Complexity Reduction for Parameter-Dependent Linear Systems

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These complex systems typically (a) have a high order and (b) are composed of many interconnected subsystems where (c) each one has many unknown parameters.
What makes a system model complex?

• Many state variables:
  - Hard to design and implement controllers;
  - Hard to simulate and store their information.

• Many model parameters:
  - Hard to implement and maintain a controller that depends on many parameters;
  - Hard to gather and store the parameter information.

What can we do about this?

• Find the closest (in some appropriate norm) reduced system (fewer states and parameters).

Farokhi, Sandberg, and Johansson (KTH)
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Related Literature

- **Model reduction:**
  Moore ’81, Glover ’84, Obinata and Anderson ’01, Antoulas ’05.

- **Parameter dependent model reduction:**
  Beck, Doyle, and Glover ’96, Beck ’97, Halevi, Zlochevsky, and Gilat ’97, Dullerud and Paganini ’00, Li and Petersen ’10, Sandberg ’13.

- **Parameter dependent linear matrix inequalities:**
  Apkarian and Tuan ’00, Parrilo and Lall ’03, Bliman ’04, Chesi, Garulli, Tesi, and Vicino ’05, Calafiore and Campi ’05, Scherer and Hol ’06

- **Gain scheduling and supervisory control:**
  Packard ’94, Rugh and Shamma ’00, Scorletti and Ghaoui ’98, Morse 96’, Leith and Leithead ’00, Dullerud and Paganini ’00.
Problem Formulation

- Given original system:

\[ G(s; \alpha) : \begin{cases} \dot{x}(t) = A(\alpha)x(t) + B(\alpha)u(t), \\ y(t) = C(\alpha)x(t) + D(\alpha)u(t), \end{cases} \]

with \( x(t) \in \mathbb{R}^n, u(t) \in \mathbb{R}^m, y(t) \in \mathbb{R}^o, \) and \( \alpha \in \mathcal{A} \subseteq \mathbb{R}^p. \) The model matrices are not necessarily linear in their parameters.
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- Find reduced system:

\[ G'(s; \alpha') : \begin{cases} 
\dot{x}'(t) = A'(\alpha')x'(t) + B'(\alpha')u(t), \\
y'(t) = C'(\alpha')x'(t) + D'(\alpha')u(t), 
\end{cases} \]

with \( x'(t) \in \mathbb{R}^{n'}(n' \leq n) \), \( u(t) \in \mathbb{R}^m \), \( y(t) \in \mathbb{R}^o \), and \( \alpha' \in \mathbb{R}^{p'}(p' \leq p) \).
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with \( x'(t) \in \mathbb{R}^{n'} (n' \leq n), u(t) \in \mathbb{R}^m, y(t) \in \mathbb{R}^o, \) and \( \alpha' \in \mathbb{R}^{p'} (p' \leq p). \)

- Complexity reduction

\[ \inf_{G'(s; \alpha')} \sup_{\alpha \in \mathcal{A}} \left\| G(s; (\alpha_i)_{i=1}^p) - G'(s; (\alpha_i)_{i=1}^{p'}) \right\|_{\infty}. \]
Complexity Reduction with General $P(\alpha)$

Lemma

For a given $G'(s; \alpha')$,

$$
\sup_{\alpha \in A} \| G(s; (\alpha_i^p)_{i=1}^p) - G'(s; (\alpha_i^p)_{i=1}^p') \|_\infty \leq \gamma,
$$

if and only if there exists $P(\alpha) = P(\alpha) \top \in \mathbb{R}^{(n+n') \times (n+n')}$ such that $P(\alpha) \geq 0$ and

$$
\begin{bmatrix}
\tilde{A}(\alpha) \top P(\alpha) + P(\alpha) \tilde{A}(\alpha) & * & * \\
\tilde{B}(\alpha) \top P(\alpha) & -I & * \\
\tilde{C}(\alpha) & \tilde{D}(\alpha) & -\gamma^2 I
\end{bmatrix} \leq 0, \quad \forall \alpha \in A.
$$

Where

$$
\tilde{A}(\alpha) = \begin{bmatrix} A((\alpha_i^p)_{i=1}^p) & 0 \\ 0 & A'((\alpha_i^{p'})_{i=1}^{p'}) \end{bmatrix}, \quad \tilde{B}(\alpha) = \begin{bmatrix} B((\alpha_i^p)_{i=1}^p) \\ B'((\alpha_i^{p'})_{i=1}^{p'}) \end{bmatrix},
$$

$$
\tilde{C}(\alpha) = \begin{bmatrix} C((\alpha_i^p)_{i=1}^p) & -C'((\alpha_i^{p'})_{i=1}^{p'}) \end{bmatrix}, \quad \tilde{D}(\alpha) = D((\alpha_i^p)_{i=1}^p) - D'((\alpha_i^{p'})_{i=1}^{p'}).$$
Lemma

For a given $G'(s; \alpha')$,

$$\sup_{\alpha \in \mathcal{A}} \|G(s; (\alpha_i)_{i=1}^p) - G'(s; (\alpha_i)_{i=1}^{p'})\|_{\infty} \leq \gamma,$$

if and only if there exists $P(\alpha) = P(\alpha)^\top \in \mathbb{R}^{(n+n') \times (n+n')}$ such that $P(\alpha) \succeq 0$ and

$$\begin{bmatrix}
\tilde{A}(\alpha)^\top P(\alpha) + P(\alpha) \tilde{A}(\alpha) & * & * \\
\tilde{B}(\alpha)^\top P(\alpha) & -I & * \\
\tilde{C}(\alpha) & \tilde{D}(\alpha) & -\gamma^2 I
\end{bmatrix} \leq 0, \ \forall \alpha \in \mathcal{A}.$$

- $P(\alpha)$ can be an arbitrary function of $\alpha$;
- The LMIs need to be checked for all $\alpha \in \mathcal{A}$. 
Lemma

Let $\mathcal{A}$ be a compact subset of $\mathbb{R}^p$. Then, for a given $G'(s; \alpha')$, we have

$$\sup_{\alpha \in \mathcal{A}} \| G(s; (\alpha_i)_{i=1}^p) - G'(s; (\alpha_i')_{i=1}^{p'}) \|_{\infty} \leq \gamma,$$

if and only if there exists $P(\alpha) \in \mathbb{R}[\alpha]^{(n+n') \times (n+n')}$, such that $P(\alpha) \geq 0$ and

$$\begin{bmatrix}
\tilde{A}(\alpha)^\top P(\alpha) + P(\alpha) \tilde{A}(\alpha) & * & * \\
\tilde{B}(\alpha)^\top P(\alpha) & -I & * \\
\tilde{C}(\alpha) & \tilde{D}(\alpha) & -\gamma^2 I
\end{bmatrix} \leq 0, \ \forall \alpha \in \mathcal{A}.$$

- $P(\alpha)$ is a polynomial matrix.
Assumptions

- The model matrices of the original system are polynomials, that is, 
  $A(\alpha) \in \mathbb{R}[\alpha]^{n \times n}$, $B(\alpha) \in \mathbb{R}[\alpha]^{n \times m}$, $C(\alpha) \in \mathbb{R}[\alpha]^{o \times n}$, and $D(\alpha) \in \mathbb{R}[\alpha]^{o \times m}$. 
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- We search over the set of polynomial matrices to find the reduced system, that is, $A'(\alpha') \in \mathbb{R}[\alpha']^{n' \times n'}$, $B'(\alpha') \in \mathbb{R}[\alpha']^{n' \times m}$, $C'(\alpha') \in \mathbb{R}[\alpha']^{o \times n'}$, and $D'(\alpha') \in \mathbb{R}[\alpha']^{o \times m}$.
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• The parameters belong to a semi-algebraic and compact set, that is, $\mathcal{A}$ is a compact subset of $\mathbb{R}^p$ where

\[ \mathcal{A} = \{ \alpha \in \mathbb{R}^p \mid q_\ell(\alpha) \geq 0, \forall 1 \leq \ell \leq L \} \text{ for } q_\ell \in \mathbb{R}[\alpha], 1 \leq \ell \leq L, \]

such that there exist sum-of-square polynomials $w_\ell$, $0 \leq \ell \leq L$, so that

\[ \{ \alpha \in \mathbb{R}^p \mid w_0(\alpha) + \sum_{\ell=1}^{L} q_\ell(\alpha)w_\ell(\alpha) \geq 0 \} \text{ is a compact set.} \]
Lemma

Let $\mathcal{A}$ be a compact semi-algebraic subset of $\mathbb{R}^p$. Then, for a given $G'(s; \alpha')$, we have

$$\sup_{\alpha \in \mathcal{A}} \left\| G(s; (\alpha_i)_{i=1}^p) - G'(s; (\alpha_i)_{i=1}^{p'}) \right\|_{\infty} \leq \gamma,$$

if there exist $\epsilon > 0$, $P(\alpha) \in S[\alpha]^{n+n'}$, and $Q_\ell(\alpha) \in S[\alpha]^{n+n'+m+o}$ for $1 \leq \ell \leq L$, such that

$$\begin{bmatrix}
\tilde{A}(\alpha)^\top P(\alpha) + P(\alpha) \tilde{A}(\alpha) & * & * \\
\tilde{B}(\alpha)^\top P(\alpha) & -I & * \\
\tilde{C}(\alpha) & \tilde{D}(\alpha) & -\gamma^2 I
\end{bmatrix} + \epsilon I + Q_0 + \sum_{\ell=1}^L Q_\ell(\alpha) q_\ell(\alpha) = 0.$$

- Set of sum-of-square matrices
  $$S[\alpha]^n = \{ X(\alpha) \mid \exists Y(\alpha) \in \mathbb{R}[\alpha]^{n \times n} \text{ s.t. } X(\alpha) = Y(\alpha)^\top Y(\alpha) \};$$
Lemma

Let $A$ be a compact semi-algebraic subset of $\mathbb{R}^p$. Then, for a given $G'(s; \alpha')$, we have

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- Fewer conditions to check at the price of getting a sufficient condition;
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  $S[\alpha]^n = \{X(\alpha) \mid \exists Y(\alpha) \in \mathbb{R}[\alpha]^{n \times n} \text{ s.t. } X(\alpha) = Y(\alpha)^\top Y(\alpha)\}$;
- Fewer conditions to check at the price of getting a sufficient condition;
- A numerical procedure for the complexity reduction problem given in the paper.
Solving the complexity reduction problem with general or polynomial $P(\alpha)$ is very difficult.

The proposed method, which relies on sum-of-square $P(\alpha)$, does not scale well with the number of parameters and the system dimension.

- If we use the structure of the system to represent the feasible set with smaller LMIs, the computational complexity grows slower;
- Try to exploit sparsity patterns or symmetry structures in future to develop better numerical algorithms;
- cf., Kojima, Kim, and Waki '05, Waki, Kim, Kojima, and Muramatsu '06, Nie '09, Hancock and Papachristodoulou '11.
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- exponentially in terms of the number of the parameters;
- polynomially in terms of the dimension of the system and the order of the polynomials.

What can we do about it?

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- cf., Kojima, Kim, and Waki ’05, Waki, Kim, Kojima, and Muramatsu ’06, Nie ’09, Hancock and Papachristodoulou ’11.
We compare our method with the approach presented in Beck, Doyle, and Glover ’96.

Consider

\[ G(\alpha) : \begin{cases} 
\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} &= \begin{bmatrix} 0.5\alpha_1 & 0.1 \\ 0.3 & 0.5\alpha_2 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(k), \\
 y(k) &= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}, 
\end{cases} \]

where

\[ \mathcal{A} = \{ \alpha \in \mathbb{R}^2 | 1 - \alpha_i^2 \geq 0 \text{ for } i = 1, 2 \} . \]
Numerical Example

For cases where $n' = 2$ and $n' = 1$:

- $G'(\zeta; \alpha_1)$: Use the presented method with polynomial degrees $d_A = d_B = d_C = d_D = 1$, $p' = 1$, $d_P = d_{Q_0} = 2$, and $d_{Q_1} = d_{Q_2} = 0$.

- $G_r(\zeta; \alpha_1)$: Use the method of Beck, Doyle, and Glover ’96 to extract the reduced system for $p' = 1$. 
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- $G_r(z; \alpha_1)$: Use the method of Beck, Doyle, and Glover '96 to extract the reduced system for $p' = 1$.

We choose the set of model matrices that are affine in the parameters to make the comparison fair. Using the presented algorithm, we can indeed achieve a lower error if we decide to pick a richer set of model matrices.
Numerical Example

For cases where $n' = 2$ and $n' = 1$:

- $G'(\tilde{\theta}; \alpha_1)$: Use the presented method with polynomial degrees $d_A = d_B = d_C = d_D = 1$, $p' = 1$, $d_P = d_{Q_0} = 2$, and $d_{Q_1} = d_{Q_2} = 0$.

- $G_r(\tilde{\theta}; \alpha_1)$: Use the method of Beck, Doyle, and Glover '96 to extract the reduced system for $p' = 1$.

<table>
<thead>
<tr>
<th></th>
<th>$n' = 2$ $p' = 1$</th>
<th>$n' = 1$ $p' = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\max_{\alpha \in A} | G_r(\tilde{\theta}; \alpha_1) - G(\tilde{\theta}; \alpha_1, \alpha_2) |_{\infty}$</td>
<td>0.14</td>
<td>0.27</td>
</tr>
<tr>
<td>$\max_{\alpha \in A} | G'(\tilde{\theta}; \alpha_1) - G(\tilde{\theta}; \alpha_1, \alpha_2) |_{\infty}$</td>
<td>0.09</td>
<td>0.19</td>
</tr>
<tr>
<td>Improvement</td>
<td>35%</td>
<td>30%</td>
</tr>
</tbody>
</table>
Numerical Example: Power Network

Fig: Schematic diagram of the power network in our numerical example.

\[
\begin{align*}
\dot{\delta}_1(t) &= \omega_1(t), \\
\dot{\omega}_1(t) &= \frac{1}{M_1} \left[ P_1(t) - c_{12}^{-1} \sin(\delta_1(t) - \delta_2(t)) - c_1^{-1} \sin(\delta_1(t)) - D_1 \omega_1(t) \right], \\
\dot{\delta}_2(t) &= \omega_2(t), \\
\dot{\omega}_2(t) &= \frac{1}{M_2} \left[ P_2(t) - c_{12}^{-1} \sin(\delta_2(t) - \delta_1(t)) - c_2^{-1} \sin(\delta_2(t)) - D_2 \omega_2(t) \right],
\end{align*}
\]

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$M_1$</th>
<th>$M_2$</th>
<th>$c_{12}$</th>
<th>$c_1$</th>
<th>$c_2$</th>
<th>$D_1$</th>
<th>$D_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal Value (p.u.)</td>
<td>0.026</td>
<td>0.032</td>
<td>0.40</td>
<td>0.50</td>
<td>0.50</td>
<td>0.0064</td>
<td>0.0064</td>
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\[ \dot{\delta}_2(t) = \omega_2(t), \]
\[ \dot{\omega}_2(t) = \frac{1}{M_2} \left[ P_2(t) - c_{12}^{-1} \sin(\delta_2(t) - \delta_1(t)) - c_2^{-1} \sin(\delta_2(t)) - D_2\omega_2(t) \right], \]

where
\[ A = \{ \alpha \in \mathbb{R}^2 \mid 0.1^2 - \alpha_i^2 \geq 0 \text{ for } i = 1, 2 \}. \]
Numerical Example: Power Network

After linearizing the system, we get

\[
G(s; \alpha) : \begin{cases}
\dot{x}(t) = A(\alpha_1, \alpha_2)x(t) + Bu(t), \\
y(t) = Cx(t) + Du(t).
\end{cases}
\]

where

- \(u(t)\) is mechanical power injected to the first generator;
- \(y(t)\) is the changes in the angle of the terminal voltage of the first generator.
Numerical Example: Power Network

Fig: Schematic diagram of the power network in our numerical example.

Let us fix $d_A = 1$, $d_B = d_C = d_D = 0$, $d_P = 3$, and $d_{Q_i} = 2$ for $i = 0, 1, 2$. Then, use the proposed algorithm to extract

$$G'(s; \alpha_1) : \begin{cases} \dot{x}(t) = A'(\alpha_1)x(t) + B'u(t), \\ y(t) = C'x(t) + D'u(t). \end{cases}$$

For this reduced system, we get

$$\sup_{\alpha \in A} \|G'(s; \alpha_1) - G(s; \alpha)\|_\infty = 0.15,$$

which corresponds to $0.3 \sup_{\alpha \in A} \|G(s; \alpha)\|_\infty$. 
Conclusions

- Complexity reduction = Model reduction + Parameter reduction;
- Computational method for complexity reduction based on sum-of-squares;
- Illustrated on a numerical example.

Future Work:
- A less computationally-intensive algorithm using randomized algorithms;
- Apply to Controller reduction problem.

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