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UNSTEADY INTERACTIVE BOUNDARY LAYER FLOWS

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ABSTRACT

A method has been developed to calculate unsteady boundary layer flows using vorticity–stream function formulation with an appropriate viscous–inviscid interaction law to extend the calculations to large but finite Reynolds numbers. The problems considered are the impulsively moved circular cylinder and the flow induced when a line vortex is introduced into the neighborhood of a circular cylinder. The results for the first case are in good agreement with the previous ones.

INTRODUCTION

It has been well established by Goldstein (1948) for steady boundary layer flows that flow separation is accompanied by the appearance of a singularity at the point of vanishing skin friction. For unsteady flow in which separation, or flow breakaway, occurs the boundary layer solution develops a singularity at a finite time. Van Dommelen and Shen (1980, 1982) have studied this breakdown and have analysed the structure of singularity at the point of breakdown. Cowley (1983) and Ingham (1984) have confirmed this working within Eulerian framework.

The problems we have considered are (i) the impulsively moved circular cylinder and (ii) the flow induced when a line vortex is introduced into the neighbourhood of a circular cylinder. The breakdown of our boundary layer calculations in second example are in accord with Van Dommelen & Shen. This type of breakdown has been observed by Doligalski & Walker (1984) and Ersoy & Walker (1986). Chuang & Conlisk (1989) have carried out interactive calculations for the convected vortex, their solution eventually fail although their calculations proceed beyond the boundary layer breakdown point. We experience no breakdown in our interactive calculations although we cannot extend our results to the point where vortex shedding from the cylinder takes place.

THE GOVERNING EQUATIONS

The unsteady composite formulation, in boundary layer scalings, for incompressible two–dimensional flow, in polar coordinates (y,θ) with vorticity–stream function formulation, is

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial \theta} + v\frac{\partial w}{\partial y} = \frac{\partial^2 w}{\partial y^2} \tag{1a}$$

$$w = -\frac{\partial^2 \psi}{\partial y^2} \tag{1b}$$

with the initial and boundary conditions

$t = 0$, w, ψ are obtained from Rayleigh solutions,
$y = 0$, $\psi = 0$, and w is derived from the no–slip condition $u = 0$ at $y = 0$,
$y \to \infty$, $\frac{\partial \psi}{\partial y} \to -u_s(\theta,t)$, $w \to 0$

$$u_s(\theta,t) = u_{sp}(\theta,t) - \frac{\epsilon}{2\pi}\int_0^{2\pi} \frac{(d(u_s\delta)/d\theta)\sin(\alpha-\theta)}{1-\cos(\alpha-\theta)}\,d\alpha$$

$$\text{and}\quad \delta = \int_0^{\infty}\left(1-\frac{u}{u_s}\right)dy \tag{2}$$

and finally w and ψ are periodic in θ with respect to 2π.

Here w is vorticity, ψ is stream–function, (u,v) are the velocities in (y,θ) direction, t is time, u_{sp} is the slip velocity predicted by potential flow, the integral in $u_s(\theta,t)$ is Cauchy principal value integral, $\epsilon = Re^{-\frac{1}{2}}$, where Re is the Reynolds number.

RESULTS AND DISCUSSIONS

Our starting solution is obtained by Rayleigh solution at $t = 0.01$ for both boundary layer and interactive studies. To advance the solution beyond this time we have used finite difference scheme. For boundary layer calculations, central difference scheme has been used in both θ and y directions. Upwind/downwind scheme in θ–direction and central difference scheme in y–direction is used in interactive calculations. To enhance the resolution in the neighbourhood of the point where breakdown occurs, say $\theta = \theta_s$, we have introduced grid stretching into our boundary layer calculations. For impulsively moved circular cylinder

$$\frac{d\theta}{d\alpha} = \left\{1+\frac{\cos 2\alpha_s}{2(1+\lambda)^2}\right\}^{-1}\left\{1-\frac{\cos(\alpha-\alpha_s)}{1+\lambda}\right\}\left\{1-\frac{\cos(\alpha+\alpha_s)}{1+\lambda}\right\}$$

and for line vortex in the presence of a circular cylinder

$$\frac{d\theta}{d\alpha} = 1-\frac{\cos(\alpha-\alpha_s)}{1+\lambda}$$

where λ is a parameter, $\alpha = \alpha_s$ corresponds to $\theta = \theta_s$.

The impulsively moved circular cylinder

For the flow around circular cylinder which is impulsively set into motion at time $t = 0$ with speed U_o we take $u_{sp} = \sin\theta$.

It is the viscous displacement velocity V_∞ that might be expected to reveal in a dramatic fashion the presence of a singularity in the solution. Van Dommelen & Shen (1980) show that as $t \to t_s$, $V_\infty \sim C(t_s - t)^{-7/4}$ where C is a constant. Further, as in figure 1, the angular region over which the rapid size in V_∞ takes place diminishes to extent as $t \to t_s$. If $\Delta\theta_s$ is a measure of this it can be shown that $\Delta\theta_s = 0\{(t_s-t)^{3/2}\}$. This demonstrates the need for high resolution close to $\theta = \theta_s$ as $t \to t_s$. We have extended our results to $t = 2.94$ and for $t \geq 2.7$ our plot of $\log V_\infty$ versus $\log(t_s - t)$ yields a straight line of slope $-7/4$, in accord with the singularity by van Dommelen & Shen, with $t_s = 3.01$.

Our interactive results for V_∞ are shown in figure 2(a–c) for Re = 10^6, 10^5 and 10^4 respectively. There is no evidence of the singular behaviour at $t \approx 3$ which is shown to be developing in the boundary layer calculation (figure 1) and in all the cases the extrema are reduced. However beyond t = 3 new features emerge. For Re = 10^5 (figure 2b) in addition to the large positive and negative peaks in V_∞ an additional region in which V_∞ is positive exists for $t > 3.2$. This is due to the existence of individual co-rotating eddies as described by Henkes & Veldman (1987). For Re = 10^4 the flow development is dominated by the rear stagnation point eddy with no apparent individualization of primary eddy. This qualitative change in the behaviour of the flow is in accord with the experimental results of Bouard & Coutenceau (1980) and Ta Phuoc Loc & Bouard (1985).

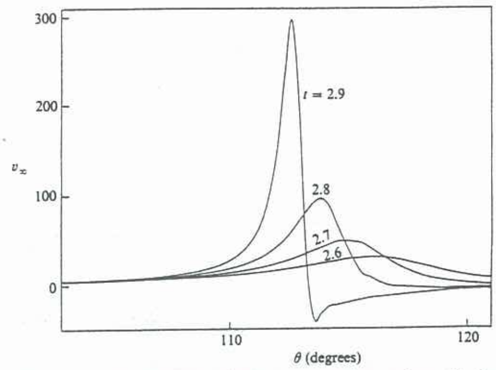

FIGURE 1. The viscous displacement velocity v_∞, at various values of t, from the boundary- layer calculation.

The Vortex and Cylinder

For the flow induced when an inviscid line vortex with circulation Γ is introduced at time t = 0 in the vicinity of, and parallel to, circular cylinder

$$u_{sp} = \frac{1}{2\pi}\left\{1 - \frac{R^2 - 1}{R^2 + 1 - 2R\cos(\theta - H)}\right\}$$

where $H = \pi - \frac{t}{2\pi R^2(R-1)}$.

The vortex is initially at the point (R,π).

Results for boundary layer (V_∞) is shown in figure 3. The dominant feature is the development of singularity at finite time. Our results show that the singular behaviour is as predicted by von Dommelen & Shen (1980) with the breakdown time $t_s = 4.64$. During this time the vortex has moved an angular distance of only $3\frac{1}{2}^0$. This rapid breakdown of the boundary layer solutions is typical of these unsteady boundary layer situations.

In our interactive calculations we have incorporated the effect of displacement on the vortex position and ignored any viscous diffusion of the vortex in accordance with the neglect of terms of $0(\epsilon)$. The vortex position, (R, H) is determined as

$$R = R_o + \int_o^t V dt, \quad H = H_o + \int_o^t \frac{U}{R} dt$$

where

$$U = -\frac{1}{2\pi R(R^2-1)} - \frac{\epsilon}{\pi}\int_o^{2\pi} \frac{V_\infty \sin(\mu - H)}{1 + R^2 - 2R\cos(\mu - H)} d\mu$$

$$V = -\frac{\epsilon}{\pi}\int_o^{2\pi} \frac{(R - \cos(\mu - H))}{1 + R^2 - 2R\cos(\mu - H)} d\mu$$

with (R_o, H_o), the initial vortex position, equal to $(2,\pi)$ in our calculations, and V & U are the radial and transverse components of velocity of the vortex.

Figures 4(a–b) show the results of interactive calculations. We have been able to continue our calculations beyond singular point of the boundary layer solution. The most noticeable feature is a reduction in the peak of V_∞. This maximum is located just ahead of the vortex, which itself has moved an angular distance of only about 4^0 at t = 5.5, beyond that is a region of comparably large negative velocity. This distribution of the V_∞ demonstrates the existence of an eddy, arising from flow separation, just ahead of the vortex.

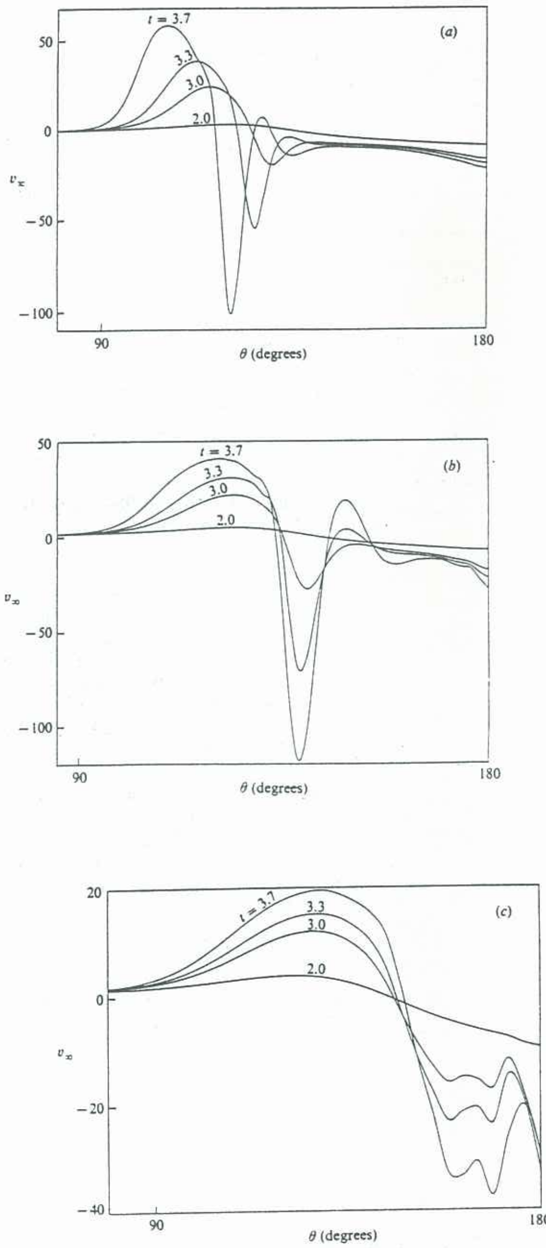

FIGURE 2. The viscous displacement velocity v_∞, at various values of t, from the interactive cal culation. (a) $Re = 10^6$, (b) 10^5, (c) 10^4.

CONCLUSIONS

In the present paper we have developed a method of calculation for unsteady high Reynolds number flows including boundary–layer case based upon vorticity–stream function formulation. The two problems we have considered are (i) the impulsive motion of a circular cylinder and (ii) the flow induced when a line vortex is introduced into the neighbourhood of a circular cylinder. Boundary–layer calculations for these problems terminate in a singular behaviour at a finite time, with a structure that is in accord with the analysis of van Dommelen & Shen (1980). The solutions have been extended to high, but finite, values of the Reynolds number using a viscous–inviscid interaction model which is based upon a thin–layer approximation. Calculations carried out using this model indicate that the boundary–layer singularity is removed.

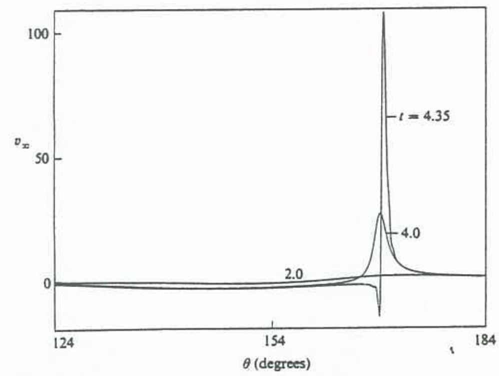

FIGURE 3. The viscous displacement velocity v_∞ at various values of t from the boundary-layer calculation.

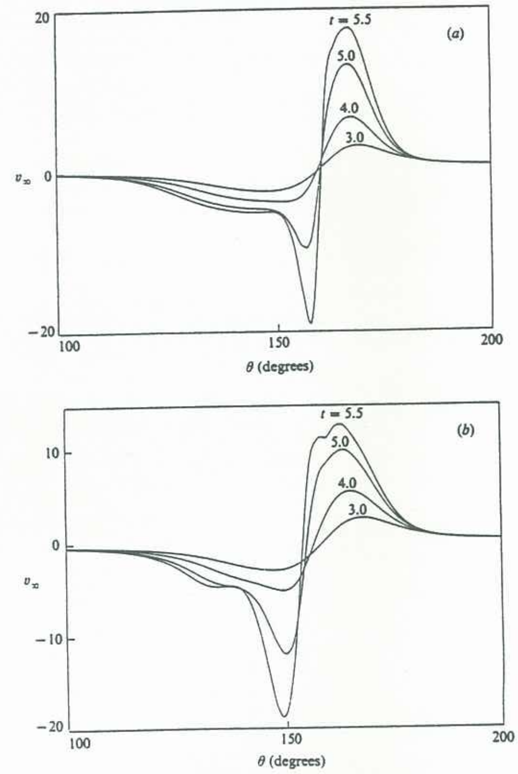

FIGURE 4. The viscous displacement velocity v_∞ at various values of t from the interactive calculation. (a) $Re = 10^6$. (b) 10^5.

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