

MEAN NORMAL PRESSURE FIELD FOR THE NEAR WALL REGION IN
 SINGLE PHASE TURBULENT FLOW

J.D. HOOPER

CSIRO Division of Mineral and Process Engineering
 Lucas Heights Research Laboratories, P.M.B. 7, Menai, NSW 2234
 AUSTRALIA

ABSTRACT

Single phase turbulent pipe flow has been shown both experimentally and analytically to generate a radial pressure field. The radial pressure field has, in the near wall region, a very large radial pressure gradient, when compared to the axial pressure gradient creating the flow. Similarity of the wall region of developed turbulent pipe flow to boundary layer flows not near separation or in strongly favourable pressure gradients, suggests that the mean normal pressure field is a feature of a single phase turbulent boundary layers.

INTRODUCTION

Axially developed single phase turbulent pipe flow has been shown, initially by Patterson et al (1967), to sustain a radial pressure field. The radial pressure field for the flow region away from the pipe wall was experimentally shown by this study to be of the same order of magnitude as that calculated from the radial momentum equation. However, the near wall region of the flow was not investigated.

A further experimental study of the radial pressure field in developed pipe flow was made by Hooper (1980), and again a radial pressure field was detected. The magnitude of the mean radial static pressure variation was calculated by using the turbulence intensities established by Laufer (1954), and the ratio of the maximum radial pressure gradient to the axial pressure gradient was estimated. This ratio is surprisingly high, being at least an order of magnitude greater for the transition region than the axial pressure gradient sustaining the axial flow.

The paper presents an analysis of the normal gradient of the mean static pressure field, which, it is argued, is a little recognised feature of the near wall region of turbulent single phase flow.

COMPUTATION OF THE NEAR WALL PRESSURE FIELD

The presence of a static pressure field normal to the wall for a turbulent single phase boundary layer was noted by Townsend (1956), where

$$dp/dy = -\rho d(\overline{v^2})/dy \quad (1)$$

The boundary layer approximations used to derive this equation are not required in axially symmetric developed single phase turbulent pipe flow.

The mean static pressure field (Laufer, (1954)) is given by:

$$P(x, r, \theta) = Cx + f(r) \quad (2)$$

The radial momentum or Navier-Stokes equation, when reduced to allow for axial flow development and azimuthal symmetry shows:

$$-\frac{df(r)}{dr} - \frac{\rho}{r} \frac{d(\overline{rv^2})}{dr} + \rho \frac{\overline{w^2}}{r} = 0 \quad (3)$$

Integration of equation (3), using established values of the non-axial turbulence intensities v' and w' shows the radial pressure field to be:

$$P_w - f(r) = \rho \left[\overline{v^2} - \int_R^r \frac{\overline{w^2} - \overline{v^2}}{r} dr \right] \quad (4)$$

The turbulence intensities are generally normalised by the wall friction velocity v^* , which is related to the constant axial pressure gradient C by:

$$C = 2 v^{*2} / R \quad (5)$$

Equation (3) restated in terms of the wall co-ordinates and scaled show the ratio of the wall pressure gradient dp/dy to the constant axial pressure gradient C (equation (6)).

$$\frac{1}{C} \frac{df(y)}{dy} = \frac{-R}{2v^{*2}} \left[\frac{d\overline{v^2}}{dy} + \frac{\overline{w^2} - \overline{v^2}}{R-y} \right] \quad (6)$$

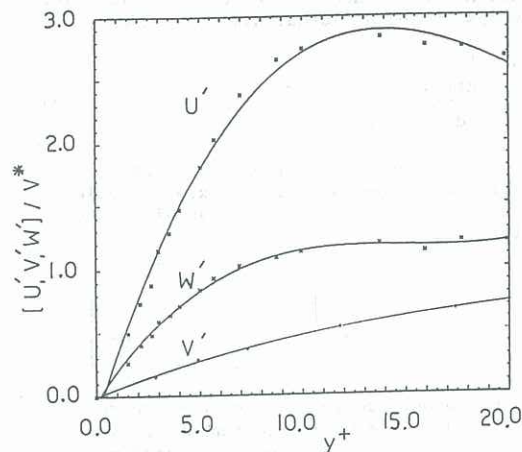
The accuracy of calculating the radial pressure gradient in the near wall region of developed turbulent single phase pipe flow is thus governed by a knowledge of the turbulence intensities v' and w' .

The experimental determination of these components of the Reynolds stresses in the near wall region is difficult. Laufer (1954) made measurements at two Reynolds numbers, 50,000 and 500,000. The lower Reynolds number study showed the behaviour of v' and w' in the transition region.

The near wall region of turbulent single phase flow has been extensively studied since the pioneering work of Laufer. A brief but not comprehensive review of this subject shows a variety of measurement techniques, and an increasing interest in the physical structure of

the near wall flow. Sirkar and Hanratty (1970) used an electrochemical technique to gather data on the limiting behaviour of w' at the wall. Wallace and Brodley (1972) used hot film probes to perform a structural study of the turbulent velocity components near the wall. In a survey paper, Willmarth and Bogar (1977) also presented a structural study, and showed the presence of large, localised pressure excursions migrating across the wall.

Eckelmann (1974), and Kreplin and Eckelmann (1979) used a high aspect ratio rectangular duct with oil as the working fluid to measure u' , v' and w' in the near wall region. The measurement probe was a hot film, and this data is in good agreement with the work of Laufer (1954). The u' , v' and w' distribution in the oil duct established by Kreplin and Eckelmann (1979) is shown by Figure 1, for the region to $20y^+$. A least squares cubic spline curve fitted to the turbulence intensity distributions is used to predict, by Equation (6), the corresponding ratio between the wall pressure gradient and the constant pipe axial pressure gradient C .



Turbulence intensities u' , v' and w' normalised by wall friction velocity v^* , after Kreplin and Eckelmann (1979).

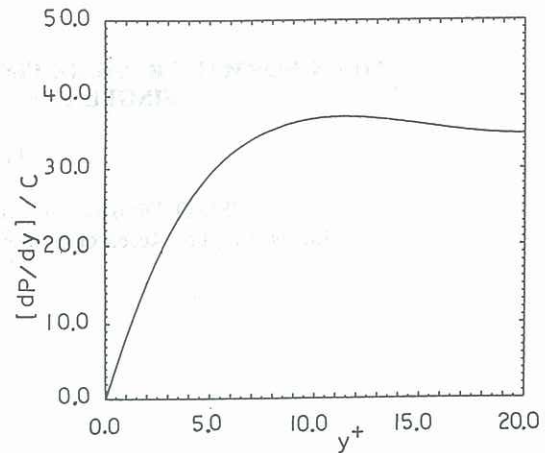
Figure 1

The results of the prediction are shown by Figure 2, for a 200 mm diameter duct using atmospheric pressure air as the working fluid and operating at a nominal Reynolds number of 100,000. Both the axial and radial pressure gradients are negative, and Figure 2 shows the static pressure to fall with increasing wall distance for the given y^+ range. The prediction very near to the wall ($y^+ < 3.0$) is not reliable, as the experimental data (Figure 1) does not allow an accurate prediction of the predominant

term of equation (6), $\frac{d\overline{v^2}}{dy}$ for this region.

However, the maximum radial mean static pressure gradient in the near wall region is approximately 40 times higher than the constant axial pressure gradient C generating the flow.

This pressure ratio is obviously dependent on an accurate knowledge of the relevant Reynolds normal stresses near to the wall. However, its existence is established in the radial momentum equation, and is a significant feature of turbulent pipe flow when compared to laminar flow conditions. It is interesting to

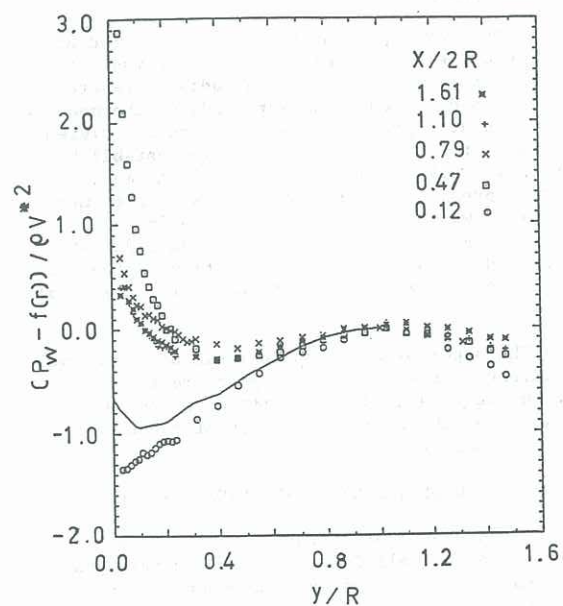


Ratio of the mean static pressure gradient normal to the pipe wall to the constant axial pressure gradient C .

Figure 2

speculate on the structural features of wall turbulence which may be responsible for this effect. Perry (1986) in a keynote paper summarised the details of his own and other research into wall turbulence, and concluded that wall turbulence consisted of a forest of hairpin vortices lifting from the wall. These vortices initially appear as a cylindrical structure rolling along the wall, with the vorticity confined to the azimuthal axis for pipe flow. Each of these structures, appearing at random time intervals, carries an intense pressure field with it along the wall. It may be that the time average of this pressure field at a point on the wall is associated with the mean radial static pressure field.

EXPERIMENTAL RESULTS



Experimental mean radial static pressure distribution in a 127mm diameter pipe as a function of the distance to the exit, Reynolds number 86,500.

Figure 3

The integration of the radial pressure gradient, normalised by the local pipe wall shear stress (equation (4)) shows the variation of the mean static pressure field across a circular duct. The results of Hooper (1980) are shown by Figure 3, for a 127 mm diameter pipe suing air as the working fluid, and operating at a Reynolds number of 86,500. The measurements were made using a Furness micromanometer and a 1.6 mm diameter static pressure probe. Also shown is the numerically integrated mean radial pressure curve, using previously determined normal Reynolds stress components.

The measured mean radial static pressure distribution is seen to be sensitive to the distance from the pipe exit, and is generally of lesser magnitude than the derived curve.

Problems of probe spatial averaging effects, and the sensitivity of the static probe to the non-axial turbulence components v' and w' are discussed by Hooper (1980). The experimental results were shown to be sensitive to both the probe diameter and geometry, and the data cannot be regarded as an absolute determination of the mean radial static pressure distribution. Additionally, to achieve symmetry of the pressure field around the pipe centre line, it was necessary to support the axial probe by a radial traverse arm at least 1.2 pipe diameters downstream of the measurement plane. However, the data substantiates the existence of a radial mean static pressure distribution in developed pipe flow, although the wall region was not resolved.

CONCLUSION

The similarity of the near wall region of a turbulent single phase boundary layer to develop pipe flow, suggests that the predicted strong mean static pressure gradient normal to the wall is present for unseparated turbulent single phase flows. This little recognised feature of turbulent flow has been derived for axisymmetric pipe flow without recourse to boundary layer approximations. The structural features of near wall turbulent flow responsible for this effect have as yet to be identified, but may be related to the known hairpin vortices in this region.

Notation:

C	Constant axial static pressure gradient.
u, v, w	turbulent velocity components
$f(r)$	radial mean static pressure function

v^*	wall friction velocity
ρ	fluid density
P	mean static pressure field
R	pipe radius
y	wall distance ($R-r$)
y^+	dimensionless wall distance.
Superscript '	denotes r.m.s. value
Subscript w	denotes wall value

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