On the Capacity of MIMO Cellular Networks with Macrodiversity

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Abstract—We consider the information theoretic limits of the uplink in a MIMO cellular network with two types of macrodiversity. In the first case, the whole network acts as a multi-access channel where every base station antenna array is part of a central receiver (global macrodiversity). In the second case, we partition the network into clusters of cooperating base stations (local macrodiversity) thereby eliminating the growth in complexity at the receiver with network size. Macrodiversity is shown to provide very significant gains in sum capacity over single cell processing schemes where co-channel interference is the dominant impairment. We also explore the rate at which the sum capacity of the local macrodiversity scheme approaches that of global macrodiversity as the cluster size increases.

I. INTRODUCTION

There has been a great demand for an increase in the spectral efficiency of wireless networks. The use of multiple antennas at each end in a wireless link has been shown to provide promising gains in spectral efficiency of these networks [1], [2]. Indeed MIMO will be an important component of future generation wireless networks [3].

In cellular networks, co-channel interference is the dominant impairment and reduces the capacity of the overall network. MIMO has been shown to provide multiplexing and diversity gains and with the available degrees of freedom it has the potential to null out the co-channel interference not only from intra-cell but also from inter-cell users. In this case a number of recent works have considered single user decoding using MIMO in the presence of co-channel interference from surrounding cells [4]–[6].

Traditional cellular networks have used cell planning and reuse partitioning to minimize the effect of co-channel interference. On the other hand, information theoretic studies of these networks have shown this approach to be suboptimal in terms of the spectral efficiency of the overall network ([7]–[9] to name a few). Hanly and Whiting [7] introduced the concept of base station (BS) cooperation to jointly decode all the users in the cellular network (macrodiversity). They dispersed with the cellular structure altogether and considered the entire network of BSs as antenna elements of a central receiver spread throughout the network. With only a single antenna at each BS and no multi-path fading in the model, it was shown that reuse partitioning was not able to achieve the capacity of a general cellular network. The concept of macrodiversity was independently studied in [10] and later extended to multi-path fading in [9]. In [11], the sum capacity of a general macrodiversity network with MIMO has been characterized for an orthogonal multi-access system such that there exists only one active user in each cell with full frequency reuse. The model is then extended for multiple number of users per cell recently in [12]. Some parallel work in this case considers joint processing of an infinite circular array of cells for single antenna at each user and BS in [13].

In a cellular network with \(M\) cells and \(K\) users per cell, the maximum likelihood decision has complexity up to exponential in the product of \(M\) and \(K\); there is no known algorithm with complexity less than this, except in special cases such as the linear array, see [14]. In terms of implementing the central (global) receiver, a number of practical issues such as delay, synchronization, decentralization etc. have to be resolved as well. Since all these problems require a solution involving the base stations, which are not as limited in resources as the user handsets, many of these issues can be solved. Further, if we drop the requirement of an exact maximum likelihood decision, it may be that one can operate at close to optimal performance, with manageable complexity. For example, recent works have considered message passing algorithms, in which messages are passed between adjacent BSs, so distributing the computations involved in joint, cooperative decoding [14]–[16]. These works have shown close to MAP performance after only a few message passings.

In the present paper, we take a different approach. We divide the network into a number of disjoint local networks, thereby eliminating the growth in the complexity with network size, \(\mathcal{M}\). We measure the tradeoff between capacity and local network size as a function of the number of users per cell and the level of inter-cell interference.

We consider an asymptotic regime where the number of users in each cell and the number of receive antennas at each base station grow large simultaneously keeping their ratio fixed. We use recent results from random matrix theory [17], [18] to characterize our analytical expressions for the spectral efficiency. Although the theory used here has been derived in [18], our context is totally different from that of [18].

II. WYNER CIRCULAR CELLULAR ARRAY MODEL

Wyner [10] provided a very tractable model to capture the effect of inter-cell interference in the cellular network. The linear model consists of an infinite number of cells arranged on a line. The received signal at a BS consists of the active users...
(transmitting on the uplink) of that cell plus a gain $\alpha \in [0, 1]$ times the signals of the active users from adjacent cells. Thus the interference is considered to be caused by adjacent cells' users only, and the parameter $\alpha$ provides a simple way to model the level of inter-cell interference.

We adopt this model to the MIMO setting of the present paper. We consider a finite circular network of $M$ cells such that each BS in a cell can hear signals from the users of exactly two adjacent cells (to avoid edge effects). We consider the asymptotics in which the antenna arrays at the BSs, and the number of users in each cell, grow large together.

We have $K$ active users in each cell with $t$ antennas per user and $N$ antennas at each BS. The received signal at the BS in cell $m$ is then given by:

$$y_{m} = \alpha H_{m(m-1)}x_{m-1} + H_{mn}x_{n} + \alpha H_{m(m+1)}x_{m+1} + n_{m}$$

where $H_{mn}$ represents the (matrix) channel gain from the $K$ users in cell $m$. The channel gain from inter-cell users in cells $m-1$ and $m+1$ is the average gain $\alpha$ times the fast fading gain matrices $H_{m(m-1)}$ and $H_{m(m+1)}$ respectively. Each fast fading channel gain matrix $H_{mi}$ ($i$ can have values of $m-1$, $m+1$ and $m$) is an $N \times K$ matrix. The entries of each of these matrices form an i.i.d Gaussian collection with zero mean, independent real and imaginary parts, each with variance $1/2$. $x_{m}$ is the transmit vector from users in cell $m$, $y_{m} \in \mathbb{C}^{N \times 1}$ is the received signal vector and $n_{m} \in \mathbb{C}^{N \times 1}$ is noise vector and has independent, circularly symmetric complex Gaussian entries; thus, $n_{m} \sim N(0, I_{N})$.

We consider that only the BSs are aware of the channel state information (CSI). We have a per user power constraint such that all the users are transmitting independent signals to the BSs, with the same average transmit power $P$. The spatial covariance matrix of the transmitted signals of the users within each cell is $Q_{i} = \mathbb{E}[x_{i}x_{i}^{\dagger}] = \frac{P}{t} I_{K_{i}}$ [19].

We are interested in characterizing the information theoretic limits of the modified Wyner cellular network using macrodiversity. In the first case, we consider global macrodiversity where the whole network acts as a multi-access channel. The spectral efficiency of this architecture has already been considered in our earlier work in [12] and so here we only provide a review of the related results.

III. GLOBAL MACRODIVERSITY

We now consider the model in which the BSs are cooperating and sharing data to jointly decode all users in the cellular array. Thus, there exists a hyper-receiver that has knowledge of the code books of all the users and has access to data from all the BSs. We consider the received signals at all $M$ BSs in our circular array and from equation (1) we can write it in block form as

$$y = Hx + n$$

where $y = [y_{1}^{T}, \ldots, y_{M}^{T}]^{T}$ is the concatenation of the received signal vectors at all the BSs. Similarly $x = [x_{1}^{T}, \ldots, x_{M}^{T}]^{T}$ is a vector which is a concatenation of the transmitted signals of all the users in the array. $H$ is the overall channel matrix from the users to the BSs in the system and is given as

$$H = \begin{bmatrix}
H_{11} & \alpha H_{12} & 0 & \cdots & \alpha H_{1M} \\
\alpha H_{21} & H_{22} & H_{23} & \cdots & 0 \\
0 & \alpha H_{32} & H_{33} & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\alpha H_{M1} & 0 & 0 & \cdots & H_{MM}
\end{bmatrix}$$

and $n = [n_{1}^{T}, \ldots, n_{M}^{T}]^{T}$ is the noise vector.

Using our earlier result in [11] we can thus write

$$I(x; y, H) = \mathbb{E} \left[ \log \det(I_{MN}) + HQH^{\dagger} \right]$$

where the capacity is achieved with circularly symmetric complex Gaussian transmit signal vector $x$ with zero mean and diagonal covariance matrix [19]

$$Q = \text{diag}(\frac{P}{t} I_{K_{1}}, \ldots, \frac{P}{t} I_{K_{M}}).$$

The relevant large-system result is Theorem 2.54 in [17]. Applying this result to the matrix $HQH^{\dagger}$, we obtain the main result of this section:

Result 1: [12] The asymptotic sum capacity $C_{MAC}$ is given by

$$\lim_{N \to \infty} \left( \frac{1}{MN} D_{MAC} - \min(1, \beta) \log(N) \right) = C_{MAC}$$

where

$$C_{MAC} = \begin{cases}
\log((1+2\alpha^{2})^{\beta}(1-\beta)) + \log(\frac{P}{t} \alpha^{2} \beta) & \beta > 1 \\
\log((1-\beta) (1+2\alpha^{2})^{\beta} + 1) + \log(\frac{P}{t} \beta) & \beta < 1 \\
\log(\frac{P}{t} \beta^{2} \alpha^{2}) & \beta = 1
\end{cases}$$

and $\frac{P}{t} = \frac{2\alpha}{\beta}$.

Using the above result, global macrodiversity has been shown to provide a huge gain in the spectral efficiency of the cellular network as compared to the traditional approach of single cell processing with different reuse partitioning factors [12]. With full reuse in each cell, the gain from macrodiversity increases as the inter-cell interference factor $\alpha$ increases. This is in contrast to the single cell processing schemes where increasing $\alpha$ reduces the spectral efficiency of the network.

IV. LOCAL MACRODIVERSITY

Although global macrodiversity is optimal in an information theoretic sense, the question is how to implement the hyper receiver taking into account practical issues such as synchronization, decentralization and delay. We take a simplified approach to decentralize the joint decoding of all the users in the network. We divide the network into a number of smaller
networks each with an equal number of cells. We limit the cooperation among the BSs only to the cells of each local network. Thus the BSs in each local network will jointly decode the users of that network.

With full frequency reuse in each cell, the cells at the edge of each local network will cause co-channel interference in the adjacent cells which belong to other local networks. To avoid this effect we consider a model where the cells at the edge of each local network do not transmit, i.e., their users will remain silent. However their BSs will be cooperating to jointly decode the users of the remaining cells in the local network.

Let us consider one such local network of \( m \) cells in our circular array of \( M \) cells with \( m < M \). We label the cells of the local network in a set \( N \) as \( N = \{1, 2, \ldots, m\} \). In order to avoid co-channel interference in the adjacent networks the users of cells 1 and \( m \) do not transmit. We denote the set of cells with active users by \( \mathcal{L} = \{2, \ldots, m-1\} \). The BSs of the cells in set \( N \) will be cooperating to jointly decode the users of this local network. The received signal at these \( m \) BSs can be given as

\[
y = \mathbf{H} x + n
\]

where \( \mathbf{y} = [y^T_1, \ldots, y^T_m]^T \) is the concatenation of the received signal vectors and \( \mathbf{n} = [n^T_1, \ldots, n^T_m]^T \) is the concatenation of the noise vectors at \( m \) BSs. \( \mathbf{x} = [x^T_2, \ldots, x^T_{m-1}]^T \) is a concatenation of the transmitted signal vectors of all the users of the cells contained in the set \( \mathcal{L} \).

The mutual information in this case can be given as

\[
I(\mathbf{x}; \mathbf{y}, \mathbf{H}) = E \left[ \log \det(\mathbf{I}_{mN} + \mathbf{HH}^H) \right]
\]

where \( \mathbf{Q} = \frac{P}{m} \mathbf{I}_{(mN)} \) and \( P' = \frac{m-2}{m} P \). This is because \( m-2 \) out of \( m \) cells’ users are transmitting and so the power of each user will be \( \frac{m-2}{m} \) times that when all the cells’ users are transmitting.

We can now characterize the sum capacity of local macrodiversity in a similar fashion to global macrodiversity in the previous section.

**Result 2:** Denote the sum capacity of the local network with cluster size \( m \) as \( C_{\text{local}} \). As the number of users in each cell and the number of antennas at each BS in a cell get large keeping their ratio fixed \( \frac{N}{m} = \beta \), \( C_{\text{local}} \) is given as

\[
\lim_{N \to \infty} \left( \frac{1}{mN} C_{\text{local}} - \min(\beta, \frac{m-2}{m}) \log(N) \right) = \frac{1}{m} \mathcal{C}
\]

where

\[
\mathcal{C} = \sum_{n \in \mathcal{L}} \log \left( 1 + \frac{\beta P' \alpha_n^2}{\omega_n} \right) \frac{1 + \beta \log(1 + \omega_n)}{1 + \beta \log(1 + \omega_n)}
\]

\[
\omega_n = \sum_{l \in \mathcal{L}} \frac{\beta \alpha_l^2}{\omega_l} + \frac{1}{1 + \beta \omega_l}
\]

with \( \omega_n \) and \( \omega_l \) satisfying the following equations

\[
\omega_n = \sum_{l \in \mathcal{L}} \frac{\beta \alpha_l^2}{\omega_l} + \frac{1}{1 + \beta \omega_l}
\]

\[
\omega_l = \sum_{n \in \mathcal{L}} \frac{\beta \alpha_n^2}{\omega_n} + \frac{1}{1 + \beta \omega_n}
\]

respectively for each value of \( \ell \in \mathcal{L} \).

The objective of the current paper is to characterize the sum capacity of the local macrodiversity network, as presented in the previous result. We do not focus on the rates achievable by individual users. Within our current setup, some users will not actually achieve zero rate (since users in edge cells in a cluster do not transmit). Without going into detail, we hint at a simple scheme whereby all users could achieve equal rates.

Suppose time is divided equally into \( m \) transmit time slots (where \( m \) is the number of cells in each local cluster). After each time slot, each local network shifts by one cell in such a way that every cell spends one time slot in each of the \( m \) possible cell positions within a cluster, including the edge positions. In this way, every user will be able to achieve the same rate.

We explain in detail this concept with the help of a simplified example of a cell size \( 3 \) local network.

**Example: Local network with Three Cells \((m = 3)\)**

This is the most simplified case where we have every third cell active at a particular time instant. Suppose we consider the users in cell \( m \) be active in a given time slot with data received at BSs in cell \( m-1, m \) and \( m+1 \) as shown in Figure 2(a). BS \( m \) will receive no interference from cells \( m-1 \) and \( m+1 \) as they will not be active during that time slot. The three BSs (at cells \( m-1, m \) and \( m+1 \)) will cooperate to jointly decode the cell \( m \) users. Using Result 2 from the current section, we can write the sum capacity in this case as

\[
\frac{1}{3N} C_{\text{local}} = \min(1, \beta) \log(N) = \frac{1}{3} \mathcal{C}
\]

where \( C_{\text{local}} \) is the sum capacity of the local network under consideration and

\[
\mathcal{C} = \left( 2 \log \left( \frac{\beta P' \alpha_{m-1}}{\omega_{m-1}} \right) + \frac{1}{1 + \beta \omega_{m-1}} \right) + \frac{1}{1 + \beta \omega_{m-1}} + \frac{1}{1 + \beta \omega_{m+1}}
\]

\[
\omega_{m-1} = \sum_{n \in \{m-1, m, m+1\}} \frac{\beta \alpha_n^2}{\omega_n} + \frac{1}{1 + \beta \omega_n}
\]

and where \( \beta = (\beta - 3) \) and \( P' = 3P \). \( u \) satisfies the following fixed point equation

\[
u = \frac{P'}{1 + \beta \omega_n}
\]
This result can also be obtained directly from using Theorem 2.45 in [17] as the channel matrix in this case is just of $3N \times K$ dimensions.

After the first time slot, each local network shifts by one cell either clockwise or anti-clockwise. For example if we assume clockwise direction and cell $m$ is active in the current time slot, then in next time slot, cell $m+1$ users will be active as shown in Figure 2(b). Now the local network will consist of cells $m$, $m+1$ and $m+2$ for the given time slot. A similar shift would occur for the next time slot leading to the local network shown in Figure 2(c).

A similar concept known as inter-cell time sharing between adjacent cells was introduced by Shamai and Wyner in [8]. The scheme reduces the inter-cell interference of adjacent cells in the Wyner model by allowing the adjacent cells to transmit only for a fraction of the frame of transmission times. The odd-numbered users transmit for a fraction of the frame at the start of frame and even-numbered cells for the same fraction but towards the end of the frame. The scheme has been shown to greatly increase the spectral efficiency of CDMA type systems both with and without fading.

V. NUMERICAL RESULTS

We now use the above theory to numerically evaluate the asymptotic spectral efficiency of the local macrodiversity network and gain some insights about the design of such a cellular network. We compare our results with global macrodiversity and single cell processing schemes. We consider 10 receive antennas at each BS in the network and a single transmit antenna at each user. All the users in the network are considered to be transmitting at equal power of 20 dB above noise level. Defining $\beta$ to be the ratio of the number of users in each cell to the number of antennas at the BS, we plot the sum capacity normalized by the product of the number of cells and $N \log N$ against $\beta$. We vary the number of users in each cell from 1 to 100 and corresponding $\beta$ varies from 0.1 to 10.

In the case of local macrodiversity, we consider cluster sizes of 3, 4 and 10. The users of the cells at the edge of each local network will not be transmitting. We consider three different values of inter-cell data factor $\alpha$ as shown in Figure 3. The dashed lines correspond to the analytical results (considering a normalization using value of $N = 10$) and the dot signs (circle, plus etc.) are for Monte-carlo simulations obtained by averaging over 1000 realizations of the channel matrix.

As shown, the normalized sum capacity increases for both macrodiversity (global and local) networks as $\alpha$ increases. This is because increasing $\alpha$ increases the receiver diversity and the amount of received power collected from each user. The normalized sum capacity of the local macrodiversity network is smaller than the one for global macrodiversity. As the size of local network increases, the normalized capacity approaches that of global macrodiversity. Even with reasonable cluster size of 10 cells, little is lost compared to global macrodiversity, especially when $\alpha = 1$.

In the case of global macrodiversity, the normalized sum capacity increases approximately linearly with the number of users when $\beta < 1$. When $\beta > 1$, there is only a logarithmic gain. The curve exhibits a knee at $\beta = 1$ and which makes this an attractive operating point as beyond that ($\beta > 1$) we only get a logarithmic gain in spectral efficiency at the cost of exponentially increasing complexity of the receiver. This knee effect is also evident in local macrodiversity networks but its location depends on the size of local network. For example in case of a local network of three cells ($m = 3$), the knee occurs at $\beta = 3$. For the given 10 antennas at each of the three BSs, the local receiver can jointly decode a maximum of 30 users in the active cell to operate at the knee. As the size of a local network increases, the knee shifts from $\beta = 3$ toward $\beta = 1$.

We also compare our results with the single cell processing schemes studied in [12]. The spectral efficiency of local macrodiversity is larger than single cell processing (SCP) schemes for almost all values of $\alpha$. The exception occurs at low $\alpha$ ($\leq 0.2$) and low $\beta$ ($\leq 1.5$) where the SCP scheme with half reuse performs better than local macrodiversity with $m = 3$. However in a multiuser system, operating at low $\beta$ is very inefficient as we are wasting the available system resources by not operating close to the knee.

VI. CONCLUSIONS

We have considered the information theoretic limits of cellular networks employing macrodiversity. We provide analytical expressions for the sum capacity of the whole network where all the users are jointly decoded by a centralized receiver (global macrodiversity). Owing to the complex nature of this receiver due to centralized processing of all the users, we propose local macrodiversity; the whole network is divided into small macrodiversity networks with each one jointly decoding the users only local to itself. We keep the cells at the edge of each local network silent (users not transmitting) to avoid co-channel interference among these networks. It turns out that local macrodiversity provides higher spectral efficiency gains than single cell processing schemes. These gains approach the spectral efficiency of global macrodiversity for quite moderately sized local macrodiversity networks.

REFERENCES

3: Comparison of normalized spectral efficiency for macrodiversity (global) and local and single cell processing (SCP) schemes.