

FISH TIDES AND THE ADDED MASS FORCE

John A.T. BYE

School of Earth Sciences
The Flinders University of South Australia
GPO Box 2100, Adelaide, SA 5001, AUSTRALIA

ABSTRACT

We show that masses moving within a fluid give rise to a conservative force, which we call the added mass force, and which complements the tide generating force due to masses moving outside the fluid. The added mass force acts parallel to the direction of motion of the moving body relative to the fluid with the magnitude and sign of the corresponding component of gravitational attraction on the mass.

The potential field of force produced by fish swimming in the ocean is derived, and shown to drive an independent circulation, which we have called the fish tide. The fish community thus has the latent ability to generate its own Ekman transport and hence establish a literal feedback loop by stimulating upwelling adjacent to a coastline.

INTRODUCTION

Dynamical oceanography is based on the solution of the equations of motion for seawater with prescribed boundary and initial conditions. In the event that these conditions can be accurately specified, and the physics of the internal thermohaline processes correctly formulated, there is an expectation that ocean models can be constructed which will reproduce the oceanic general circulation.

In these models, it is usually assumed that the population of the sea is constrained absolutely by its environment, in other words the collective energy of the biomass has a negligible effect. The fundamental law which is quoted in support of this assumption is Newton's Third Law of Action and Reaction from which it is concluded that however much power is expended by a swimming animal, it cannot exert a force on its environment.

We show below that it is quite feasible for the biomass to overcome this apparent repressive restriction, and change its

dynamical environment, presumably for the better.

THE ADDED MASS FORCE

Consider a system which consists of a powered mass (m) of density ρ' which is contained in a fluid of density, ρ , such that its reduced gravity, $g' = g(\rho' - \rho)/\rho$. Suppose initially that the mass exerts a thrust (T) vertically upwards. Then the balance of forces on the mass is,

$$T - mg' - G = 0 \quad (1)$$

where G is the drag of the fluid on the mass.

By the Third Law, the force acting on the fluid,

$$F_2 = -T + G = -mg' \quad (2)$$

Equivalently, we may partition T into two parts, as follows,

$$(I) T_1 = -F_1 \text{ where } F_1 = -G \quad (3)$$

$$(II) T_2 = -F_2 \text{ where } F_2 = -mg' \quad (4)$$

and $T = T_1 + T_2$ in which the system (I) exerts no net force on the fluid, and the system (II) exerts the force F_2 . In (3), the partial thrust (T_1) is balanced by the fluid drag, and generates no pressure field in the far fluid, whereas the partial thrust (T_2) can generate pressure and stress fields in the far fluid.

The fluid motion generated by T_1 may be called the massless solution as F_1 is independent of m , and that generated by T_2 may be called the dragless solution as F_2 is independent of G . The existence of F_2 is consistent with Darwin's Theorem (Darwin, 1953), that the added mass for a body translating in an infinite expanse of perfect fluid equals the drift volume times the density of the fluid. This is because the dragless

solution maintains a backward thrust on the fluid equal to the gravitational force acting on the mass. A suitable term for F_2 would be 'the added mass force'.

Consider next a powered mass which instantaneously exerts a thrust at an angle (β) to the horizontal, and travels relative to the fluid at an angle (α) to the horizontal. Then the components of the balance of forces on the mass parallel, and normal (in the vertical plane) to its direction of travel are respectively,

$$T \cos(\beta - \alpha) - mg' \sin \alpha = G \quad (5)$$

$$T \sin(\beta - \alpha) - mg' \cos \alpha = 0 \quad (6)$$

with a zero balance normal to the mass in the horizontal plane. On eliminating the angle ($\beta - \alpha$) we obtain,

$$T^2 = (mg' \cos \alpha)^2 + (mg' \sin \alpha + G)^2 \quad (7)$$

Equation (7) yields, for the partial thrusts; massless ($m=0$) $T_1 = G$ ($\beta = \alpha$) and for dragless ($G=0$), $T_2 = mg'$ ($\beta = \pi/2$). The magnitudes of T_1 and T_2 are the same as in the vertical configuration, but T_1 acts along the direction of travel, and T_2 in the vertical.

On resolving along α , the added mass force acting on the fluid,

$$E_2 = -mg' \sin \alpha \quad (8)$$

which acts parallel to the direction of relative motion of the moving body. This result may also be obtained directly by applying the Third Law to (5).

The condition $T = mg'$ is sufficient for the powered mass to hover in the fluid, i.e. to establish a stable powered mass - fluid system, and $T > mg'$ enables a range of relative motion to be executed in order to impart a desired added mass force (E_2) on the fluid which generates currents through a system of shearing stresses and pressure gradients. Since E_2 is a conservative force, we may define the potential, Ω , such that,

$$E_2 = -\nabla \Omega \quad (9)$$

where $\Omega = mg'z + const$, and in a local co-ordinate frame (ξ, μ, λ), where 0ξ lies along the direction of travel, and 0μ and 0λ are orthogonal co-ordinates in the transverse plane,

$$\frac{\partial \Omega}{\partial \xi} = mg' \sin \alpha, \quad \frac{\partial \Omega}{\partial \mu} = \frac{\partial \Omega}{\partial \lambda} = 0$$

and relative to the Earth,

$$\begin{aligned} \frac{\partial \Omega}{\partial x} &= mg' \sin \alpha \cos \alpha \cos \gamma \\ \frac{\partial \Omega}{\partial y} &= mg' \sin \alpha \cos \alpha \sin \gamma \\ \frac{\partial \Omega}{\partial z} &= mg' \sin^2 \alpha \end{aligned} \quad (10)$$

where $0x$ lies towards the East, $0y$ lies towards the North, and $0z$ is vertically upwards, and the mass travels relative to the fluid at an angle (γ) to $0x$. The significant force (as with astronomical tidal forcing) is the horizontal component of E_2 , which is maximised for motion along $\alpha = \pm \pi/4$.

THE FISH TIDE

Using (10), the added mass force for the biomass in the ocean is easily incorporated in the equations of motion. Suppose that there are $n(z)$ individuals (i) in a unit volume of ocean at z , then the force/unit volume

$$\text{acting on the water is } -\sum_i^{n(z)} (\nabla \Omega)_i$$

This force is of a similar nature to the astronomical tidal force.

It is common practice in ocean modelling to integrate the conservation equations for horizontal motion over the depth of the water column (Phillips, 1966).

The resulting force/unit area acting on the water column,

$$X = \tau_s - \tau_b + \rho g H \nabla_h \zeta - \sum_i^N (\nabla_h \Omega)_i \quad (11)$$

where τ_s and τ_b are the surface and bottom

shearing stresses, $\nabla_h = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y} \right)$,

$\zeta = -\frac{\Omega_o}{g} - \frac{p_a}{\rho g} + const$ is the static ocean

level due to the astronomical tide of potential (Ω_o) and the atmospheric pressure (p_a), and the last term is the added mass force due to the biomass consisting of N individuals/unit area. The circulation produced by this term, can be called the fish tide. The magnitude of the fish tide depends on two properties of the biomass - its mass and its reduced gravity.

The Ekman transport due to a fish community

Suppose that a community of fishes wishes to exert a unidirectional force on the ocean. Fig. 1 shows a suitable swimming pattern. On the assumption that each fish descends relatively light ($g' < 0$) due to air

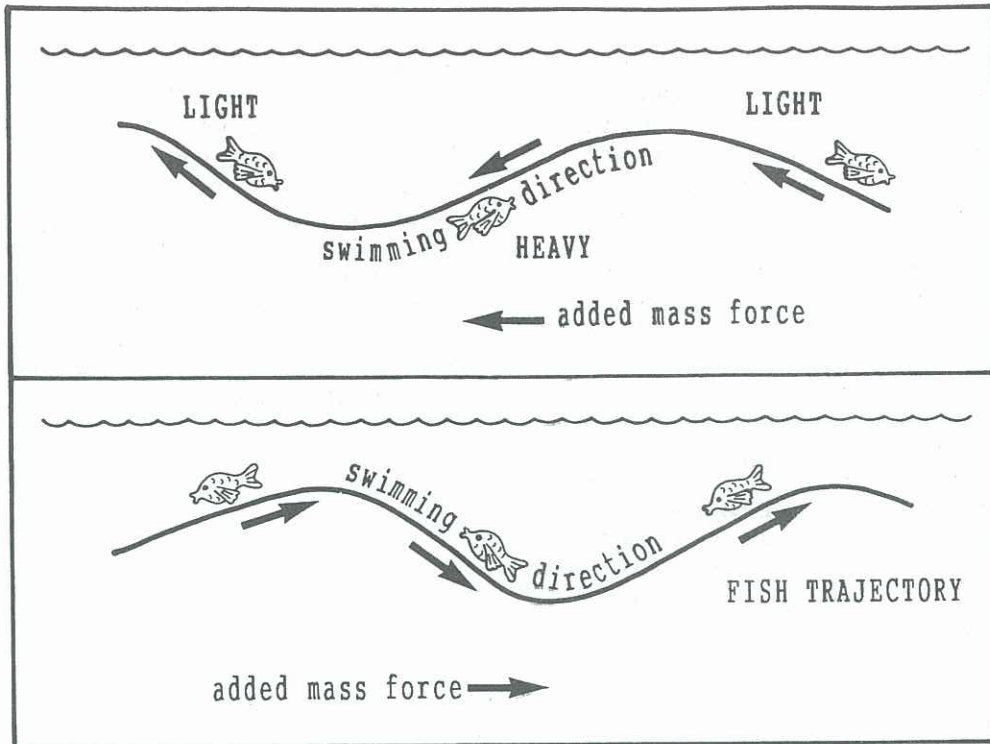


FIGURE 1 Undulating swimming by fish in still water.

ingestion, and ascends relatively heavy ($g' > 0$) due to air utilisation, the fish would do work on the ocean, and a force would be always exerted opposite to its undulating swimming motion. The wavelength and direction of this motion being at the discretion of the fish.

The implications of this ability are remarkable. On assuming (for simplicity) that each fish swims with a quasi-horizontal trajectory ($\cos \alpha_i \approx 1$) we have that

$$\sum_i^N (\nabla_h \Omega)_i - \sum_i^N (mg' \sin \alpha)_i \quad (12)$$

The magnitude of (12) can be estimated using typical values: $|g'| \sim 1 \text{ ms}^{-2}$, which corresponds with a 10% difference in density between the fish and seawater (McNeill, 1968), $|\sin \alpha| \sim 0.1$ which would occur during vertical migrations of 100m in a 1km swimming trajectory, and

$$\sum_i^N m_i \sim 0.1 \text{ kg m}^{-2} \text{ which is observed}$$

(Peters, 1983, Sheldon et al 1970) in regions of the World ocean important to fisheries. On combining these values we obtain,

$$\left| \sum_i^N (\nabla_h \Omega)_i \right| \approx 10^{-2} \text{ Nm}^{-2} \text{ This estimate is}$$

equivalent to a surface shearing stress due to a breeze (Phillips, 1966), and since the fish tidal forcing rapidly decreases with depth, it would be expected to act in a similar manner. Thus the fish community has the ability to generate its own Ekman transport (Phillips, 1966), and hence establish a (literal) feedback loop by stimulating upwelling adjacent to a coastline.

The important question is on which scale of averaging (if any) this added mass force could be exerted, through the action of coherently oriented swimming patterns. Two favourable situations exist on the scale of averaging of fish communities (which is much greater than that of individual swimming gyrations) namely, in which (i) a community remains approximately in the same location in the presence of a steady current, and (ii) a community migration occurs. These scenarios suggest that a significant dynamical coupling between biomass and ocean is possible.

A detailed understanding of this system would require the initiation of a very demanding physical oceanographic and marine biological sampling programme. We suggest intuitively however, that a desirable situation (with the potential for a rich ecological diversity) exists if the dynamical effects of the biomass and of the atmosphere are of the same order of magnitude. Our calculations indicate that the World Ocean was evolving towards this state prior to the advent of the fisherman.

CONCLUSION

An interpretation of Newton's laws in terms of an added mass force has been found useful in describing the effects of moving masses within a fluid, supported under the gravitational field of the Earth (or other body). The properties of the added mass force are that it acts parallel to the direction of the moving body relative to its fluid with the magnitude and sign of the corresponding component of gravitational attraction of the mass.

For a mass exterior to the fluid, the added mass force is the astronomical tidal force whose magnitude is also proportional to the mass of the moving body, and whose direction is determined by the position of the external body as it moves relative to the fluid.

In both situations, the added mass force is a conservative force with an independent existence quite distinct from that of thrust and drag forces, and also the current field. Its importance lies in the rhythm that it imparts to the fluid. The disintegration of the orbiting external body into an encircling ring would remove the tidal signal. The replacement of a community of interior masses in coherent motion, by a rain of passive matter similarly would remove any distinct added mass signal, and in the limit of completely dissolved debris, the added mass force would be identical with the buoyancy force of the fluid itself.

Properties of the added mass force for the biomass-ocean system have been illustrated as the author's personal field of interest is physical oceanography. It is probable, however, that the existence of the added mass force, hidden within classical mechanics, has many implications, especially in relation to the life sciences. Tides have traditionally been perceived as a fateful force imposed from beyond, they may also be the agent of free choice from within.

ACKNOWLEDGMENTS

Thanks are expressed to Dr G P Harris and Dr R T Wells for providing essential biological references.

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