EQUILIBRIUM OF FLOW IN AXIAL FLOW FANS DESIGNED FOR CONSTANT LIFT-DRAG RATIO

P. S. Barna

School Mech. Engineering,
University of New South Wales,
Kensington, N.S.W.

Synopsis – The well-known design method based on free vortex flow through axial flow fans is accompanied by radial equilibrium, constant circulation along the fan blades, and constant axial flow velocity across the fan annulus. These conditions are satisfied if the efficiency along the blade is constant and it may be shown that this can be attained only by varying the lift-drag ratio. The resulting disadvantage is that the blades must work on a lower lift-drag ratio at the hub than at the tip.

An alternative method of design assumes constancy of lift-drag ratio along the blade which corrects the above disadvantage and allows the blades to work at the same lift-drag ratio at the hub as at the tip. This may result in a higher average overall efficiency at the design point. The radial equilibrium, however, is somewhat disturbed, and the axial flow velocity is no longer constant along the blades.

In this paper, for constant pressure rise along the blade, the equilibrium equation is solved by assuming that blade element efficiencies vary linearly over small intervals of flow ratios which is normally the case in conventional axial flow fan designs. Solution of the equation then leads to a simple expression giving the axial velocity distribution; more particularly, it shows that axial velocity is greater at the tip than at the hub, the difference being approximately proportional to the difference in blade element efficiency prevailing at hub and tip.

LIST OF SYMBOLS

a, b, c	Constants (defined in the text)
A	Design parameter
C_L, C_D	Lift and drag coefficients
p_{01}	Total pressure upstream from rotor
p_{02}	Total pressure downstream from rotor
Δp_0	Total pressure rise across rotor

r	Radius
R_H, R_T	Hub and tip radius (resp.)
V_r	Radial velocity
V_x	Axial fluid velocity at radius r
V_{xm}	Mean axial velocity
V_{xH}, V_{xT}	Axial velocity at hub and tip (resp.)
V_t	Velocity of rotor at radius r $(r\omega)$
V_φ	Whirl velocity of fluid leaving the rotor
γ	Lift drag ratio (C_L/C_D)
λ	Flow ratio $(V_x/\omega r)$
σ	Ratio of r to R_T
σ_0	Hub-tip (radius) ratio
η	Efficiency of the blade element (hydraulic)
η_H, η_T	Efficiency at the hub and tip (resp.)
ϱ	Density of fluid
ω	Angular speed of rotor

1. INTRODUCTION

Among the many possibilities, the most conventional method of blade design for axial flow fans specifies a whirl velocity distribution downstream from the rotor which follows the free vortex law, i.e. $V_\varphi = c/r$. It may be shown, that if the blading design satisfies this specification the axial velocity remains constant (with radius) and the streamlines align parallel with the axis. In other words, the flow in transit through the rotor remains in radial equilibrium and the centrifugal force due to whirl is balanced by a radial pressure gradient at all radii. Furthermore, the total pressure rise developed is also constant with radius.

The less conventional blade design methods specify whirl velocity distributions other than the free vortex. A characteristic feature of such designs is that the flow in transit through the rotor is displaced and radial equilibrium is attained only at some distance downstream from the blade trailing edge. The axial velocity no longer remains constant and varies with radius. The total pressure developed may or may not vary with radius depending on the whirl velocity distribution.

The question of how far downstream (or upstream) radial equilibrium is attained has been theoretically studied by various investigators [(1)] on simplified models called actuator discs. In these models, the blade row (moving or stationary) is replaced by a disc of infinitely small axial thickness across which a sudden change in tangential (whirl) velocity and vorticity takes place. The results of such analysis show an exponential type of attenuation of the perturbation axial velocity. In the simple radial equilibrium theory, however, the changes are confined to the transit region and are considered completed by the time the flow leaves the blade trailing edges.

2. BASIC EQUATIONS

In the absence of body forces, for the *radial* direction, Euler's equation of motion for axi-symmetric steady flow of a nonviscous fluid expressed in cylindrical polar coordinates r, φ x, is given by

$$V_r\frac{\partial V_r}{\partial r}+V_x\frac{\partial V_r}{\partial x}-\frac{V_\varphi^2}{r} = -\frac{1}{\varrho}\frac{\partial p}{\partial r} \tag{1}$$

The absolute velocity at blade exit

$$V^2 = V_r^2+V_x^2+V_\varphi^2$$

Differentiating the latter with respect to r gives

$$\frac{1}{2}\frac{\partial V^2}{\partial r} = V_r\frac{\partial V_r}{\partial r}+V_x\frac{\partial V_x}{\partial r}+V_\varphi\frac{\partial V_\varphi}{\partial r} \tag{2}$$

Subtracting Eq. (2) from Eq. (1) yields

$$V_x\frac{\partial V_x}{\partial r}+V_\varphi\frac{\partial V_\varphi}{\partial r}+\frac{V_\varphi^2}{r}-V_x\frac{\partial V_r}{\partial r}=\frac{1}{\varrho}\frac{\partial p}{\partial r}+\frac{1}{2}\frac{\partial V^2}{\partial r} \tag{3}$$

The basic assumption of the radial equilibrium type of design is that the radial velocity V_r is zero at entry and exit from a blade row; thus all terms containing derivatives of V_r are zero. If the flow is incompressible then the total pressure p_0 is given by

$$p_0 = p+\frac{1}{2}\varrho V^2 \tag{4}$$

Further, considering that

$$\frac{1}{2r^2}\frac{\partial}{\partial r}(r^2V_\varphi^2) = V_\varphi\frac{\partial V_\varphi}{\partial r}+\frac{V_\varphi^2}{r}, \text{ and } \frac{1}{2}\frac{\partial V_x^2}{\partial r} = V_x\frac{\partial V_x}{\partial r},$$

Equation (3) may finally be written as

$$\frac{\partial V_x^2}{\partial r} + \frac{1}{r^2}\frac{\partial}{\partial r}(rV_\varphi)^2 = \frac{2}{\varrho}\frac{\partial p_0}{\partial r} \tag{5}$$

Equation (5) will be used to determine the radial distribution of the axial velocity downstream from the rotor.

The *theoretical* total head rise in a rotor is given by the Euler equation

$$V_t V_\varphi = \frac{\Delta p_E}{\varrho} \tag{6}$$

where $\Delta p_E/\varrho$ is referred to as the "Euler head".

In a frictional flow, the total head rise Δp_0 is less than Δp_E due to losses; the blade element efficiency is normally defined as

$$\eta = \frac{\Delta p_0}{\Delta p_E} \tag{7}$$

where p_0 is the Pitot head of the stream.

Thus the actual Pitot head *rise* across the rotor

$$\Delta p_0 = \varrho V_t V_\varphi \eta = \varrho \omega r V_\varphi \eta \tag{8}$$

3. CONSIDERATION OF THE PRESENT PROBLEM

In order to obtain both a desired free whirl distribution ($rV_\varphi = C$) and a constant pressure rise, the blade element efficiency must be kept constant along the blade in accordance with Eq. (8).

It may be shown[(2)] that the blade element efficiency is a function of both the flow ratio $\lambda = V_x/\omega r$ and the lift drag ratio $\gamma = C_L/C_D$. More particularly, assuming small whirl velocities, the efficiency

$$\eta = \frac{\lambda(\gamma - \lambda)^*}{1 + \lambda\gamma} \tag{9}$$

In Fig. 1 based on Eq. (9) curves of η against λ are shown for constant γ values. It appears that for η to be constant, γ must vary. Since, in going from the blade tip towards the hub λ increases, correspondingly γ must decrease for free vortex (whirl) distribution.

* Equation (9) is derived for free whirl distribution and is assumed to hold for small deviations from this condition.

On the other hand if γ is kept constant, η will increase towards the hub resulting in an increase of the average (overall) efficiency of the fan. In this case, however, the whirl velocity distribution no longer follows the law of the free whirl and is given by

$$V_\varphi = \frac{\Delta p_0}{\varrho \omega r \eta} \tag{10}$$

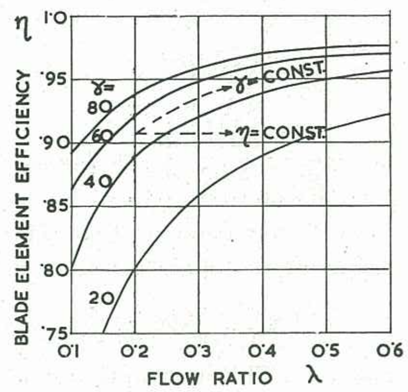

Fig. 1. Variation of blade efficiency with flow ratio for constant lift-drag ratio.

In the case of free whirl distribution and constant pressure rise, derivatives of both p_0 and rV_φ in Eq. (5) vanish, hence

$$\frac{\partial V_x^2}{\partial x} = 0$$

and thus $V_x = \text{const.}$

In the case of constant lift drag ratio, however, $rV_\varphi = f(r)$ and as a result, the axial flow velocity must vary along the blade in accordance with Eq. (5)

4. CALCULATION OF THE AXIAL VELOCITY DISTRIBUTION

Since the total head downstream from the fan for a constant pressure rise is given by

$$p_{02} = p_{01} + \varrho V_t V_\varphi \eta = \text{const.}$$

provided p_{01}, is constant, the term $\partial p_0/\partial r$ in Eq. (5) vanishes. From Eq. (10)

$$V_\varphi r = \frac{\Delta p_0}{\varrho\omega\eta}$$

Hence

$$\frac{1}{r^2}\frac{\partial}{\partial r}(V_\varphi r)^2 = -\frac{2A^2\partial\eta/\partial r}{r^2\eta^3}$$

where

$$A = \Delta p_0/\omega r$$

Thus

$$\frac{\partial V_x^2}{\partial r} - \frac{2A^2}{r^2\eta^3}\frac{\partial\eta}{\partial r} = 0 \tag{11}$$

In order to simplify the integration of Eq. (11) assume the efficiency of the blade element to follow a linear relationship

$$\eta = a - br \tag{12}$$

where a and b are constants. This assumption holds fairly accurately for hub-tip ratios encountered in conventional fan designs. Since $\partial\eta/\partial r = -b$

$$\frac{\partial V_x^2}{\partial r} + \frac{2A^2 b}{r^2\eta^3} = 0 \tag{13}$$

Integration[3] of Eq. (13) with respect to r between limits R_T and R_H leads to

$$V_{xT}^2 - V_{xH}^2 = 2A^2b^2\left[\frac{1}{2a^2}\left(\frac{1}{\eta_T^2} - \frac{1}{\eta_H^2}\right) + \frac{2}{a^3}\left(\frac{1}{\eta_T} - \frac{1}{\eta_H}\right) - \right.$$

$$\left. - \frac{1}{a^3 b}\left(\frac{1}{R_T} - \frac{1}{R_H}\right) - \frac{3}{a^4}\log\frac{\eta_T R_H}{\eta_H R_T}\right] \tag{14}$$

At any radius r the blade efficiency is obtained from the assumed linear relationship

$$\eta = \eta_H - \frac{\eta_H - \eta_T}{R_T - R_H}(r - R_H)$$

Introducing $\sigma_0 = R_H/R_T$, and upon substitution into the expression for η one obtains,

$$\eta = \eta_H + \frac{\eta_H - \eta_T}{1-\sigma_0}\sigma_0 - \frac{\eta_H - \eta_T}{R_T(1-\sigma_0)} r$$

hence the constants in Eq. (12) are now as follows:

$$a = \eta_H + (\eta_H - \eta_T)\frac{\sigma_0}{1-\sigma_0}$$

$$b = \frac{\eta_H - \eta_T}{R_T(1-\sigma_0)} \tag{15}$$

It is noted that the order of $(\eta_H - \eta_T)$ is about 3–7 per cent in conventional designs. Since in Eq. (14) the product

$$A^2b^2 = \left(\frac{\Delta p_0}{\varrho\omega R_T}\right)^2\left(\frac{\eta_H - \eta_T}{1-\sigma_0}\right)^2$$

and contains the square of $(\eta_H - \eta_T)$, all terms in Eq. (14) become small in comparison with the term having b in the denominator and may therefore be neglected.

Hence

$$V_{xT}^2 - V_{xH}^2 \doteqdot 2\left(\frac{\Delta p_0}{\varrho\omega R_T}\right)^2 \frac{\eta_H - \eta_T}{a^3\sigma_0} \tag{16}$$

Assuming that the mean velocity

$$V_{xm} = \frac{V_{xT} + V_{xH}}{2}$$

and that for a first approximation the value of $a \doteqdot 1$, one obtains

$$V_{xT} - V_{xH} \doteqdot \left(\frac{\Delta p_0}{\varrho\omega R_T}\right)^2 \frac{\eta_H - \eta_T}{\sigma_0 V_{xm}} \tag{17}$$

In order to obtain the velocity distribution the integration of Eq. (13) is performed between the limits r and R_H and with the approximation set out previously, one obtains

$$V_x^2 - V_{xH}^2 = 2\frac{A^2b}{a^3}\left[\frac{r - R_H}{R_H r}\right]$$

Introducing $\sigma = r/R_T$ and substituting for A and b,

$$V_x^2 - V_{xH}^2 = \frac{2}{a^3}\left(\frac{\Delta p_0}{\varrho\omega R_T}\right)^2 \frac{\eta_H - \eta_T}{\sigma_0(1-\sigma_0)}\left(1 - \frac{\sigma_0}{\sigma}\right) \tag{18}$$

The calculation of V_{xH} is carried out in the conventional fashion, as follows: it may be shown[4] that for linear velocity distribution the radial position where the local axial velocity equals the specified upstream or mean velocity V_{xm} is given by

$$\sigma_m = \frac{2}{3}\left[1+\frac{\sigma_0^2}{1+\sigma_0}\right]$$

Putting $\sigma=\sigma_m$ and $V_x=V_{xm}$ into Eq. (18).

$$V_{xm}^2 - V_{xH}^2 = \frac{2}{a^3}\left(\frac{\Delta p_0}{\varrho\omega R_T}\right)^2 \frac{\eta_H-\eta_T}{\sigma_0(1-\sigma_0)}\left(1-\frac{\sigma_0}{\sigma_m}\right) \tag{19}$$

Once the value of V_{xH} is found, the axial velocity distribution may be obtained with the aid of Eq. (18). The difference in efficiency is readily found from Eq. (9). However, for rapid estimation of the axial velocity difference between tip and hub, Eq. (17) may be used as a first approximation.

CONCLUSIONS

(a) The design of an axial flow fan rotor for constant lift-drag ratio and total pressure rise along the blade results in axial velocities which are greater at the tip and smaller at the hub than the mean velocity. With the method of calculation developed the axial velocity distribution can rapidly be established.

(b) For a specified pressure rise and tip speed the factor which affects this difference is the hub-tip ratio (and the value of the lift-drag ratio γ). For small values of σ_0 the difference $V_{xT}-V_{xH}$ is larger, whilst for larger σ_0 values the difference may become quite negligible. Calculations show that for conventional designs the difference may be as much as 5 per cent.

(c) Since the overall efficiency of a fan unit consisting of rotor, guide vane and tail fairing decreases with increasing σ_0, designers favour keeping σ_0 as low as possible. This being the case, a design for high efficiency would justify the accurate assessment of the axial velocity distribution.

(d) Designers who base their calculations on constant lift-drag ratios along the blade may assume at first approximation that the axial flow velocity is constant. They are well advised, however, to take into consideration these variations when preparing for their final lay-out.

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