onset time becomes earlier, the transition becomes stronger and takes longer and the peak activity increases. At around 25,000 a single frequency oscillation appears in the flow and only dies out after the transition has finished. At Gr $= 50,000$ the flow behaves as if it is near a resonance and oscillates between two different flow states with multiple timescales for the motion. Note that Lee and Korpela did not examine flows for Gr $> 25,000$.

HEAT TRANSFER CHARACTERISTICS

The time history of the Nu (dimensionless heat transfer rate) for Gr $= 12,000$ is shown in Figure 3. Initially there is a rapid drop and a small rebound as the single cell flow pattern develops ($t < 10$). It is then constant until $t = 67$ when it begins to increase. This corresponds to the peak in DV. The cells do not all form simultaneously, but rather starting from the outer ends. So the heat transfer increases steadily as each pair of cells becomes established. As the new flow settles down the Nu becomes constant again, but at a higher level. This clearly shows that the 5 cell state has a higher heat transfer rate (1.3584) compared to the single cell state (1.3265). This represents an enhancement of the heat transfer rate by 2.40% for this Gr.

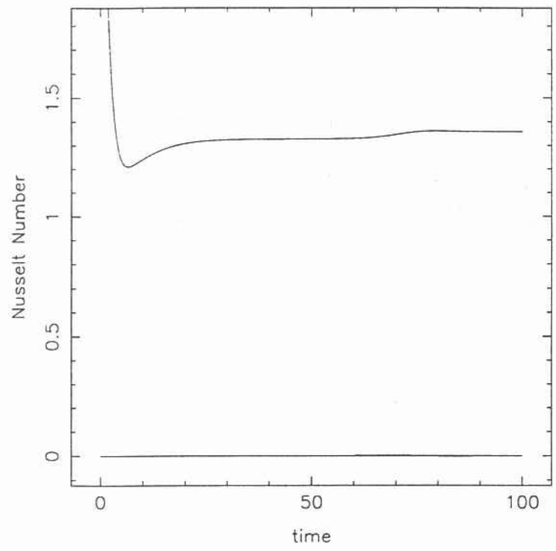

Figure 3: TIME SERIES OF THE NU FOR Gr = 12000

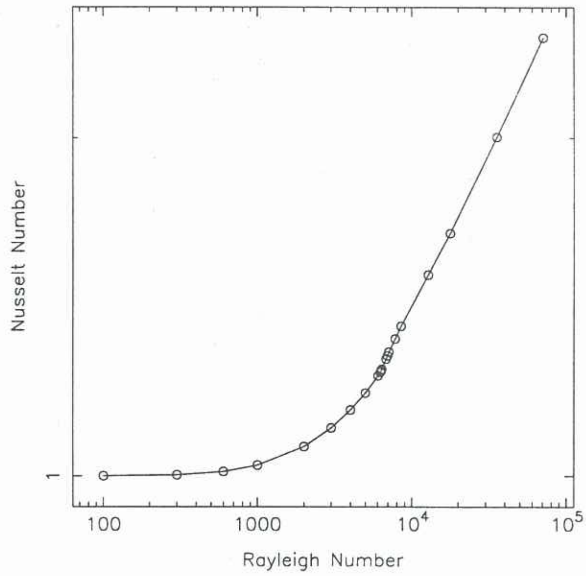

Figure 4: NU BEHAVIOUR WITH INCREASING RA

The multi-cell flow pattern has an enhanced heat transfer capacity because the cells act as a short circuit. In a single cell flow, the heat is conducted out into the boundary layer and is then convected all the way up the long side, across the top and much of the way down the cold side before being completely absorbed. The heat path is very long, so the thermal resistance is quite high. In the multi-cell flow, some of the heat diffuses from the wall boundary layer and the nearby vertical jet of hot air into the air in the cells. It is then convected around to the side by the recirculating motion in the cells. From here it diffuses into the opposite boundary layer to be absorbed by the cold wall. The heat paths from one wall to the other around the cells are appreciably shorter than around the perimeter and therefore have lower thermal resistance. As the Gr increases and the cells become stronger progressively more heat follows these shorter paths. The majority of the flow and heat transport still occur in the main outer recirculation around the boundaries, so the enhancement to the heat transfer is only modest.

Figure 4 shows the behaviour of the Nusselt number for the final steady flow, with increasing Ra(=Gr Pr). For Gr<100 the heat transport is entirely conductive. For $300 < \text{Gr} < 5000$ the Nu begins to increase, slowly at first, but accelerating. Above Gr$= 5000$ the Nu growth quickly becomes linear with log Ra. At higher Gr there are additional transitions to 5, 4 and 3 cells. By Gr $= 100,000$ the transition has been inhibited entirely, leaving a single cell flow. Despite all these changes in the topological structure of the flow and the modest enhancement of the heat transfer by the multiple cells, the Nu curve shows only very small variations from a linear shape.

CONCLUSION

Our simulations show:

- For Gr$< 8,800$ the final flow consists of a single stable elongated recirculation cell.
- For $8,900 < \text{Gr} < 25,000$, the single cell flow pattern is unstable. After a period of apparently steady behaviour there is an abrupt transition to a multi-cell flow.
- For $25,000 \leq \text{Gr} < 50,000$ the intermediate single cell flow no longer develops. The flow evolves directly to its final state.
- This transition produces a modestly enhanced Nu and is accompanied by a surge in activity.
- The time taken for the transition to occur is short for Gr$> 11,000$. The onset time increases exponentially as Gr approaches the critical Gr, of around 8,900, from above.
- The lowest Gr transitions are to 6 cell flows and have extremely long onset times. For Gr> 9800 the transition is to a 5 cell state. For Gr$\geq 18,000$ the number of cells decreases to 4 and then 3. By Gr $= 100,000$ there is a single cell.
- For most of the common range, our results are consistent with those of Lee and Korpela. The differences in the critical Gr and 6 cell flows relate to the very long onset times that occur for lower Gr and which could not be detected by their short simulations.

REFERENCES

Cleary, P. W., 1995, "Numerical modelling of natural convection", Technical Report DMS - C 95/44, CSIRO Division of Mathematics and Statistics.

Lee, Y., and Korpela, S. A., 1983, "Multicellular natural convection in a vertical slot", *J. Fluid Mech.*, Vol. 126, pp. 91-121.