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SPECTRAL STUDY OF WEAKLY COMPRESSIBLE TURBULENCE

F. BATAILLE and J.P. BERTOGLIO

Laboratoire de Mecanique des Fluides
Ecole Centrale de Lyon, Ecully, FRANCE

Nowadays, the study of compressible turbulence has a renewed interest because of the existence of ambitious aeronautical programs. For Mach numbers built on the mean velocity which are higher than 4 or 5, it cannot be assumed that the divergence of the fluctuating field remains equal to zero and compressibility effects do affect the dynamics of turbulence. Recently, a lot of work has been performed to forecast and understand the dynamics of compressible turbulence. We can, in particular, quote studies using direct numerical simulations from Blaisdell (1990), Normand and Lesieur (1992), Erlebacher et al. (1987), Passot and Pouquet (1987).

The aim of the present work is to investigate the extent to which two-point closure theories can be extended to isotropic compressible turbulence.

The fluid is assumed barotropic in order to avoid introducing the energy equation. A statistical average is used and the density is decomposed into a mean part and a fluctuating part :

$$\rho = <\rho> + \rho' \qquad (1)$$

In order to avoid excessive complexity, the fluctuating density ρ' is neglected as compared to $<\rho>$. This approximation requires that the turbulent Mach number satisfies $Mt^2 << 1$ with the turbulent Mach number defined as :

$$Mt = \sqrt{q^2/C_o} \qquad (2)$$

where C_o is the velocity of sound and q^2 is twice the turbulent kinetic energy. Equations are written in the framework of the E.D.Q.N.M. theory (Marion, 1988), in the particular case of homogeneous and isotropic turbulence. The resulting set of equations is composed by:

- an equation on the spectrum of the turbulent kinetic energy linked to the solenoidal part of the velocity field, which corresponds to the velocity fluctuations perpendicular to the wave vector $\underline{K}$. This equation is :

$$\frac{\partial}{\partial t} E^{SS}(K,t) = -2\nu K^2 E^{SS}(K,t) + 8\pi K^2 T^{SS}(K,t) \qquad (3)$$

- an equation on the spectrum of the turbulent kinetic energy linked to the "purely compressible" part of the velocity field, i.e. linked to the fluctuation along the direction of the wave vector:

$$\frac{\partial}{\partial t} E^{CC}(K,t) = -2\nu' K^2 E^{CC}(K,t) + 4\pi K^2 T^{CC}(K,t) - 2\frac{K}{<\rho>} E^{CP}(K,t) \qquad (4)$$

- an equation on the spectrum of the potential energy linked to the pressure :

$$\frac{\partial}{\partial t} E^{PP}(K,t) = 2\frac{K}{<\rho>} E^{CP}(K,t) \qquad (5)$$

- and an equation on the pressure-velocity correlation spectrum :

$$\frac{\partial}{\partial t} E^{CP}(K,t) = -\nu' K^2 E^{CP}(K,t) + 2\pi K^2 T^{CP}(K,t) - \frac{K}{<\rho>} E^{PP}(K,t) + K<\rho>C_o^2 E^{CC}(K,t) \qquad (6)$$

with, in the case of a Stokes fluid :

$$\nu' = \frac{\lambda+2\mu}{<\rho>} = \frac{4}{3}\nu \qquad (7)$$

where μ and λ are the two viscosities assumed to be uniform and constant . T^{II}, T^{CC}, T^{CP} are the transfer terms arising from the non linear terms of the basic equations. The expressions for these terms are not written here : they can be found in the thesis of Marion (1988). They are rather complex expressions, having basically the same form as the usual transfer term for incompressible isotropic turbulence : i. e. involving the triadic interactions, but with more terms, due to the more complex character of compressible turbulence.

The results presented here correspond to cases of isotropic turbulence submitted to a forcing. In order not to perturb the evolution of the purely compressible part of the field, energy is injected on the solenoidal modes only. This is done, in a classical manner, by just using frozen values for the solenoidal energy spectrum E^{SS} in the small wave-number range (up to $K = 64\ m^{-1}$). Results are analysed for asymptotic states where the compressible mode has reached saturated values. These asymptotic states are obtained for times whic can be very long. For shorter times, it is found that the compressible quantities (energy and dissipation) sharply depend on the initial conditions. This was remarked in the direct simulations as well.

Spectra of kinetic energy, for the solenoidal and compressible modes, are given in figure 1 for different values of the turbulent Mach number. The solenoidal energy spectrum shows an inertial zone in $K^{-5/3}$ which is the same as that in the Kolmogoroff spectrum for incompressible turbulence. This spectrum is not affected when the value of the Mach number varies. It can be observed that spectra of the compressible part of the fluctuating velocity show a corresponding law with a slope ($K^{-11/3}$). This slope is found for small and moderate values of the turbulent Mach number Mt. Only when Mt is larger than 0.1, the shape of the purely compressible energy spectrum is modified. The level of the compressible energy is found to be dependent on the Mach number and varies as Mt^2.

The same slope appears for the spectrum of pressure fluctuation. We can notice that a slope of (-11/3) was found also with Large Eddy Simulation of compressible isotropic turbulence by Comte et al. (1990). When spectra are divided by the square of Mt, it appears in figure 2a that the results can be collapsed for small values of Mt (Mt < 0.03). Then, the following relation is obtained :

$$E^{CC} \alpha\ Mt^2 K^{-11/3} \qquad (8)$$

For larger but moderate values of Mt ($0.03 < Mt \leq 0.1$) the compressible energy, normalized by Mt^2, tends to decrease in the small wave-number part of the spectrum but the slope (-11/3) remains unchanged. For Mt > 0.1, the slope decreases. At Mt=1, the complete spectrum is affected (figure 2b).

The dependence on Mt^2 can also be observed in figure 3 where the acoustic energy ($\mathbb{E}^{CC} + \mathbb{E}^{PP}$) is given as a function of Mt. We, therefore, effectively obtain a straightline with a slope (+2). At Mt larger than 0.1 a departure from the Mt^2 law is observed.

The influence of the compressibility may also be observed on the dissipation. This latter is decomposed into a solenoidal part (ε_S) and a compressible part (ε_C) :

$$\varepsilon = \varepsilon_S + \varepsilon_C \qquad (9)$$

A behavior in Mt^2 is found for the compressible part of the dissipation (figure 4). For a large range of Mach numbers, the following relation is obtained :

$$\varepsilon_C = 0.5\ Mt^2\ \varepsilon_S \qquad (10)$$

which is in good agreement with results of simulations from Aupoix et al. (1990) and Sarkar et al. (1991). We also note a small departure from the Mt^2 law, at high Mach number. This departure occurs at larger turbulent Mach numbers than for the energy.

When the influence of the Reynolds number is investigated, it is found that the level of the compressible energy is affected according to a linear dependancy with Re (figure 5).

Thus, the compressible energy behaves as a function of (Mt^2, Re), as can be observed on the figure 6. Spectra of compressible energy collapse when the value of the turbulent Mach number is multiplied by 10 and when the Reynolds number is divided by 100. Finally, it should be emphasized that investigating such a range of various Reynolds numbers would have been impossible using Direct Numerical Simulations.

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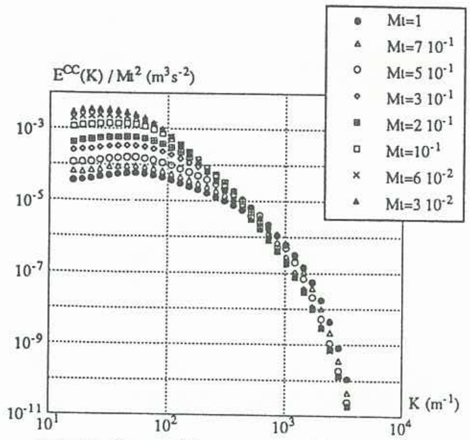

Figure 2b : Compressible energy spectra divided by Mt^2

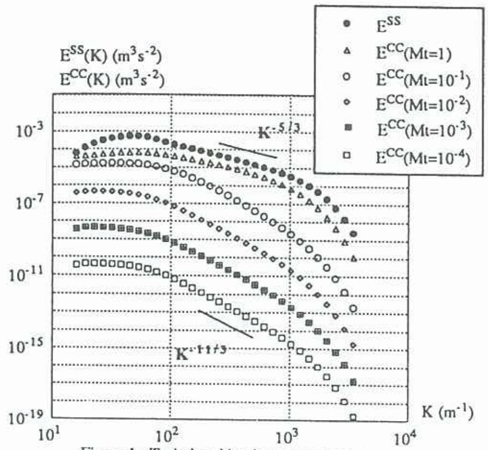

Figure 1 : Turbulent kinetic energy spectra

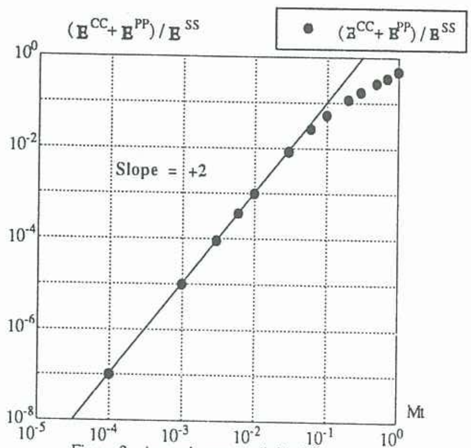

Figure 3 : Acoustic energy divided by the solenoidal energy in function of Mt

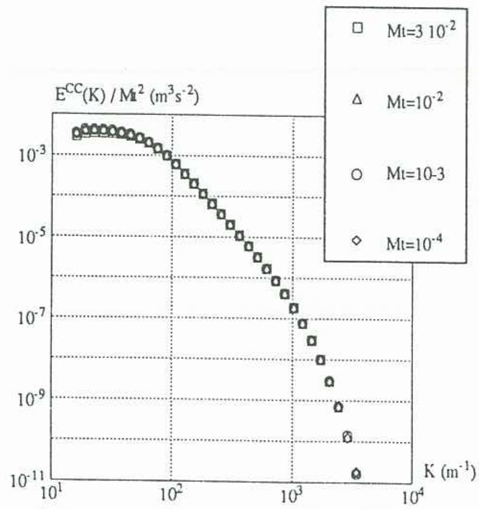

Figure 2a : Compressible energy spectra divided by Mt^2

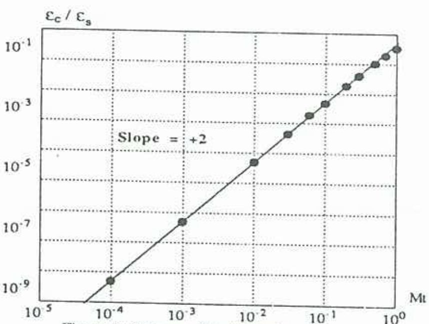

Figure 4 : Compressible dissipation divided by the incompressible dissipation

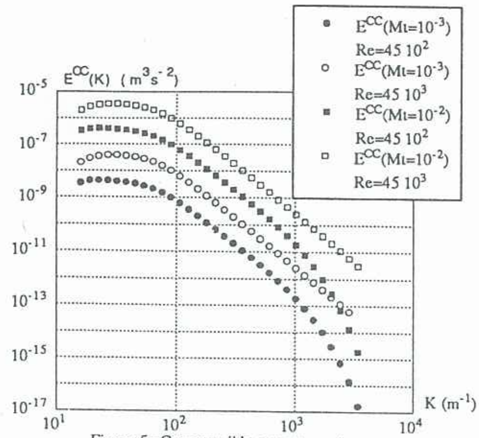

Figure 5 : Compressible energy spectra for two Reynolds numbers

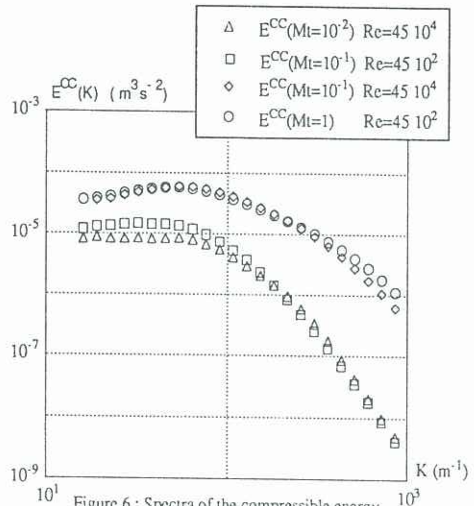

Figure 6 : Spectra of the compressible energy for different values of Mt and Re