SAIL FORCE COEFFICIENTS AND OPTIMUM APPENDAGES FOR A SAILBOAT

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ABSTRACT

Analytical formulae are given for optimum sail lift coefficients for a close-hauled upright sailboat and the maximised net driving force. The analysis also shows that the rudder should have as great a span as is possible and share a high fraction of the total appendage side force.

INTRODUCTION

Modelling the performance of yachts and sailing dinghies in straight-line sailing is a complex science which is still not fully developed. Velocity Prediction Programs (VPPs) as currently used (e.g. Schlageter & Teeters, 1993) are a development of the MIT model (Kerwin, 1978) and use detailed information on hull forces derived from tank testing and CFD analysis, but use rather arbitrarily derived information on the sail force coefficients (e.g. Milgram, et al 1993). Interaction between hull and sail design is not directly catered for. The VPPs are also complex mathematically and it is not possible to obtain direct insights into design optimisation. Here, a simple model is presented for the net driving force on an upright sailboat in straight-line sailing and it is used to optimize performance and design parameters.

NET DRIVING FORCE

Figure 1 shows the forces on an upright sailboat for close-hauled sailing in a straight line. The heading of the boat lies between the apparent wind direction and the actual trajectory of the yacht. This is due to leeway. Forces have been resolved into lift components L_j which are normal to the oncoming fluid stream (not to the lifting surface) and drag components D_j which are parallel to the stream. All of the lift on the hull has been ascribed to the keel (subscript K) and rudder (subscript R), but a separate hull drag component (subscript H) is defined. The sail subscript S) is represented as a single untwisted airfoil but the analysis also applies to multiple sail rigs. The angle β is between the apparent wind of speed U_A and the boat trajectory. The boat speed through the water is V_B. The net driving force F in the trajectory direction is given by

$$F = L_S \sin\beta - D_S \cos\beta - D_K^i - D_R^i \qquad (1)$$

where $D_K{}^i$ and $D_R{}^i$ are the lift-induced components of the drag of the underwater appendages. This net driving force is available to overcome the zero-lift component of the drag of the hull and appendages and to accelerate the boat. We are interested in maximising F for any given U_A, V_B and β by correctly choosing the lift coefficient of the sails and appendage design parameters. The side forces are assumed to be in balance so that

$$L_S \cos\beta + D_S \sin\beta = L_K + L_R \qquad (2)$$

Lift and drag coefficients, C_{Lj} and C_{Dj} are defined in the usual manner with respect to reference plan-form areas, S_j, and drag for the lifting surfaces is assumed to have parabolic dependence on lift so that

$$C_{Dj} = C_{Dj}^o + C_{Lj}^2 /(\pi A_j^*),\ j = K, R \text{ or } S \qquad (3)$$

where

$$\frac{1}{\pi A_j^*} \equiv \frac{1}{\pi A_j} + d_j \qquad (4)$$

Here the A_j are the aspect ratios of the foils defined so

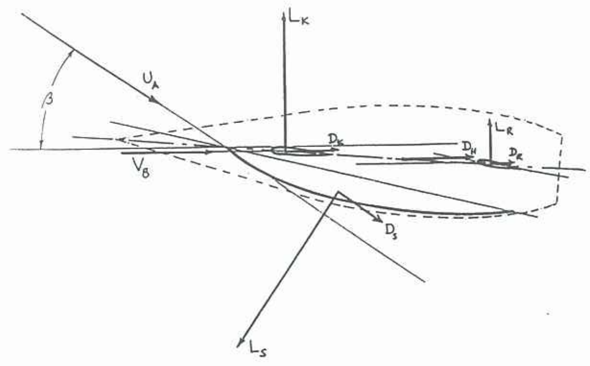

FIG 1. Forces on the foils of a sailboat.

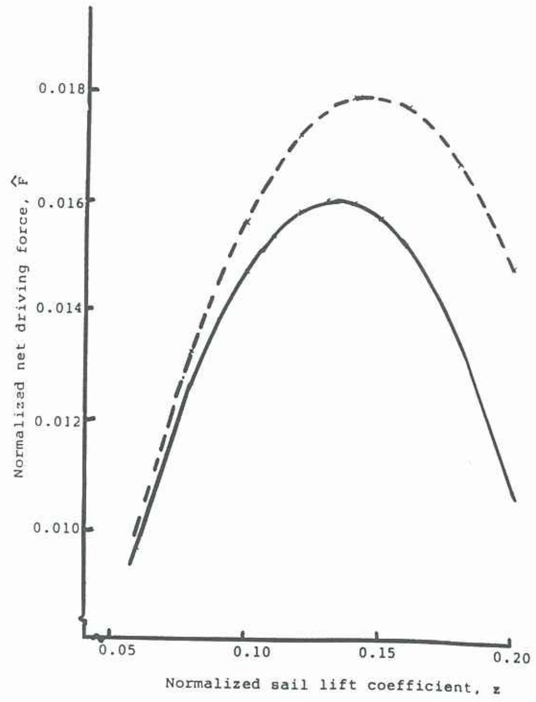

FIG 2. Normalized net driving force as a function of normalized sail lift coefficient, Eqn (10) for β = 18 deg AND $\hat{C}^o_{DS}$ = 0.005:
—— $\mathcal{F}T = 0.2$;
- - - - $\mathcal{F}T = 0.1$

as to give the rise in induced drag with lift coefficient and d_j is a coefficient that accounts for the rise in profile drag with lift coefficient. For the keel and rudder d_j can be obtained from two-dimensional section data, but it makes a small contribution and we take $A_K^* = A_K$ and $A_R^* = A_R$. For sails, best practice is for the lift coefficient to be changed by altering the camber or draft of the sail and adjusting the incidence to maintain ideal flow onto the sail at the leading edge. Highly cambered sails at their optimum incidence show much higher profile drag than sails with lower camber (Milgram, 1971, 1978), with d_j being found to be of order 0.04, which is not negligible.

Using (3) in (2) results in

$$C_{LK} = \mathcal{F}(A_K/A_s^*)[C_{LS}\cos\beta + C^o_{DS}\sin\beta + C^2_{LS}\sin\beta/(\pi A_s^*)]/(1 + by) \qquad (5)$$

$$\mathcal{F} \equiv \frac{\rho_a U_A^2 s_S^{*2}}{\rho_w V_B^2 s_K^{*2}}; \quad b = S_R/S_K \qquad (6,7)$$

$$y \equiv C_{LR}/C_{LK}; \quad s_j^* = (S_j A_j^*)^{1/2} \qquad (8,9)$$

where with the s_j^* being the effective span of the foil, the asterisk being dropped for the rudder and keel.

Using (5) and (3) in (1) results in

$$\begin{aligned}\hat{F} \equiv F/(\tfrac{1}{2}\pi\rho_a U_A^2 s_S^{*2}) &= -(\hat{C}^0_{DS}\cos\beta \\ &+ \mathcal{F}T(\hat{C}^0_{DS})^2\sin^2\beta) + \sin\beta(1 - 2\mathcal{F}T\hat{C}^0_{DS}\cos\beta)z \\ &- (\cos\beta + \mathcal{F}T\cos^2\beta + 2\mathcal{F}T\hat{C}^0_{DS}\sin^2\beta)z^2 \\ &- 2\mathcal{F}T\sin\beta\cos\beta z^3 - 2\mathcal{F}T\sin^2\beta z^4\end{aligned} \qquad (10)$$

where

$$z \equiv C_{LS}/(\pi A_S^*); \quad \hat{C}^0_{DS} \equiv C^0_{DS}/(\pi A_S^*) \qquad (11,12)$$

Also

$$T \equiv \frac{1 + ay^2}{(1 + by)^2}; \quad a = S_R A_K/(S_K A_R) \qquad (13,14)$$

Equation (10) gives a quartic equation for the normalized net driving force, $\hat{F}$, in terms of the normalized sail lift coefficient z. Figure 2 shows this for $\beta = 18$ deg and $\hat{C}^o_{DS} = 0.005$ and values of $\mathcal{F}T$ = 0.1 and 0.2. Typically $\mathcal{F}$ is of the order of 0.2 and T is a little less than unity. It is seen that there is a well-defined maximum to $\hat{F}$ giving an optimum value for the sail lift coefficient, C_{LS}. It is apparent that Eqn (10) can be used to give a theoretical value for this optimum lift coefficient. It is also apparent that reducing T can improve the net driving force. T depends on the appendage design and relative side force loading of the keel and rudder. Maximizing $\hat{F}$ for a given value of $\mathcal{F}$ and β should yield greater performance for the sailboat in both acceleration and equilibrium speed.

Optimum Sail Lift Coefficient

Taking the derivative of $\hat{F}$ with respect to z and setting it to zero yields a cubic equation in z which on solving for z gives the optimum lift coefficient to maximize $\hat{F}$. This is done with U_A, V_B and β held constant so that $\mathcal{F}$ is constant. It is assumed also that the variation of the sail lift coefficient is done so that

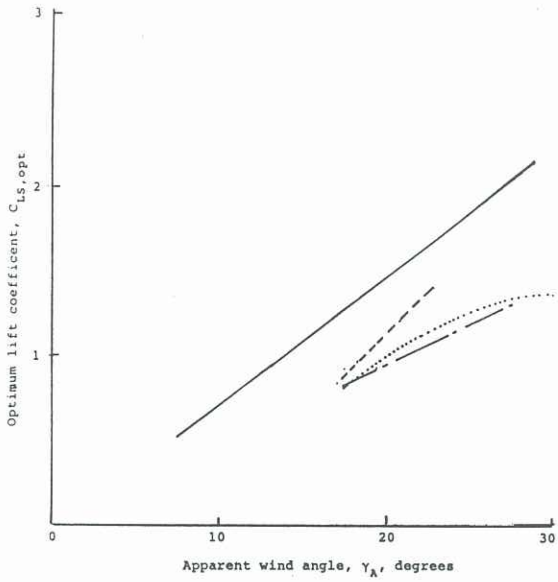

FIG 3. Maximum useable lift coefficients as a function of apparent wind angle:
——— Eqn (15);
- - - - - - Milgram et al (1993);
• • • • • • Masuyama et al (1993);
—•— Cat rig with requirement that leach of sail does not "hook" to windward (see text).

$y = C_{LR}/C_{LK}$ is not altered. We shall find the optimum for y later.

For $\beta < 25\ deg$ the term in z^4 can be neglected in Eqn (10) making the optimizing equation for z a quadratic. The radical expression in the solution of this quadratic can be expanded as a series and to sufficient accuracy we obtain expressions for the optimum lift coefficient

$$C_{LS,opt} \approx \frac{\pi A_S^* \tan\beta}{2(1 + \mathcal{F}T\cos\beta)} \left\{ 1 - \frac{3\mathcal{F}T\tan\beta\sin\beta}{2(1 + \mathcal{F}T\cos\beta)^2} \right\} \quad (15)$$

and the corresponding maximum in the driving force

$$F_{max} \approx \frac{\pi \rho_a U_A^2 s_S^{*2} \sin\beta\tan\beta}{8(1 + \mathcal{F}T\cos\beta)} \times \left\{ 1 - \frac{\mathcal{F}T\tan\beta\sin\beta}{(1 + \mathcal{F}T\cos\beta)^2} \right\} - D_S^0\cos\beta \quad (16)$$

where the D_S^0 is the zero-lift drag force of the sails. These formula predict with precision the optimum points shown in Fig 2.

A survey of two-dimensional (2-D) airfoil data (Milgram, 1971, 1978) for thin cambered foils at their ideal angle of incidence (taken where L/D is a maximum) indicates that the effect of camber on section drag coefficient contributes about 0.04 to $1/\pi A_S$ in Eqn (3). An effective aspect ratio of about 4 for induced drag contributes a further 0.08 giving a total of about 0.12, a value which is consistent with the sailing dynamometer data of Milgram et al (1993). The overall effective aspect ratio is then about 2.7. Using this and taking $\mathcal{F}T$ in Eqn (4) as 0.2 yields the optimum lift coefficients shown in Fig 3. They are plotted against apparent wind angle, γ_A which is β less leeway angle; the latter has been estimated as $\beta/8$. Also shown are the data reported by Milgram et al (1993) and values deduced from the force coefficients used by Masuyama et al (1993). It is concluded that much larger lift coefficients may be possible.

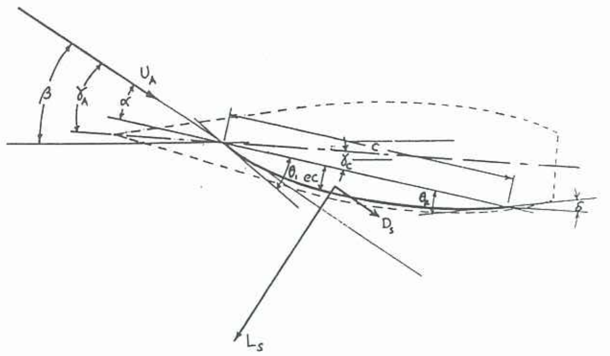

FIG 4. Definition of sail angles for a cat-rigged boat.

For thin 2-D cambered foils the data show that ideal angle of attack is at about 0.4e radians, where e is the camber or draft of the sail as a fraction of the chord; the lift coefficient for this ideal angle is about 12e. The angles, θ_1 and θ_2, that the leading and trailing edges of the sail make to the chord line are 4e radians for a circular arc profile, but depend on the section shape. Angles of attack for 3-D foils with the same lift coefficient are increased over those for 2-D by $C_{LS}/\pi A_S$ radians, where A_S is the effective aspect ratio for induced drag (Abbott & von Doenhoff, 1959). The geometrical relationships for cat-rigged boats (ie having a single sail) are shown in Fig 4. It is seen that the angle that the trailing edge makes to the centre-line of the boat, δ, is given by

$$\begin{aligned} \delta &= \theta_2 - \gamma_C = \theta_2 + \alpha - \gamma_A \\ &\approx \theta_2 + \left(\frac{0.4}{12} + \frac{1}{\pi A_i}\right) C_{LS} - \gamma_A \end{aligned} \quad (17)$$

where α is the angle of attack and γ_C the angle between the chord line and the boat centreline. If θ_2 is taken as 3e $\approx$ .25C_{LS} the old-timers requirement that $\delta \leq 0$ ('the leach should not "hook" to windward') yields $C_{LS} \leq 2.8\gamma_A$ for $A_S = 4$. This is shown on Fig 4. It is seen that it is similar to the limits on C_{LS} reported for sloop rigs with genoa jib and main sail (Milgram et al 1993 and Masuyama et al, 1993). Camber values much larger than normally used are needed to achieve the optimum lift coefficients of Eqn (15). This large camber decreases the boom angle needed to give ideal incidence, but it does give values of $\delta > 0$. Measurements taken from photographs of

1995 America's Cup yachts show $\delta \approx 10\ deg$ in light winds.

For sloop-rigged yachts there is some difficulty in achieving high C_{LS} at small values of β. In light winds efforts are made to maximise the lift coefficient. It is to be noted that the lift vector is normal to the wind and does not depend fundamentally on the orientation of the hull to the wind, only on the orientation of the sails. If lift coefficients of order 1.5 are achievable at γ_A of 25 deg they should also be achievable at γ_A of 18 deg, by rotating the boat relative to the wind, but not the sails. The boom comes up toward the weather gunwhale and the headstay needs to sag to leeward so that luffing is avoided with the high draft in the sails. Having the foot of the headstay adjustable to leeward by means of a traveller or other arrangement may be worthwhile.

OPTIMIZATION OF APPENDAGES

For constant appendage geometry, the function T may be treated as a function of y, the lift coefficient ratio (see Eqns 7, 11, 13 and 14). This function has a minimum at

$$y = y_{opt} = (C_{LR}/C_{LS})_{opt} = b/a = A_R/A_K \qquad (18)$$

The minimum value of T is obtained as

$$T_{min} = (1 + b^2/a)^{-1} = (1 + s_R^2/s_K^2)^{-1} \qquad (19)$$

It will not always be possible to operate at this optimum because of questions of the balance in yaw of the sailboat due to the rudder and keel forces involved. Some adjustment of this balance can be made by shifting the mast or otherwise changing the center of effort of the sails.

In optimising the geometry of the rudder and keel it seems important to consider this in the light of taking s_K as being fixed by other considerations. It is noted that s_K appears in the definition of $\mathscr{F}$ which is defined in Eqn (6). Often it will be maximised on its own account. Here we take it as fixed. We focus on the fraction of the total appendage lift force developed by the rudder, λ, as this is important in the overall design of the sailboat. We note

$$\lambda \equiv L_R/(L_K + L_R) = by/(1 + by) \qquad (20)$$

For conventional sailboats λ is 0.3 or lower. Some 1992 America's Cup yachts had twin steerable keels and for them λ would be near 0.5. It turns out that y can now be eliminated from Eqn (13) so that T becomes a function of λ and the rudder span:

$$T = (1 - \lambda)^2 + \lambda^2 s_K^2/s_R^2 \qquad (21)$$

This shows that the optimum design for the rudder is to have its effective span as large as is possible and for it to share a high fraction of the appendage lift. Most of the 1995 America's Cup yachts were exploring such designs.

CONCLUSIONS

Analysis of the net driving force yields algebraic formulae for the optimum sail lift coefficient for upwind sailing and for the maximised net driving force. It is found that sail coefficients currently in use are lower than optimum and efforts to set the sails at close-hauled wind angles with more camber should prove worthwhile in light winds. The analysis also shows that rudders should be as long in span as possible and should carry a large share of the side force. The analysis is restricted to the case of no heel.

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