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EFFECT OF LIQUID VISCOSITY ON BUBBLE EXPANSION AND CONTRACTION

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Abstract

With the use of high speed photography, observations of the process of bubble expansion and contraction as a result of electric spark discharge show that liquid viscosity has the significant influence on the bubble behaviour. Theoretical and numerical analyses of this behaviour are given in this paper together with some qualitative results.

1 INTRODUCTION

Most of bubble dynamic studies assume an ideal liquid. However, it is obvious that liquid viscosity exhibits an effect on bubble expansion and contraction rates, impact pressure as well as the process of material damage. The effects of liquid viscosity on the process of bubble movement have been explained by numerical computation of the Rayleigh equation. This paper describes a theoretical analysis and an experimental study of this process. A comparison is made between theoretical prediction and experimental results. Based on the comparison of the results, some tentative conclusions are presented.

2 NUMERICAL SOLUTION

If mass force and surface tension are neglected, the differential equation describing the bubble motion during bubble expansion and contraction in a viscous liquid is *(Porisky, 1952)*:

$$\frac{P_R - P_\infty}{\rho} = R\ddot{R} + \frac{3}{2}\dot{R}^2 + 4\frac{\mu}{\rho}\frac{\dot{R}}{R} \qquad (1)$$

where, P_R is the pressure inside bubble,
P_∞ is the pressure at infinity,
ρ is the density of liquid around bubble,
R is the radius of bubble,
$\dot{R}$ is the velocity of bubble wall motion,
$\ddot{R}$ is the acceleration of bubble wall motion,
μ is the dynamic viscosity of liquid.

By using the equation,

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{1}{2R^2\dot{R}}\frac{d(R^3\dot{R}^2)}{dt} \qquad (2)$$

and combining with Equation 1, the integral equation is given by:

$$\begin{aligned} &\frac{4}{3}\pi(P_\infty - P_R)({R_0}^3 - R^3) \\ &= 2\pi\rho R^3\dot{R}^2 + 16\pi\mu\int_0^t R\dot{R}^2 dt \end{aligned} \qquad (3)$$

Equation 4 can be expressed in dimensionless form as in the following:

A. For bubble expansion.

$$\frac{1}{3}(\beta^3 - 1) - \frac{1}{2}\beta^3\left(\frac{d\beta}{d\tau}\right)^2 - C\int_0^\tau \beta\left(\frac{d\beta}{d\tau}\right)^2 d\tau = 0 \qquad (4)$$

with $\beta = \frac{R}{R_0}$,
$\tau = \frac{t}{R_0}\sqrt{\frac{P_R - P_\infty}{\rho}}$,
$C = \frac{4\mu}{R_0\sqrt{\rho(P_R - P_\infty)}}$

B. For bubble contraction.

$$\frac{1}{3}(1 - \beta^3) - \frac{1}{2}\beta^3\left(\frac{d\beta}{d\tau}\right)^2 - C\int_0^\tau \beta\left(\frac{d\beta}{d\tau}\right)^2 d\tau = 0 \qquad (5)$$

with $\beta = \frac{R}{R_0}$,
$\tau = \frac{t}{R_0}\sqrt{\frac{P_\infty - P_R}{\rho}}$,
$C = \frac{4\mu}{R_0\sqrt{\rho(P_\infty - P_R)}}$

When R_0, ρ, $(P_\infty - P_R)$ are given, the relation between β and τ can be determined as a function of the liquid viscosity.

The numerical computation results have shown the importance of liquid viscosity on the behavior of bubble expansion and contraction, as shown in Fig. 1 and Fig. 2

3 EXPERIMENT

In the laboratory, the expansion and contraction processes of a spark-generated bubble were recorded by high speed photography in liquids with various viscosities. The framing speed of the high speed camera was $100,000$ frames per second, the size of frame $\phi 5mm$, the synchronous precision $10\mu s$, the diameter of the electrode made of hard alloy was $0.3mm$. The electrode was placed in a quartz glass chamber with a transparent wall. The cross section of the chamber was $10mm \times 10mm$ and the depth of liquid in the chamber was about $70mm$. The liquids with different viscosities were made

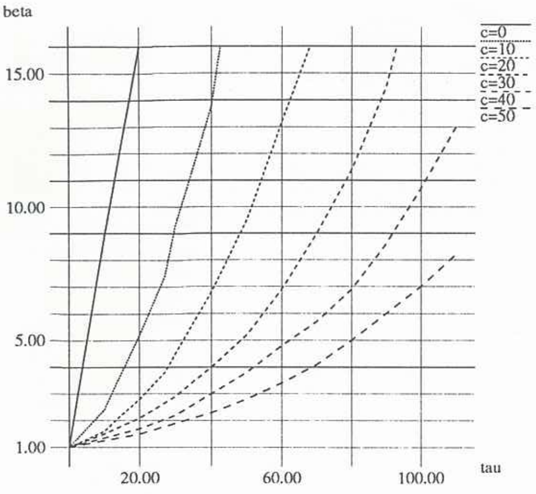

Figure 1: Bubble diameter vs time by calculations, (expansion)

with distilled water and different proportions of glycerine. Their viscosities are shown in Table 1. The temperature of the liquid was about 15^0C.

Table 1: Viscosity of Test Liquids

Distilled Water	Glycerine	Viscosity
100	0	0.0113
80	20	0.0314
60	40	0.0595
40	60	0.2052
20	80	0.67
0	100	1.10

4 RESULTS AND DISCUSSION

4.1 Effect of Viscosity on the Processes of Bubble Expansion and Contraction

The photography shows that, the more viscous the liquid, the greater the number of cycles of bubble expansion and contraction. For instance, the number of cycles with pure glycerine is about two and half, but it is only one in pure distilled water. This is because low viscosity promotes bubble instability and collapose during bubble contraction. The ratio R/R_0 against time t for different viscosites in the processes of bubble expansion and contraction during the first cycle is shown in figures 3 and 4 respectively. These figures show that the more viscous the liquid, the slower the bubble expansion and contraction, which means the two processes are slowed down by viscosity. The effect of viscosity is negligible at the begining of the bubble expansion. As the viscosity tends to infinity, the rate of the bubble contraction tends to zero.

Comparison of the theoretical curves in figure 1 and 2 with those experimental curves in figure 3 and 4, shows that there are some difference in the curve shapes. Taking the case of bubble expansion as an example, the theoretical curves are concave upwards, while the experimental curves are concave downwards. The difference could be due to the different conditions of bubble generation. The expansion is initiated by static differential pressure in the theoretical calculations, but the bubble is spark-generated in the experiment. For the later, the initial instantaneous pressure inside the bubble is very high at the instant when the electric spark discharges, that is, the bubble expansion rate is very high at the instant. Therefore, the initial acceleration of the bubble wall is very high. As a result, the curves of the bubble expansion processes are concave downwards. However, this difference has no influence on the qualitative analysis of the effects of liquid viscosity on bubble behaviour.

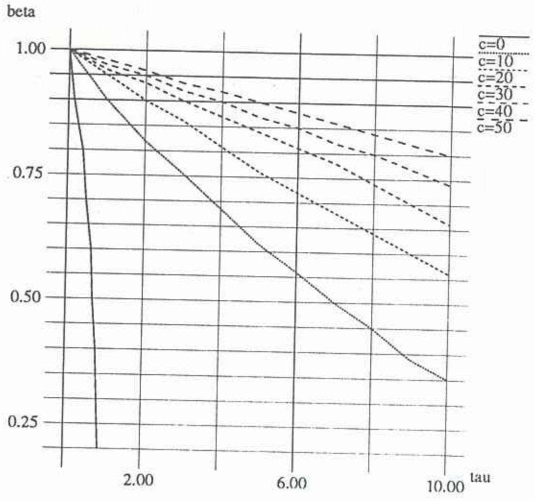

Figure 2: Bubble diameter vs time by calculations, (constration)

4.2 Effect of Viscosity on the Velocity and Acceleration in Bubble Expansion and Contraction

Based on the curves shown in figure 3 and 4, the bubble expansion or contraction rate $\dot{R}$ at any instant may be obtained for different viscous liquids. The derivative of $\dot{R}$ with respect to time, i.e $\ddot{R}$, the acceleration of the bubble wall has also been calculated. Figure 5 shows the relationship between the bubble wall velocity $\dot{R}$ and time t in various viscous liquids. Figure 6 shows the variation of the bubble wall acceleration $\ddot{R}$ with time t for the liquids, where the sign of the velocity and acceleration is postive for expansion and negative for contraction.

On examination of figure 5, it may be noted that the effect of liquid viscosity on the bubble wall velocity $\dot{R}$ during the initial expansion may be neglected. The velocity $\dot{R}$ of the bubble wall is tending to one oblique line at every instant. The six curves for the various viscous liquids are close to one straight line in the processes of bubble expansion (those curves above the abscissa). We believe that this is because the energy of every electric

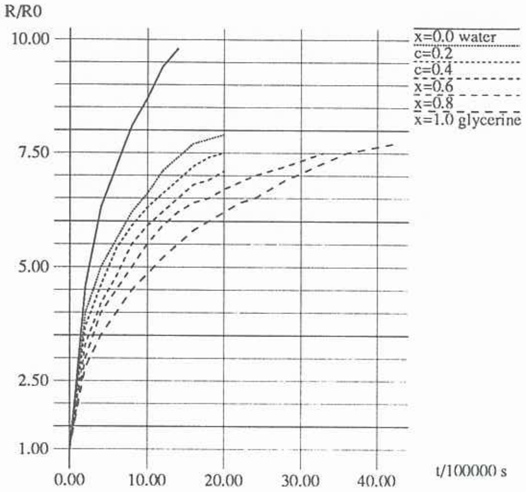

Figure 3: Bubble radius vs time by experiment, (expansion)

spark discharge are almost the same. The process of bubble expansion is the release process of electric spark discharge, the time of energy release process is different in the different viscous liquids. However, the velocity $\dot{R}$ of the bubble wall is the result of the disequilibrium force on the bubble. Different viscous liquids have different surface tension factors. The equilibrium of the bubble is only the surface tension balance with the differential pressure of the inside and outside the bubble. The outside pressure of the bubble is negligible when compare with the inside pressure of the bubble. Also, the bubble wall velocity $\dot{R}$ gets slower because the inside pressure drops as the inside vapor release to outside of the bubble. The viscous effect is smaller than the spark discharge energy at the begining of the bubble expansion, hence the velocity $\dot{R}$ are similar in the various viscous liquids. Later, the effect of viscosity tends to become more significant. The viscosity affects the process of dissipation and the cycle time of the bubble. The larger the viscous factor, the longer the cycle time of the bubble.

The effect of viscosity on the rate $\dot{R}$ of the bubble wall is quite evident when the bubble contraction. The rate $\dot{R}$ has no obvious difference at the begining several instant in the contraction process, but the rate $\dot{R}$ has very large difference before the several instant of the bubble collapse. The absolute value of the $\dot{R}$ tends to decrease with the viscosity fator μ increasing. Figure 5 shows that the bubble expansion duration increase with the viscosity factor μ. If μ is smaller, the time to reach the maximum contraction velocity is shorter and the bubble contraction is faster.

The time to reach the maximum acceleration $\ddot{R}$ is shorter if the viscosity factor μ is smaller as shown in Figure 6. The initial acceleration has no obviously difference at the same relatively instant in the shrink process. However, the acceleration $\ddot{R}$ has very large difference before several instant of the bubble collapse, and the absolute value of $\ddot{R}$ tends to decrease with the viscosity factor μ increases. There is no obvious regularity of the effect of liquid viscosity on the bubble wall ac-

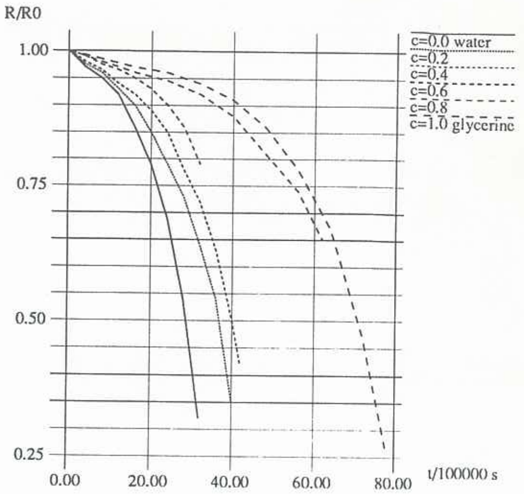

Figure 4: Bubble radius vs time by experiment, (contraction)

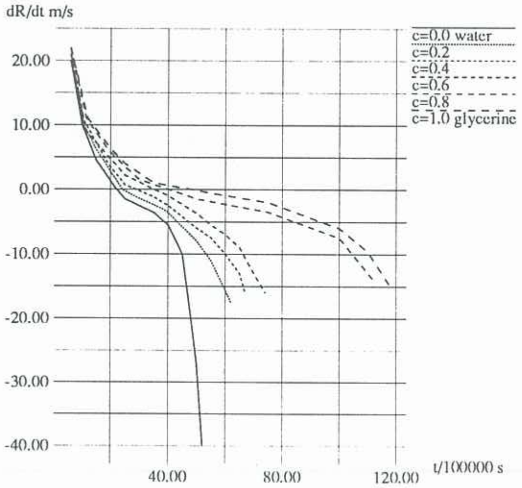

Figure 5: The velocity $\dot{R}$ vs time t

5 CONCLUSIONS

The numerical calculation and the experimental results show that the liquid viscosity affects the bubble duration, diameter, velocity and acceleration.

(1) The process of the bubble expansion and contraction become slower and the duration get longer by liquid viscosity. The cycle of the bubble is increased with the viscosity factor μ increase.

(2) The bubble wall velocities $\dot{R}$ are not much different at initial expansion in liquids of varying viscosity, but the effect of viscosity on the rate $\dot{R}$ in the bubble contraction is obvious, and the rates $\dot{R}$ tend to decrease as the viscosity factor μ increases.

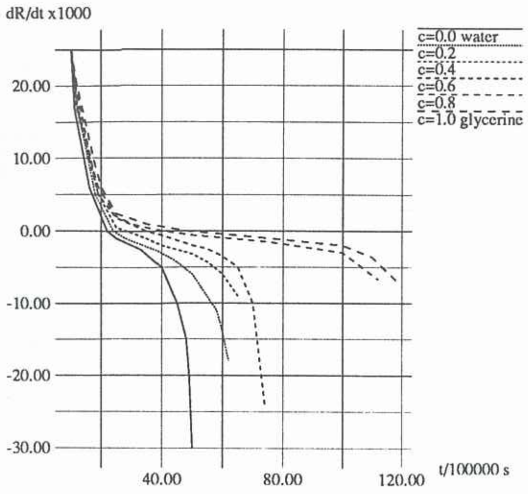

Figure 6: The acceleration $\ddot{R}$ vs time t

(3) During expansion, the acceleration $\ddot{R}$ increases as the viscosity increases at the same relative instant. The contraction acceleration $\ddot{R}$ have obvious differences before the bubble shrinks to collapse. The acceleration $\ddot{R}$ tends to decrease as the viscosity factor μ increases. The smaller the viscosity, the shorter duration to the maximum acceleration.

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REFERENCE

[1] PORITSKY, H (1952) The collapse or growth of a spherical bubble or cavity in a viscous fluid. Proc. First U.S. Natl. Congr. Appl. Mech. (ASME), 813—821.