

# Kriging for Localized Spatial Interpolation in Sensor Networks

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**Abstract.** The presence of coverage holes can adversely affect the accurate representation of natural phenomena being monitored by a Wireless Sensor Network (WSN). Current WSN research aims at solving the coverage holes problem by deploying new nodes to maximize the coverage. In this work, we take a fundamentally different approach and argue that it is not always possible to maintain exhaustive coverage in large scale WSNs and hence coverage strategies based solely on the deployment of new nodes may fail. We suggest spatial interpolation as an alternative to node deployment and present Distributed Kriging (DISK), a localized method to interpolate a spatial phenomenon inside a coverage hole using available nodal data. We test the accuracy and cost of our scheme with extensive simulations and show that it is significantly more efficient than global interpolations.

**Key words:** Wireless Sensor Networks, Coverage Holes, Interpolation, Kriging

## 1 Introduction

Monitoring physical phenomenon is an important application domain for wireless sensor networks (WSNs). The WSN data acquisition techniques accomplish a monitoring task by sampling data at different network locations over time. Currently, these techniques advocate the use of data suppression as a means of achieving energy efficiency [1]. The case for data suppression is based on the assumption that there is always an abundance of samples in a WSN and hence reporting accuracy is not compromised even if the data from a large part of the network is suppressed. However, experience with WSNs [2] and insights into future applications [3, 4] reveal that situations may arise where the assumption of an abundance of samples does not hold. Such situations arise when node failures or the sparsity of a WSN trigger *gaps* or *coverage holes* in the reported data. We argue that for an accurate representation of a physical phenomenon in such scenarios *augmenting* the reported data as well as its *suppression* is a priority. In this paper, we investigate such situations and propose novel interpolation methods to augment the reported data in the presence of gaps.

Existence of coverage holes is an important reality for WSNs and has been studied extensively [2]. Current research has predominantly focused on the identification of holes in order to alert and restore the lost coverage of a WSN [5]. However, the replacement and restoration of nodes may not be sensible in hostile environments or not

possible due to prohibitive costs. Therefore, our assertion is that coverage issues require data acquisition regimes that cope with missing data through other means than simply replacing or deploying more nodes. Thus, we need methods to interpolate the missing readings based on the available data and application specific expert knowledge.

Physical phenomenon are characterized by their spatial correlation, i.e., the fact that proximal locations have similar values and vary together. Spatial interpolation techniques can thus be used to estimate a phenomena in coverage holes. The challenge, however, is that typical interpolation techniques are not readily applicable to WSNs due to their reliance on global knowledge of the network [6]. Due to a WSNs' dynamic nature and large scale, such global information is prohibitively expensive to collect and maintain.

The key challenge that we address in this work is to perform accurate spatial interpolation for coverage holes with minimal power requirements. To maximize the use of available information we propose to first build a correlation model of the phenomenon under observation and then perform interpolation using this model. We first present the QS (Quad Suppress) algorithm, a distributed in-network aggregation algorithm for correlation modeling of a phenomenon. We then present the DISK (DISTRIBUTED Kriging) algorithm which utilizes the correlation model to perform interpolation in a fully distributed manner. With extensive simulations we show that QS and DISK are significantly more energy efficient than their global counterparts.

## 2 The Quad Suppress (QS) Algorithm

The first step towards localized spatial interpolation is to find an appropriate variogram model that best describes the spatial correlation in a dataset. The experimental variogram (EV) is a measure of spatial continuity in a spatial process defined as average squared difference between data values at a certain distance, called lag,  $h$  [7]. Assume a random variable  $Z$  represents a Gaussian spatial process and  $Z(x)$  represent its realizations at location  $x$  then its EV can be given as:

$$2\gamma(h) = \frac{1}{N(h)} \sum_{N(h)} [Z(x) - Z(x+h)]^2. \quad (1)$$

where,  $N(h)$  is the number of data pairs at distance  $h$ . A variogram model is a curve fitted on the observed EV values.

Equation 1 shows that for a certain lag  $h$ , EV construction requires the difference of the value of each node from all nodes in its EV neighborhood, i.e., present within a distance of  $h$  units from itself. Thus, a simple global approach for EV construction could be to propagate all samples to a base station. However, to reduce communication costs we propose the QS algorithm, an alternative based on in-network aggregation. In-network aggregation algorithms organize the network in a tree like fashion such that each internal node aggregates all data coming from its child nodes and communicate only the partial aggregate to its parent. However, for EV construction there is an added constraint; an internal node can aggregate a child node, say  $K$ , only if it can ensure that it is also a parent of all nodes in  $K$ 's EV neighborhood. The QS algorithm adopts a quadtree-like tree creation method to fulfill this constraint.

The QS aggregation tree is built as follows. The base station partitions the space and chooses a cell-head for each quadrant. These cell-heads recursively partition their cells choosing new sub-cell heads. The process continues until a predetermined grid resolution is reached. A random aggregation tree is then built inside each of the unpartitioned cells rooted at the corresponding cell-head. Data aggregation is performed at each internal node of this tree. The benefit of creating the aggregation tree in a quadtree-like manner is that a cell-head can locally determine whether or not it covers the EV region of a child node. It can then accordingly aggregate or forward the node's value.

Figure 1 shows the considerable difference in energy expenditure of the QS algorithm and a random tree based global data collection using 2500 samples from Digital Elevation Model (DEM) dataset [8] covering a 500 m<sup>2</sup> area of the state of Colorado, US. The significant difference in the performance of the two algorithms can be explained by their data forwarding behavior. The global data collection scheme creates the aggregation tree randomly, thus an internal node cannot determine whether or not it covers the EV neighborhood of its child nodes. Consequently, all internal nodes propagate their data to the base station in unaggregated form. On the other hand, the QS algorithm reduces the communication costs significantly by performing the aggregation in the network as soon as it realizes that a node's EV neighborhood is covered.

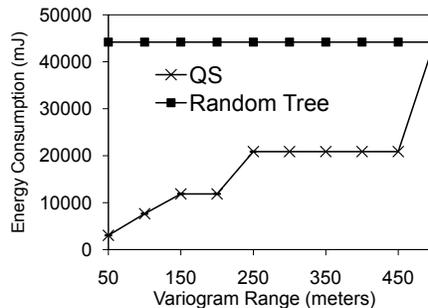


Fig. 1. Comparison of energy expenditure in QS and random tree algorithms for EV construction.

### 3 Distributed Kriging (DISK) Algorithm

Once a variogram model is established for a given phenomenon, it can then be used for spatial interpolation using Kriging. Kriging is a well known geostatistical method used to estimate unknown values of a physical process using existing knowledge about the process and a model of its spatial variation, i.e., the variogram [7]. Assume that the values  $Z(x_1), Z(x_2), \dots, Z(x_N)$  represent realizations of a spatial process  $Z$  at locations  $x_1, x_2, \dots, x_N$ , then the Kriging interpolator of  $Z$  at a point  $x_0$  is given by [9]:

$$\hat{Z}(x_0) = \sum_{i=1}^N \lambda_i Z(x_i). \quad (2)$$

where  $\lambda_i$  are the weights fulfilling the unbiasedness condition, i.e.,  $\sum_{i=1}^N \lambda_i = 1$  and the expected error is  $E[\hat{Z}(x_0) - Z(x_0)] = 0$  [7]. It can be shown that optimal weights  $\lambda_i$  for the Kriging interpolator can be computed from the following system of linear equations (SLE)

$$A = A^{-1}b. \quad (3)$$

where  $A$  is a vector comprising of Kriging weights  $\lambda_i$  and a Lagrange multiplier (added for computational reasons),  $A$  is the covariance matrix of sample locations  $x_1, x_2, \dots, x_N$  and  $b$  is a vector whose elements represent the covariance between  $x_0$  and each  $x_i \in \{x_1, x_2, \dots, x_N\}$ . All covariances are based on an appropriate variogram model defined for the spatial process in question. We first use the QS algorithm to build a variogram model for the entire WSN and distribute this model in the network. In this way all nodes can autonomously compute their correlation with any other node in the network as required during the Kriging process.

In the following subsection we explain our interpolation approach, the DISK algorithm, which in essence is a distributed and localized form of the Kriging interpolation.

### 3.1 Iterative Formulation of Kriging System of Linear Equations

The basic building block of the DISK algorithm is an iterative approach towards the Gaussian elimination method. In terms of the Kriging SLE, the Gaussian elimination method can be interpreted as the process of finding a sequence of elementary row operations, or linear maps, that transforms matrix  $A$  to its reduced row-echelon form. The basic idea of our iterative elimination approach is presented in Figure 2. If the nodes performing a Kriging operation are assumed to be aligned along a chain, each node  $k$  adds a new variable, i.e., its Kriging weight ( $\lambda_k$ ), and its corresponding linear combination ( $\sum_{i=1}^k \lambda_i c_{ki} = b_k$ ) to the Kriging SLE. In matrix terms, each node in the chain adds a new row and column to the matrix  $A$  required to be inverted while not changing the original entries. We can then order the elimination process on the basis of the following recursive formulation of matrix  $A$ :

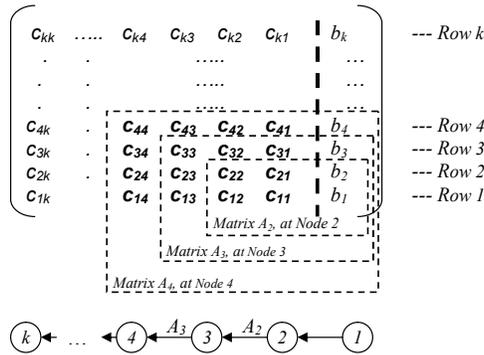


Fig. 2. Iteratively building Kriging SLE

$$A_k = \begin{bmatrix} 1 & K^T \\ K & A_{k-1} \end{bmatrix} K = \begin{bmatrix} c_{k(k-1)} \\ \dots \\ c_{k1} \end{bmatrix} A_2 = \begin{bmatrix} c_{22} & c_{21} \\ c_{12} & c_{11} \end{bmatrix} \quad (4)$$

where  $k \geq 2$ .

Now for  $A_k$ , we define  $\Phi(T_k) : A_k \rightarrow A_k^\Phi$  as a group of all linear maps enumerated by the following recursive definition:

$$\Phi(T_{k-1}) . \quad (5)$$

$$R_k \leftarrow R_k + (-k_{1i} \times R_{k-i}) \quad \forall i \in \{1, 2 \dots k-1\} . \quad (6)$$

$$R_k \leftarrow R_k \times \frac{1}{a_{11}} . \quad (7)$$

$$R_{k-i} \leftarrow R_{k-i} + (-k_{i1} \times R_k) \quad \forall i \in \{1, 2 \dots k-1\} . \quad (8)$$

where  $k \geq 2$ .  $R_i$  represents the  $i$ th row of  $A_k$  and  $a_{11}$ ,  $k_{i1}$ ,  $k_{1i}$ ,  $i = 1, 2 \dots k-1$  represent elements of matrices  $A_k$  and its corresponding  $K$  and  $K^T$  constituent vectors, respectively.  $\Phi(T_{k-1})$  represents all linear maps defined for matrix  $A_{k-1}$ , while  $\Phi(T_1)$  comprises one row operation defined as:  $R_k \leftarrow R_k \times \frac{1}{a_{11}}$ .

An immediate consequence of above formulation can be specified as the following Lemma:

**Lemma 1.** *If  $A_{k-1} = I$ , the identity matrix, and  $A_k$  defined in terms of Equation 4, then  $A_k^\Phi = I$ .*

**Theorem 1.** *Let  $A_k$  be an invertible matrix and  $\Phi(T_k) : A_k \rightarrow A_k^\Phi = I$  be the group of linear maps corresponding to  $A_k$ . The solution of SLE  $\lambda = A_k^{-1}b$  can be obtained by a recursive application of linear maps from  $T^K$  on  $b$ .*

The above formulation allows us to find the linear maps required to solve the Kriging SLE in an iterative manner. Consider the example in Figure 2. The first intermediate node (2) in the chain initiates the iterative process by computing the required transformations for  $A_2$  and transmit the composite linear map,  $T_2$ , to the node above it (node 3). Node 3 computes the row and column entries for  $A_3$  and according to the definition above, first applies the received linear map,  $T_2$  on  $A_3$  followed by linear maps that reduce all new row entries to 0, reduce the pivot element to 1 and reduce all new column entries to zero, i.e., applying Equations 6, 7 and 8 in that order. Node 3 then forwards the composite linear map  $T_3$  to the next node in chain. The iterative application of the same procedure at each intermediate node in the chain results in a final composite linear map,  $T_k$  at root node of the chain.  $T_k$  can then be applied on vector  $b$  to compute the solution of the Kriging SLE resulting in the required Kriging weighting vector:  $\lambda$ .

## 4 Experimental Study

We performed extensive simulations with the goal to evaluate the scalability of the DISK algorithm. Our simulations are based on two large datasets; a Digital Elevation

Model (DEM) dataset from the state of Colorado, US [8] and simulated traffic data for the city of Melbourne, Australia. Along with DISK, we also simulate a Global Kriging algorithm (GK) that assumes the full knowledge of node and coverage hole locations at the base station and performs its interpolation by propagating the required data to the base station. In the DEM dataset experiments, we estimate the altitude at various points while in the traffic dataset experiments, we estimate the number of cars at various locations. We measure the cost of a technique as the overall node energy used in data transmission. The accuracy is based on cross-validation of interpolation with the known values and is computed as the root mean square error (RMSE).

We define the Kriging neighborhood as the set of nodes located within a predefined distance to a point being interpolated. We measure the relationship between the network area and the Kriging neighborhood as a ratio between their sizes (in terms of number of nodes), referred to as the *network to neighborhood area ratio* (NNR).

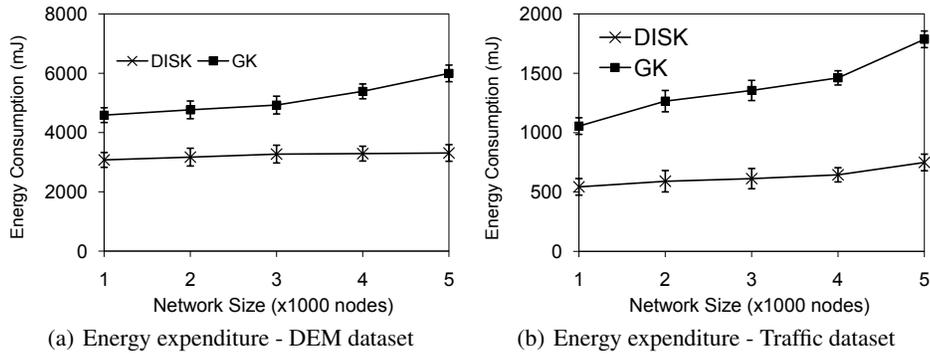
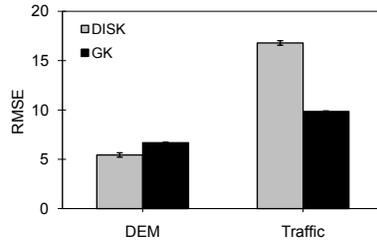


Fig. 3. DISK and GK approaches for various network sizes

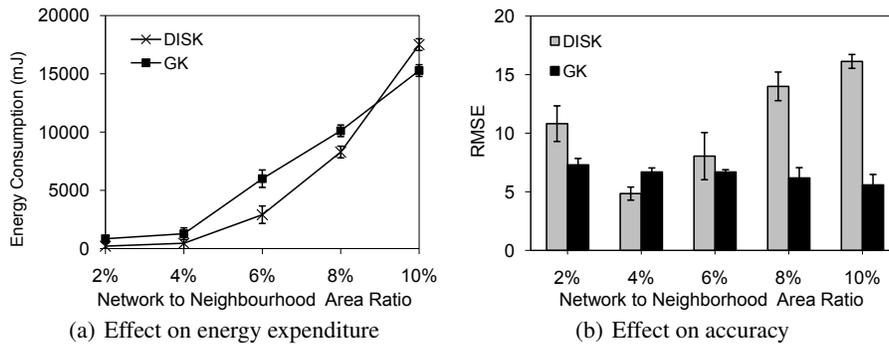
*The effect of network size.* In this experiment we analyze the scalability of DISK and GK with increasing network size. In each step, we expand the deployment area and the Kriging neighborhood such that the NNR stays constant at 5%. It can be observed from Figure 3 that DISK scales well with increasing network size while GK increases in cost rapidly. For the network of 1000 nodes, DISK uses about 60% of energy used by GK while for the 5000 nodes network its energy usage reduces to about 48% of GK.

Figure 4 shows a comparison of the accuracy of DISK and GK algorithms. Although both techniques interpolate using the same Kriging neighborhood, the difference in accuracy can be attributed to the variogram model used in each case. DISK uses the localized variogram model built through the QS algorithm while GK creates the variogram model centrally after collecting all data. In the case of highly correlated DEM data the localized variogram model enables DISK to achieve slightly better accuracy than GK. In this case, lower estimation accuracy of GK can be attributed to the use of global information which adds noise to its estimation. On the other hand, DISK suffers in the traffic dataset due to low levels of spatial correlation in this data.



**Fig. 4.** Accuracy of Disk and GK. Root mean square error (RMSE) is computed in terms of meters and number of cars for DEM and traffic data respectively.

*The effect of the Kriging neighborhood size.* In this experiment we increase the NNR by increasing the Kriging neighborhood area while the network size is kept fixed at 5000 nodes. Figure 5 presents the result of these experiments for DEM data. Increasing the neighborhood area results in an increase in the number of nodes in the neighborhood. Consequently, the communication cost of GK increases as there is more data to propagate to the base station. Similarly, the communication cost of DISK also rises due to the involvement of more nodes in the Kriging process. However, we observe that for accurate Kriging the NNR should not be increased beyond a certain value as further samples add noise to the estimation and lower the accuracy. For DEM data, the most accurate estimation results are obtained for 4% NNR where the cost of DISK is only about 40% of GK. A similar behavior with respect to energy expenditure was observed in our experiments with the traffic dataset (not shown here for the brevity of presentation).



**Fig. 5.** Effect of size of the Kriging neighborhood

## 5 Related Work

The problem of phenomenon estimation in WSNs for a region of interest not fully covered by a WSN is explored in [10]. The major limitations of [10] in comparison to

DISK is its global nature. This method assumes complete knowledge of the network at the base station and pumps all of the data from sampled nodes to the sink. The Distributed Regression method [6] for global kernel regression closely compares to DISK. This method is based on the notion that a number of regional correlation structures can be identified inside a given deployment. It is thus suggested that message passing for a global regression task can be optimized by distributing the global computation among the constituent regions. Although we propose the idea of distributed interpolation on a regional basis as well, the concept of a region in our method is fundamentally different from [6]. For DISK, the interest in a region is not based upon its correlation pattern but its vicinity to a coverage hole. Moreover, the iterative Gaussian elimination step in DISK makes the computation more localized than the distributed regression method.

## 6 Conclusions and Future Work

Coverage holes are a reality for WSNs and it is often important to estimate the information within a coverage hole. Kriging is a well-established interpolation method that particularly suits this problem as spatial correlations in measurements is common in WSNs. The challenge, however, is to perform Kriging with minimal communication and computation and with high accuracy in a WSN. We address this challenge by proposing QS and DISK algorithms that enable distributed and localized variogram modeling and Kriging. In extensive simulations we show that our methods are significantly more energy efficient than global interpolations. In future, we plan to investigate the applications of DISK to form energy efficient node redeployment strategies where the movement of nodes can be guided by phenomenon estimation in a region of interest.

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