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NUMERICAL ANALYSIS OF SUBSONIC FREE JETS ISSUING FROM SPOOL VALVE ORIFICES

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ABSTRACT

The main purpose of the present study is to investigate the influence of the compressibility on the jet flow from spool valve orifices by numerical analysis. In the analysis, SFC(Stream Function Coordinate)which has the velocity potential and the stream function as the independent variables is used. The method is one of the inverse formulation in fluid mechanics and can be applied to the problem with unknown shape of the physical boundary such as free surfaces. The basic assumptions in the present calculation are that the flow is two-dimensional, steady, inviscid and isentropic. The basic equations are led and the finite-difference solutions of the subsonic jet flow from spool valve orifices are obtained. In the results, the influence of the compressibility on the free surface profile and velocity, pressure and density distributions was obtained.

NOTATION

a : local sonic velocity
H : clearance between spool and sleeve
L : valve opening
M : local Mach number
p : pressure
q : velocity
u, v : velocity components in the x and y directions
W : jet width
x, y : Cartesian coordinates
θ : angle between the velocity and the x-axis
θ' : jet angle at the far downstream position
κ : ratio of specific heats
ξ : coordinate in the ϕ direction
ρ : density
ϕ : velocity potential
ψ : stream function
Superscript
$*$: non-dimensional quantity
Subscript
∞ : quantity on a free surface

INTRODUCTION

At the present time spool valves are universally used as a control valve for pneumatic and hydraulic power applications. The study of flow in the valves has been of considerable interest in a simulation of practical flow conditions.

In the early study, analysis of free jets issuing from spool valve orifices was performed by conformal mapping. In the analysis, the incompressible potential flow is assumed (Mises, 1917). Recently, the influence of the viscosity on the jet flow was studied by solving the Navier-Stokes equations and continuity equation for two-dimensional incompressible viscous flow numerically (Tsukiji and Takahashi, 1983). However the effect of a compressibility must be considered to simulate the jet flow in the pneumatic spool valves. Some theoretical studies were directed to compressible jet flow through nozzles. Due to the complicated geometry, compressible flow in pneumatic spool valves has not been studied.

The main purpose of the present investigation is to comprehend the influence of the compressibility on the jet flow issuing from spool valve orifices by numerical analysis. In the first stage of analysis assuming a two-dimensional, steady, inviscid and isentropic flow, the governing equations were derived by stream function - velocity potential coordinate system which is sometimes called SFC(Stream Function Coordinate) (Dulikravich, 1991a and 1991b, Huang and Dulikravich, 1986). The present method is one of the inverse formulation in fluid mechanics and can be applied to the flow problem with unknown shape of the physical boundary such as free surfaces. In the next, the finite-difference equations transformed from the governing equations were solved by a successive over relaxation scheme. Before the calculation for the compressible flow we confirmed reliability of calculated results in the case of the incompressible flow, by comparing those with the results obtained using conformal mapping. For the compressible flow, jet angles and jet width obtained were compared with the incompressible flow solutions and the influence of the compressibility on the jet shape became evident under conditions of subsonic flow. Furthermore, the density, pressure and velocity distributions were calculated for different freestream Mach numbers and positions of the spool. The influence of the freestream Mach number and the positions of the spool on the flow characteristics was investigated. In designing the pneumatic spool valve, the flow characteristics obtained in the present study are available. The present method is very useful for investigating compressible jet flow issuing from orifices of complex shapes.

MATHEMATICAL FORMULATION

The stream function ψ and the velocity potential ϕ are defined by Eqs. (1) ~ (4).

$$\frac{\partial \psi}{\partial x} = -\frac{\rho}{\rho_\infty} v \quad \cdots\cdots (1)$$

$$\frac{\partial \psi}{\partial y} = \frac{\rho}{\rho_\infty} u \quad \cdots\cdots (2)$$

$$u = \frac{\partial \phi}{\partial x} \quad \cdots\cdots (3)$$

$$v = \frac{\partial \phi}{\partial y} \quad \cdots\cdots (4)$$

where the subscript ∞ indicates free-stream values, ρ is density, and u and v are the velocity components in the x and y directions, respectively.

Continuity equation and irrotational equation in the ϕ - ψ coordinate are shown as follows :

$$\frac{\partial}{\partial \phi}(\ln q)+\frac{1}{\rho}\frac{\partial \rho}{\partial \phi}+\frac{\rho}{\rho_\infty}\frac{\partial \theta}{\partial \psi}=0 \tag{5}$$

$$\frac{\partial \theta}{\partial \phi}-\frac{\rho}{\rho_\infty}\frac{\partial}{\partial \psi}(\ln q)=0 \tag{6}$$

where q and θ are the velocity and the angle between the velocity vector and the x axis.

The energy equation is

$$p/\rho^{\kappa}=const. \tag{7}$$

in the case of the isentropic flow, where p and κ are pressure and ratio of specific heats.

From Eq.(7), we obtain the following :

$$\frac{dp}{d\rho}=\kappa\frac{p}{\rho}\equiv a^2 \tag{8}$$

where a is local sonic velocity.

Substitution of Eq.(8) into Eq.(7) yields

$$a=a_\infty\left(\frac{\rho}{\rho_\infty}\right)^{(\kappa-1)/2} \tag{9}$$

On the other hand, the Bernoulli equation without body forces can be written as

$$\frac{1}{2}q^2+\frac{a^2}{\kappa-1}=const. \tag{10}$$

From Eqs.(9) and (10), the following result is obtained :

$$\frac{1}{2}q^2+\frac{a^2_\infty}{\kappa-1}\left(\frac{\rho}{\rho_\infty}\right)^{\kappa-1}=\frac{1}{2}q^2_\infty+\frac{a^2_\infty}{\kappa-1} \tag{11}$$

The equations of motion are expressed as follows :

$$q\frac{\partial q}{\partial \phi}=-\frac{1}{\rho}\frac{\partial p}{\partial \phi} \tag{12}$$

$$q\frac{\partial q}{\partial \psi}=-\frac{1}{\rho}\frac{\partial p}{\partial \psi} \tag{13}$$

Substitution of Eqs.(8) and (12) into Eq.(5) yields

$$\left(1-M^2\right)\frac{\partial}{\partial \phi}(\ln q)+\frac{\rho}{\rho_\infty}\frac{\partial \theta}{\partial \psi}=0 \tag{14}$$

where M is Mach number, which is defined by

$$M\equiv q/a \tag{15}$$

Furthermore, eliminating θ from Eqs.(6) and (14) yields

$$\frac{\partial}{\partial \phi}\left[\frac{\rho_\infty}{\rho}\left(1-M^2\right)\frac{\partial}{\partial \phi}(\ln q)\right]+\frac{\partial}{\partial \psi}\left[\frac{\rho}{\rho_\infty}\frac{\partial}{\partial \psi}(\ln q)\right]=0 \tag{16}$$

In the same way, eliminating q yields

$$\frac{\partial}{\partial \phi}\left(\frac{\rho_\infty}{\rho}\frac{\partial \theta}{\partial \phi}\right)+\frac{\partial}{\partial \psi}\left(\frac{1}{1-M^2}\frac{\rho}{\rho_\infty}\frac{\partial \theta}{\partial \psi}\right)=0 \tag{17}$$

In the present calculation, q and θ are obtained by solving Eqs.(16) and (17), and the density ρ is calculated from Eq.(11). From Eq.(10) the sonic velocity a is given by :

$$a^2=\frac{\kappa-1}{2}\left(q^2_\infty-q^2\right)+a^2_\infty \tag{18}$$

NUMERICAL CALCULATION

In the present analysis, the governing equations, in dimensionless form, were obtained from Eqs.(11), (16), (17) and (18), and were transformed to the finite-difference equations. The equations were solved by a successive over relaxation scheme. In the calculated results, velocity $q^*(=q/q_\infty)$, density $\rho^*(=\rho/\rho_\infty)$ and pressure $p^*(=p^*/p_\infty)$ show non-dimensional quantities. Mach number on the free surface is defined by $M_\infty=q_\infty/a_\infty$.

The free jet flow issuing from the spool valve orifice is shown in Fig.1. The positions of mesh points in the $\xi(\phi^*)$ and ψ^* directions are expressed by subscripts $i\ (1\leq i\leq m)$ and $j(1\leq j\leq n)$ respectively. In this figure, L is valve opening and H is clearance between spool and sleeve. $\phi^*=-\infty\sim+\infty$ is transformed into $\xi=-1\sim+1$ by $\xi=tanhA\phi^*$. In the present analysis, A=1, κ=1.4, m=41 and n=21 were selected.

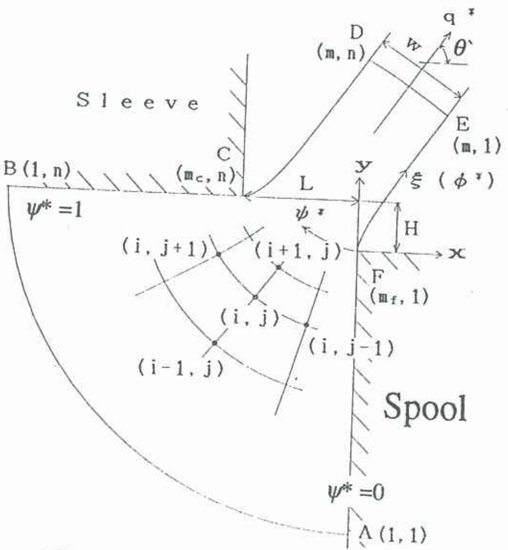

Fig.1 Spool valve orifice

RESULTS AND DISCUSSION

Incompressible Flow

Before the calculation for the compressible flow, the calculated jet shape for the incompressible flow ($M_\infty=0$) is compared with the result obtained by conformal mapping to estimate the accuracy in the present method. The shape of the jet issuing from the spool valve orifice in the case of H/L=0.02 is shown in Fig.2. We find that the jet shape calculated by the present method is very close agreement with the analytical result obtained by conformal mapping. Furthermore, the errors of the jet angle θ` at the far downstream position between the analytical results and the calculated results are shown in Table.I. The errors in all cases are below 1%. The good numerical results were obtained by the present method. From these results, we find that the present method is available to calculate the compressible jet flow.

Compressible Flow

Jet width. The relation between the orifice shape H/L and the jet width W/L is shown in Fig.3. The jet width increases with Mach number M_∞. In the results, the compressibility

Table.I Jet angles (M_∞=0)

H / L		0.8312	0.7087	0.6032	0.5098	0.4258	0.2774	0.2102	0.02610
conformal mapping	θ` (deg)	47.86	50.26	52.55	54.75	56.87	60.89	62.79	68.12
present calculation	θ` (deg)	47.46	49.85	52.16	54.40	56.55	60.65	62.60	68.04
errors	(%)	0.8378	0.8142	0.7410	0.6477	0.5583	0.3846	0.3072	0.1142

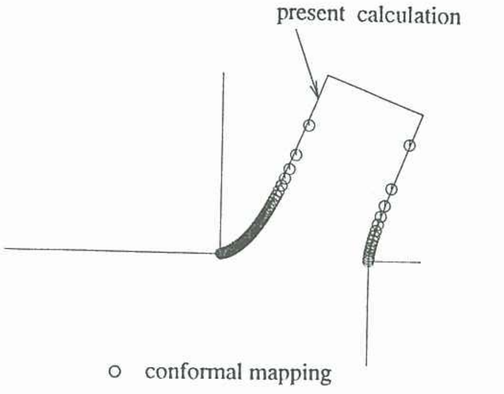

Fig.2 Jet shape (H/L=0.02)

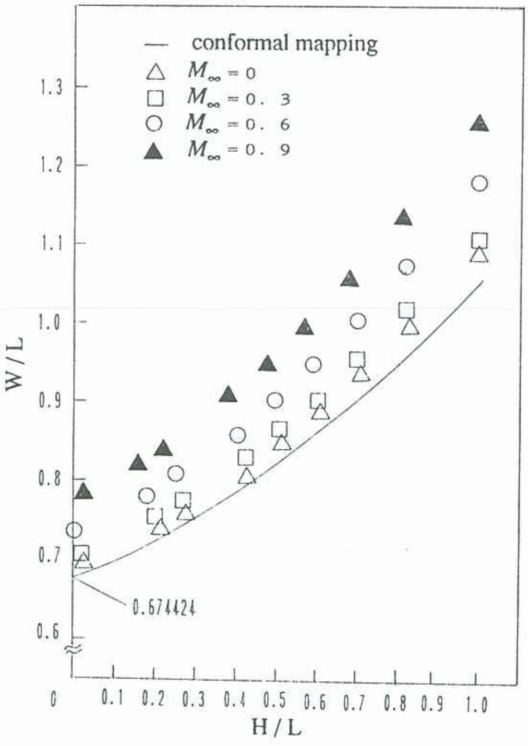

Fig.3 Jet width

has the great influence on the jet width and increases the jet width. However, the difference of the jet width between the Mach number 0 and 0.3 is very small because the incompressible flow can be assumed in the case of Mach number $M_\infty < 0.3$ in general. Here, the jet width calculated by the conformal mapping shows the value at the far downstream position, however the present results are the value at the position (i = 40). Therefore there is a difference between the analysis results and the present results in the case of M_∞ =0.

Jet angle. The relation between the orifice shape H/L and the jet angle θ` is shown in Fig.4. In the case of Mach number 0.3, the influence of the compressible on the jet angle is small as mentioned in the section of Jet width. And the jet angle increases gradually with the Mach number. The jet angle for the Mach number 0.9 is about 72.25 degree in the case of H/L=0.0163.

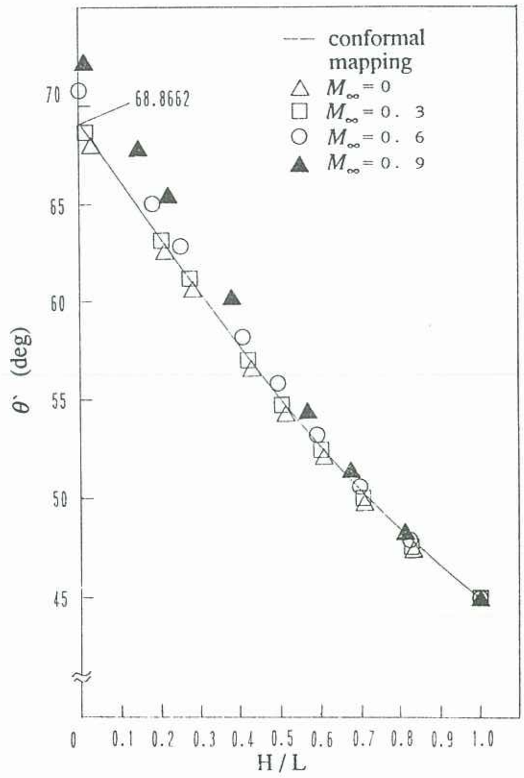

Fig.4 Jet angles

Pressure distributions on the wall $\overline{AF}$. The pressure distributions on the wall $\overline{AF}$ are shown in Fig.5. The pressure on the wall $\overline{AF}$ decreases with the increase of the valve

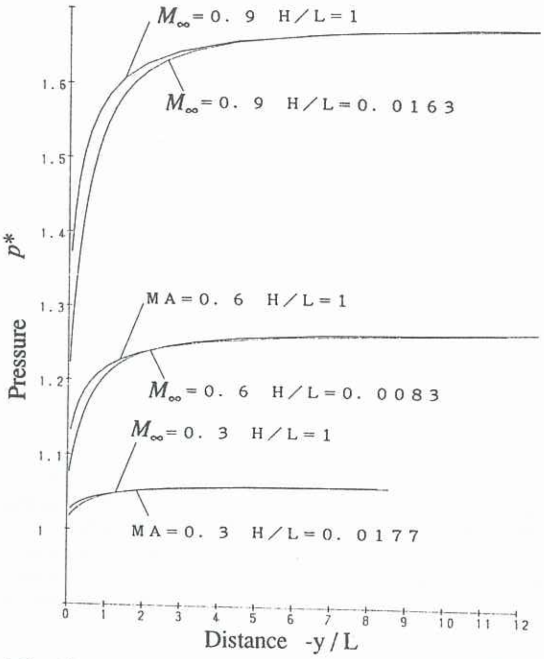

Fig.5 Pressure distributions on the wall $\overline{AF}$

opening for the same Mach number near the orifice. Furthermore, in the case of the same valve opening, the pressure drop increases with the Mach number near the orifice.

Velocity and density distributions. The velocity and density distributions are shown in Figs.6 and 7. In these figures, the values of density and velocity become almost constant in the ψ^* direction near the ψ^* =0.55 line as the Mach number increases.

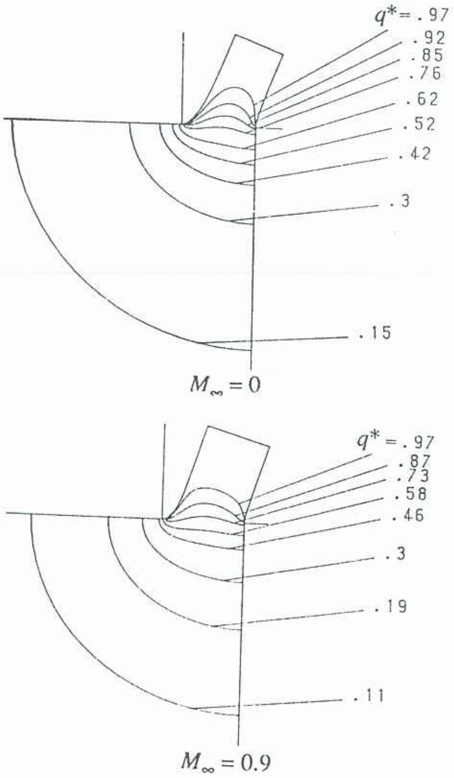

Fig.6 Velocity contours (H/L≒0)

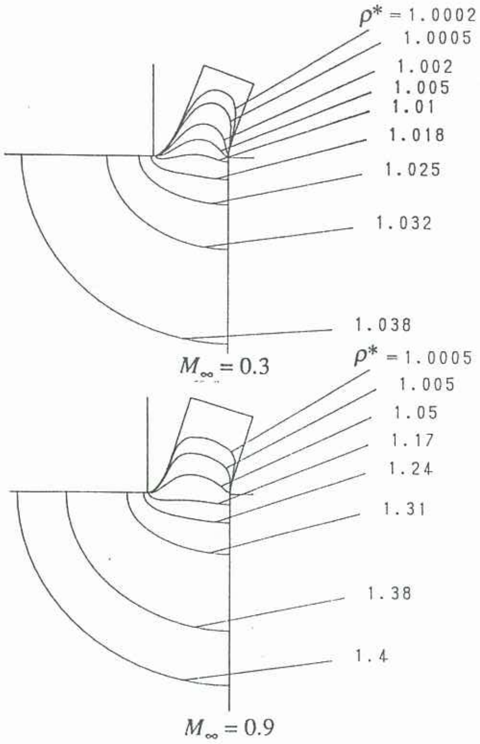

Fig.7 Density contours (H/L≒0)

CONCLUSIONS

The following conclusions can be drawn.

(1) The jet width and jet angle increase with the Mach number. Specially when the Mach number is beyond 0.3, the compressibility has great influence on the jet angle and the jet width.

(2) The pressure on the wall $\overline{AF}$ decreases with the increase of the valve opening for the same Mach number and the pressure drop increases with the Mach number in the case of same valve opening, near the orifice.

(3) The values of density and velocity become almost constant in the ψ^* direction near the ψ^* =0.55 line as the Mach number increases.

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