K-Coverage in Regular Deterministic Sensor Deployments

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***Abstract*—**
K-coverage is necessary for the proper functioning of many applications, such as intrusion detection, data gathering, and object tracking. It is also desirable in situations where a stronger environmental monitoring capability is desired, such as military applications. In this paper, we study the problem of k-coverage in deterministic homogeneous deployments of sensors. We examine the three regular sensor deployments – triangular, square and hexagonal deployments – for k-coverage of the deployment area, for $k \geq 1$. We compare the three regular deployments in terms of sensor density. For each deployment, we compute an upper-bound and a lower-bound on the optimal distance of sensors from each other that ensure k-coverage of the area. We present the results for each k from 1 to 20. It is also shown that the required number of sensors to k-cover the area using uniform random deployment is approximately 3-10 times higher than regular deployments.

I. Introduction

Coverage problem is a fundamental issue that needs to be addressed in deployment of every sensor network. In general, coverage can be considered as the measure of quality of service of a sensor network [1]. The goal is to have each location in the physical space of interest within the sensing region of at least one sensor. The sensor coverage problem has been addressed and reviewed in many surveys [2][3].

Many practical applications such as event monitoring applications require guaranteed higher degree of coverage. In such applications, it is essential to place sensors such that every point of the target area can be monitored by more than one sensor. *K-coverage* is a more general concept of coverage, where each point in the field is covered by at least *k* sensors. *K*-coverage is required in sensor network applications due to various reasons such as multiple-sensor data fusion, increased accuracy, fault tolerance, reliability, or robustness.

High coverage degree is useful for multiple-sensor data fusion. Data fusion techniques combine data from multiple sensors to achieve more specific inferences that could be achieved by using a single sensor [4]. For example, a target position estimation may be accomplished by a triangulation or a least-squares computation over a set of sensor measurements [5]. The triangulation technique computes the position of an object by measuring the distances or bearings from multiple reference positions using various ranging techniques [6].

High degree of coverage is particularly essential for applications that demand a high degree of accuracy. For example, reliable detection may be achievable with a relatively coarse space-time resolution, whereas classification, needed for tracking multiple targets, typically requires processing at a higher resolution depending on the desired accuracy of classification [7]. Increasing k provides a more precise target location estimation in sensor networks, by more fine-grained partitioning of the sensor field. For example, in a 1-coverage of an area, we can only detect in which sensor region a target is located. Whereas in higher coverages of the area, the location of the target can be reduced to a certain intersection of at least k sensor regions. It is proven that k-coverage of a target improves the estimate of its location or velocity by a factor of $\sqrt{k}$, if detection data are fused in an optimal manner [4].

The coverage requirement also depends on the number of faults to be tolerated. Practically speaking, networks with a higher degree of coverage are more reliable as they are more robust to sensor failures and errorneous sensor measurements. Reliability is an important issue mainly in the applications in which failed sensors cannot be easily diagnosed and replaced, such as in sensor networks for planet exploration [8].

We aim to investigate the k-coverage problem in deployments of sensors. There are two fundamentally different ways to deploy sensors, deterministic and random deployments [9]. In a deterministic deployment, the sensors can be placed exactly where they are needed, while in a random deployment, sensors are usually placed according to a uniformly random distribution. A deterministic sensor placement may be feasible in friendly and accessible environments, while random sensor distribution is generally considered in remote or inhospitable areas, or for military applications [2].

In this paper, we first investigate regular deterministic sensor deployments (Section III). Regular sensor deployments are of particular importance in many applications mainly because they provide a uniform and high consistent partitioned space. For example, a uniform partitioned space can be utilized in navigation applications in order to minimize the orientation error in navigation tasks. Then, via simulations, we show that the required number of sensors to provide k-coverage in regular sensor deployments is approximately 3-10 times lower than random deployments (Section IV).

II. Related Work

A number of previous works [10] [1] are proposed to check if k-coverage of the target area is possible with the already deployed sensors. Given a set of sensors deployed in a target area, their goal is to determine whether every point in the area is covered by at least k sensors, where k is a given parameter. One naive solution is to find out all sub-regions divided by the sensing boundaries of all n sensors, and then check if each sub-region is k-covered. This could be difficult and computationally expensive since there may exit as many as $O(n^2)$ sub-regions divided by the circles. Also, it may be difficult to calculate these sub-regions [10].

Instead of determining the coverage of each sub-region, Huang and Tseng [10] look at how the perimeter of each sensor's sensing region is covered. They proved that when no two sensors are located in the same location, the whole network area is k-covered if and only if the perimeter of each sensor in the network is k-covered. They present polynomial-time algorithms, in terms of the number of sensors, to determine whether a sensor's perimeter is k-covered or not.

Many studies address the problem of selecting a minimum number of sensors to activate from an already densely deployed set of sensors such that the field remains k-covered and all selected sensors are connected. This also leads to an effective approach for energy conservation in wireless sensor networks because a subset of denslely deployed sensors are selected to stay active at any time interval, while other sensors are scheduled to sleep.

Kumar *et al.* [11] consider three kinds of sensor deploymemts on a unit square – a $\sqrt{n}\times\sqrt{n}$ grid, random uniform (for all n points), and Poisson (with density n). They computed the number of sensors, given the sensing radius (r), network life-time (p), and coverage (k), in order to guarantee that all the points in the field are k-covered. In their sleeping model, time is divided into periods and each sensor independently decides whether to remain awake for each period (with probability p) or go to sleep. Using this model, they find that the number of sensors needed in the grid deployment is of the same order as in random deployments.

It has been shown that selecting a minimum subset of sensors to k-cover a field from an already deployed set of sensors is NP-hard [12]. [12], [13], [14], [15], [16] present *approximation* algorithms to solve the connected k-coverage problem using a minimum number of sensors.

However, the proposed works do not answer the sensor placement problem to provide k-coverage, for $k \geq 2$. Kim *et al.* [17] addressed the problem of placing sensors to provide 3-coverage of the entire target area satisfying the minimum separation requirement, which is the minimum required distance between the sensors. They propose two methods, overlaying and TRE-based methods. The overlaying method overlays the 1-coverage optimal placement solution three times ensuring minimum separation among the sensors in different layers. The TRE-based method firsts forms a 3-covered region called TRE (Triple-Rounded-Edge area), which is an intersection of coverage circles of three sensors equally separated by d from each other, and then places the TREs repeatedly to cover the whole target area. They proved that the TRE-based method gives a better coverage redundancy than the overlaying method when the minimum required distance between the sensors is not greater than $0.232R$, there R is the sensors' sensing range.

In this paper (Section III), we evaluate the k-coverage problem in regular deterministic sensor deployments. To the best of our knowledge, this is the first analytical work on k-coverage problem in regular deterministic sensor deployments. We also compare the regular deterministic sensor deployments with the uniform random sensor deployment in terms of sensor density (Section IV).

III. K-Coverage in Regular Deterministic Sensor Deployments

A tiling of a two dimensional plane with a geometric shape with no overlaps and no gaps is called a *tessellation*. It is well-known that there are only three regular tessellations – tessellations composed of regular polygons – tiling the plane, which consist of equilateral triangles, squares and regular hexagons. In regular deterministic sensor deployments, the sensors can be placed at the polygon's vertices of a regular tessellation covering the whole sensor field [18]. Figure 1 illustrates the three regular sensor deployments, which are called triangular, square and hexagonal deployments throughout this paper.

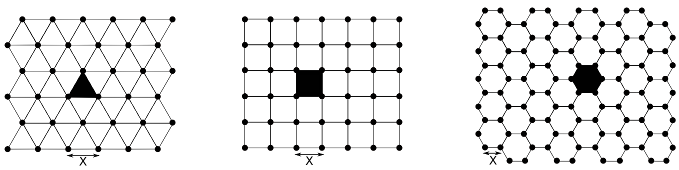


Fig. 1. Regular sensor deployments using (a) regular triangles (b) squares (c) regular hexagons

It is well-known that the optimal sensor deployment for 1-coverage is the triangular deployment, in which the sensors are placed $X = R\sqrt{3}$ away from each other, as shown in Figure 2. This deployment achieves the minimum overlapping of sensor regions and hence, requires the minimum number of sensors [19]. In this paper, we aim to find the optimal regular sensor deployment to k-cover the sensor field, for $k \geq 2$. The optimal deployment is assumed to be the one with the minimum required number of sensors. In each regular deployment, the side of the polygon constituting the deployment is shown by X (Figure 1) and is refered to by the *deployment-side*, throughout this paper. The sensor density in each deployment is determined by the value of the its deployment-side. Therefore, in next section, we find an upper- and a lower-bound on the optimal value of the deployment-side, X, that provides k-coverage in any of the three regular deployments.

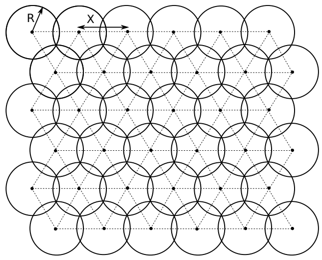


Fig. 2. Optimal 1-coverage of a deployment area

A. Proof of an upper- and a lower-bound on the value of X

1) Assumptions: We adopt the following assumptions and notations throughout the discussions in this section.

- Sensors can be deployed anywhere in a deployment area.
- Sensors can monitor a circular region centered at the sensor's location, whose radius R equals the sensing range of the sensor.
- All sensors have the same sensing range, R.
- To eliminate the effect of area boundaries when evaluating the sensor placement algorithms, we assume that the size of the deployment area is sufficiently larger than the size of sensing region of each individual sensor.

2) Problem Statement: In each deployment, the problem of coverage of the deployment area reduces to the problem of coverage of a single regular polygon constituting the deployment (shown in dark in Figure 1), due to the symmetric and periodic deployment scheme. Furthermore, each constituting regular polygon can be further divided into six, eight and twelve right triangles of the same shape and size, in triangular, square and hexagonal deployments, respectively (Figures 3(a) to 3(c)). The constituting triangles are the smallest constituting polygons of each deployment that are similar in terms of their shape and size as well as the relative placement of sensors to their vertices. As a result, the optimal k-coverage of the deployment area can be further reduced to the optimal k-coverage of a constituting triangle Δ_{abc}, or simply Δ (Figure 3), for each deployment.

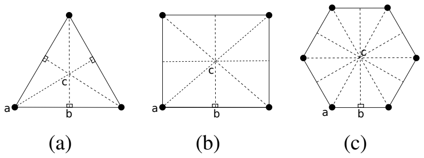


Fig. 3. The constituting triangles of (a) triangular (b), square (c) hexagonal deployments

Following the above discussion, finding an optimal k-coverage of the sensor field can be stated as follows. *In a regular deployment of sensors with sensing range of R, we aim to find the optimal deployment-side, X_{opt}, such that the triangle Δ is k-covered, for any k greater than one.*

3) Lower- and upper-bounds on the value of X_{opt} for k-coverage of triangle Δ: Now, we compute a lower-bound and an upper-bound on the value of X_{opt} for k-coverage of the triangle Δ. First in Lemma 2, we prove that if X is set to any value greater than the computed upper-bound, X_H^k, the triangle Δ is not fully k-covered. Then, in Lemma 3, we prove that if X is set to the computed lower-bound, X_L^k, the triangle Δ is at least k-covered. The following notations and definitions as well as Lemma 1 are used in the proofs in Lemmas 2 and 3.

Notation 1: The distance between Sensor S_i and vertices a, b and c of triangle Δ are denoted by D_i^a, D_i^b and D_i^c, respectively.

Notation 2: D_i^{max} is the maximum distance of Sensor S_i to vertices of triangle Δ, i.e. $D_i^{max} = max(D_i^a, D_i^b, D_i^c)$.

Definition 1: For vertex a of Δ, we define an ordered set DA as $DA = (D_{i_1}^a, D_{i_2}^a, \ldots, D_{i_n}^a)$, where n is the number of sensors in the field and $\forall\ 1 \leq k \leq n : D_{i_k}^a \leq D_{i_{k+1}}^a$. The ordered sets of DB and DC are defined similarly.

Notation 3: DA_k, DB_k and DC_k are the k^{th} elements of the ordered sets of DA, DB and DC, respectively.

Notation 4: D_k^{abc} is defined as the maximum of the k^{th} elements of the ordered sets of DA, DB and DC; i.e. D_k^{abc} = $max(DA_k, DB_k, DC_k)$.

Definition 2: We define an ordered set of D_i^{max} values as: $DX = \{D_{i_1}^{max}, D_{i_2}^{max}, \ldots, D_{i_n}^{max}\}$, where n is the number of sensors in the field and $\forall\ 0 \leq k < n : D_{i_k}^{max} \leq D_{i_{k+1}}^{max}$.

Lemma 1: If for Sensor S_i, $D_i^{max} = R$, then S_i covers the whole triangle Δ.

Proof: If R equals D_i^{max}, by Notation 2, Sensor S_i covers the three vertices of triangle Δ. As a result, the whole triangle Δ is covered by the region of Sensor S_i. ■

Based on these lemmas and definitions, Lemma 2 and Lemma 3 define an upper-bound and a lower-bound on the value of X_{opt} for the k-coverage of triangle Δ.

Lemma 2: Suppose that the deployment-side, X, is set to X_H^k such that D_k^{abc} equals R. Then, triangle Δ is not fully k-covered when X is greater than X_H^k.

Proof: For all values of X greater than X_H^k, R becomes less than D_k^{abc}. By Notation 4, D_k^{abc} = $max(DA_k, DB_k, DC_k)$. By Definition 1 and Notation 3, if R is less than DM_k, for $m \in \{a, b, c\}$, then vertex m of Δ is covered by less than k sensors. Therefore, triangle Δ is not fully k-covered. ■

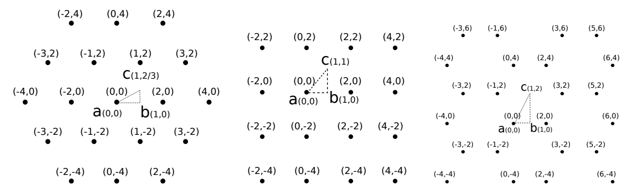
Lemma 3: Suppose that the deployment-side, X, is set to X_L^k such that $D_{i_k}^{max}$ equals R. Then, triangle Δ is at least $k+m$-covered, where j is the greatest non-negative integer such that $D_{i_k}^{max} = D_{i_{k+m}}^{max}$ and $D_{i_{k+m}}^{max} \neq D_{i_{k+m+1}}^{max}$.

Proof: If $D_{i_k}^{max}$ equals R, by Lemma 1 triangle Δ is covered by all sensors whose corresponding values in DX (Definition 2) are less than or equal to $D_{i_k}^{max}$. By Definition 2 DX is sorted and by considering the lemma's condition, there are $k+m$ such sensors. Therefore, all points in triangle Δ are definitiely covered by $k+m$ sensors. Therefore, the trianlge Δ is at least $k+m$-covered. ■

B. Calculation of the lower- and upper-bounds

Based on Lemmas 2 and 3, an upper-bound and a lower-bound on the optimum value of X to k-cover the deployment area can be computed in any of the three regular deployments.

Based on definitions and lemmas in Section III-A3, X_H^k and X_L^k values are computed using the distances of sensors to the vertices of a constituting triangle Δ and the sensing range of the sensors, R. To compute the euclidean distances, without loss of generality, it is assumed that vertex a of triangle Δ is placed at coordinate (0,0). Figures 4 shows the coordinates of some sensors in the field for triangular, square and hexagonal deployments. Please note that x and y scales both equal X for square deployment, while for triangular and hexagonal deployments, x and y scales equal X and $\frac{\sqrt{3}}{2}X$, respectively.



(a) (b) (c)

Fig. 4. Sensor coordinates in the deployment area

Using the Euclidean distance formulae, the distance from Sensor $S_i(x, y)$ to vertices a, b and c of triangle Δ , represented by D_a^i, D_b^i, D_c^i as in Notation 1, can be computed as shown in Equations 1, 2 and 3 for triangular, square and hexagonal deployments, respectively.

$$D_a^i = \frac{X}{2}\sqrt{x^2 + \frac{3}{4}y^2} \qquad D_b^i = \frac{X}{2}\sqrt{x^2 + y^2 - 2x + 1}$$
$$D_c^i = \frac{X}{2}\sqrt{x^2 + \frac{3}{4}y^2 - 2x - y + \frac{4}{3}} \tag{1}$$

$$D_a^i = \frac{X}{2}\sqrt{x^2 + y^2} \qquad D_b^i = \frac{X}{2}\sqrt{x^2 + \frac{3}{4}y^2 - 2x + 1}$$
$$D_c^i = \frac{X}{2}\sqrt{x^2 + y^2 - 2x - 2y + 2} \tag{2}$$

$$D_a^i = \frac{X}{2}\sqrt{x^2 + \frac{3}{4}y^2} \qquad D_b^i = \frac{X}{2}\sqrt{x^2 + \frac{3}{4}y^2}$$
$$D_c^i = \frac{X}{2}\sqrt{x^2 + \frac{3}{4}y^2 - 2x - 3y + 4} \tag{3}$$

Using the calculated values of D_a^i, D_b^i and D_c^i for each deployment and by Notation 4 and Definition 2, the values of D_k^{abc}, $D_{i_k}^{max}$ are computed for every k from 1 to 20. Then, using Lemmas 2 and 3, the values of X_H^k and X_L^k are calculated for every k from 1 to 20, which can be represented as:

$$R = \frac{X_L^k}{2}\sqrt{\alpha_H^k} \qquad R = \frac{X_H^k}{2}\sqrt{\alpha_L^k} \tag{4}$$

Generally, the relation between R and X (X^k) to provide k-coverage in any of the deployments can be shown as:

$$R = \frac{X^k}{2}\sqrt{\alpha^k} \tag{5}$$

The value of α^k shows the relation between the sensors range R and the deployment-side X. When α^k equals α_H^k for a given deployment, the area is at least k-covered and when α^k is less than α_L^k, the area is not fully k-covered. The values of α_H^k and α_L^k for the three regular deployments, for k for 1 to 20, are shown in columns 2 to 7 of Table I.

We use the sensor density λ, as defined in [18], to evaluate the three regular deployments. Let A_p denote the area of the constituting polygon, N_p the number of nodes composing the polygon, and N_n the number of polygons that share a node, then λ can be computed as $\lambda = N_p/A_pN_n$. Therefore, the sensor density for triangular, square and hexagonal deployments can be computed as follows, where X is the deployment-side.

$$\lambda_t = \frac{3}{\frac{\sqrt{3}}{4}X^2 \times 6} \qquad \lambda_s = \frac{4}{X^2 \times 4} \qquad \lambda_h = \frac{6}{\frac{3\times\sqrt{3}}{2}X^2 \times 3} \tag{6}$$

The sensor densities of triangular, square and hexagonal deployments when α^k equals α_H^k (X=X_L^k) are shown by λ_H^t, λ_H^s and λ_H^h, respectively. Similarly, the sensor densities of triangular, square and hexagonal deployments when α^k equals α_L^k are shown by λ_L^t, λ_L^s and λ_L^h, respectively. Note that the sensor density in each deployment is inversely proportional to the value of X^2 (Equation 6) . Using Equations 4 and 6, the ratio of the sensor densities of the three deployments are analyzed and discussed in the next section (Table I).

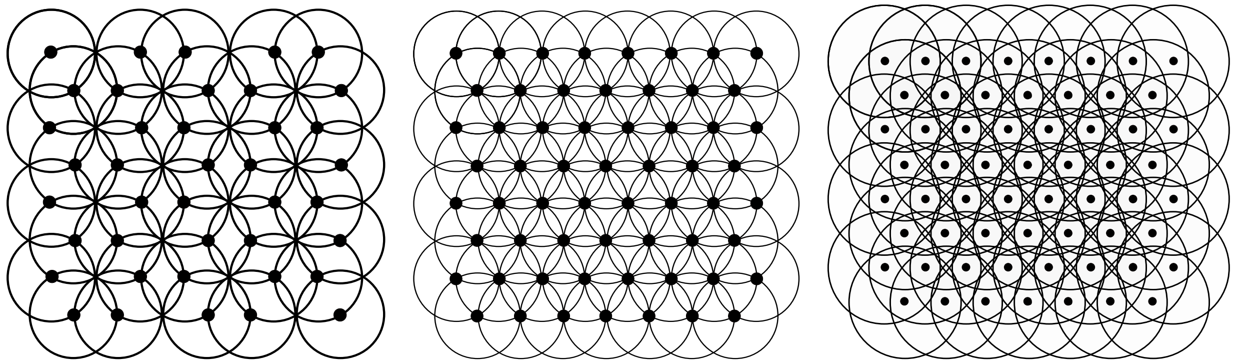
C. Analysis and comparison

For a given regular deployment, the optimum value of α to provide k-coverage, α_{opt}^k, is defined to be the value that provides full k-coverage of the deployment area with the minimum number of sensors. By Lemmas 2 and 3 and by Equation 4, the value of α_{opt}^k lies between the two values of α_L^k and α_H^k, for each k in any regular deployment.

The full k-coverage of an area in any of the deployments can be achieved by setting the α^k value to the upper-bound value α_H^k (Lemma 2). Thus, the narrower the gap between α_L^k and α_H^k, the lower is the increase of the sensor density comparing to the optimal case. Columns 14, 15 and 16 of Table I, $\frac{\lambda_H^t}{\lambda_L^t}$, $\frac{\lambda_H^s}{\lambda_L^s}$ and $\frac{\lambda_H^h}{\lambda_L^h}$, show the worst case increase in the sensor densities if α^k is set to α_H^k. Therefore, the worst-case increase in the sensor densities is 33%, 60% and 71% for triangular, square, and hexagonal deployments, respectively.

Moreover, as shown in Table I, for some values of k, the lower- and upper-bounds of α^k met, which gives us the optimum value of α^k in that deployment ($\alpha_H^k = \alpha_L^k = \alpha_{opt}^k$). For example, to achieve a 1-coverage of the deployment area in triangular, square and hexagonal deployments, the deployment-side, X, is best to be set to $2R/\sqrt{1.33}$, $2R/\sqrt{2}$ and $2R/\sqrt{4}$ (Equation 5), respectively.

Using Table I, we can also compare the three regular deployments in terms of their required sensor densities to provide k-coverage, for every value of k from 1 to 20. Column pairs of (8,9), (10,11), (12,13) show the ratio of the sensor density of the triangular, square and hexagonal deployments, when $\alpha^k = \alpha_H^k$, to the sensor densities of the other two deployments when $\alpha^k = \alpha_L^k$. For example, the values of $\frac{\lambda_H^t}{\lambda_L^s}$ and $\frac{\lambda_H^t}{\lambda_L^h}$ are shown in columns 8 and 9 of Table I, respectively.



(a) (b) (c)

Fig. 5. Optimal regular deployments to provide optimal (a) 2-coverage (b) 3-coverage (c) 5-coverage

Therefore, as an example, for any k with corresponding values of $\frac{\lambda_H^t}{\lambda_L^s}$ and $\frac{\lambda_H^t}{\lambda_L^h}$ less than one, the triangular deployment provides the optimum k-coverage of the area in terms of the sensor density. As a result, we can conclude that the optimum regular deployment to k-cover the area is triangular for k=1, 3, 5, square for k=7, 13, 14 and hexagonal for k=2.

For k equal to 1, 2, 3, 5, 7, 14, the optimum deployment-side is also known for the given deployment, because $\alpha_H^k = \alpha_L^k$. Figure 5 (a-c) show the optimal regular deployments to k-cover an area for k=2, 3 and 5, which are the hexagonal deployment with deployment-side of $2R/\sqrt{4}$, triangular deployment with deployment-side of $2R/\sqrt{4}$ and triangular deployment with deployment-side of $2R/\sqrt{7}$, respectively. Generally, Table I assists in comparing the three regular deployments to k-cover an area for any given k between 1 and 20, even for the values of k for which finding the optimal regular deployment is not possible.

D. Validation of the theoretical results

The theoretical results presented in Section III-B is verified via simulations. In these simulations, the sensors were deployed in a square area of side L equal to 1800, while the sensors' sensing range was set to 80. All three regular deployments, triangular, square and hexagonal, were deployed and the simulations were run for every k from 1 to 20. We observed a match between our theoretical derivations and the simulation results. The simulation area was at least k-covered when the deployment-side, X, was set to $X_L^k = 160/\sqrt{\alpha_H^k}$ (Equation 4), and the area was not fully k-covered when X became greater than $X_H^k = 160/\sqrt{\alpha_L^k}$ (Equation 4).

Furthermore, the optimum deployment-side of each deployment, $X_{opt}^k = 160/\sqrt{\alpha_{opt}^k}$, for the given area was computed as follows. For every k in each deployment, the value of α^k was changed from α_H^k to α_L^k (shown in Table I) by steps of 0.1. The sensors were deployed within the square area using the deployment-side values corresponding to α^k values, using Equation 5. The maximum value of X^k that provided a full k-coverage of the area was the optimum value of X^k, X_{opt}^k, in our setup. Last three columns of Table I show the optimum value of α^k – α_{opt}^k – to provide k-coverage for every k from 1 to 20 for the three deployments. The optimal values are used in the next section to compare the sensor densities of the regular deployments with uniform random deployment.

IV. Unifrom Random Deployment

In this section, we compare the number of sensors required to provide k-coverage in the three regular deployments in a given area with the number of sensors required to provide k-coverage in uniform random sensor deployment for values of k from 1 to 20. The minimum required number of sensors to k-cover an area in uniform random deployment is computed via simulations for each k. The sensors were deployed in a square area of side L equal to 1800, while the sensors' sensing range were set to 80. Under uniform random distribution, the sensors were distributed uniformly over the deployment area until every point in the area was covered by k sensors. Each sensor had an equal likelihood of being at any location in the area. We performed 100 iterations for each k. Figure IV shows the average number of required sensors to k-cover an area, for k from 1 to 20. The results are shown along with the required number of sensors to achieve k-coverage in the same area in the three regular deployments (using the α_{opt}^k values in Table I). As shown in this figure, the required number of sensors in regular deployments is 3-10 times lower than the required number of sensors in random deployments for different values of k.

V. Conclusion

Regular sensor deployments are of particular importance in many applications mainly because they provide a uniform and high consistent partitioned space. In this paper, we compared the three regular sensor deployments, triangular, square and hexagonal deployments, based on the required sensor density to k-cover the deployment area, for $k \geq 1$. For each deployment, we computed an upper-bound and a lower-bound on the optimal distance of sensors from each other that ensure k-coverage of the area. Further, we showed that the regular sensor deployments are preferable to uniform random deployment in terms of the sensor density for k-coverage of an area, for $k \geq 1$.

	triangular		square		hexagonal		densities comparison									triangular	square	hexagonal
k	α_H^k	α_L^k	α_H^k	α_L^k	α_H^k	α_L^k	$\frac{\lambda_H^t}{\lambda_L^s}$	$\frac{\lambda_H^t}{\lambda_L^h}$	$\frac{\lambda_H^s}{\lambda_L^t}$	$\frac{\lambda_H^s}{\lambda_L^h}$	$\frac{\lambda_H^h}{\lambda_L^t}$	$\frac{\lambda_H^h}{\lambda_L^s}$	$\frac{\lambda_H^t}{\lambda_L^t}$	$\frac{\lambda_H^s}{\lambda_L^s}$	$\frac{\lambda_H^h}{\lambda_L^h}$	α_{opt}^k	α_{opt}^k	α_{opt}^k
1	1.33	1.33	2	2	4	4	0.77	0.50	1.30	0.65	2.00	1.54	1.00	1.00	1.00	1.33	2	4
2	4	4	4	4	4	4	1.15	1.50	0.87	1.30	0.67	0.77	1.00	1.00	1.00	4	4	4
3	4	4	5	5	7	7	0.92	0.86	1.08	0.93	1.17	1.08	1.00	1.00	1.00	4	5	7
4	5.33	5.33	8	5	12	7	1.23	1.14	1.30	1.48	1.50	1.85	1.00	1.60	1.71	5.33	5.6	7.9
5	7	7	10	10	13	12	0.81	0.88	1.24	1.08	1.24	1.00	1.00	1.00	1.08	7	10	12
6	9.33	7	10	10	16	12	1.08	1.17	1.24	1.08	1.52	1.23	1.33	1.00	1.33	7.33	10	12
7	9.33	9.33	10	10	16	16	1.08	0.87	0.93	0.81	1.14	1.23	1.00	1.00	1.00	9.33	10	16
8	12	12	13	10	16	16	1.39	1.13	0.94	1.06	0.89	1.23	1.00	1.30	1.00	12	11.1	16
9	12	12	16	13	19	16	1.07	1.13	1.15	1.30	1.06	1.13	1.00	1.23	1.19	12	13	17.4
10	13	12	17	16	28	16	0.94	1.22	1.23	1.38	1.56	1.35	1.08	1.06	1.75	12.3	16	20.2
11	16	13	18	16	28	19	1.15	1.26	1.20	1.23	1.44	1.35	1.23	1.13	1.47	13.5	16.3	21.3
12	16	13	20	16	28	19	1.15	1.26	1.33	1.37	1.44	1.35	1.23	1.25	1.47	13.8	17.7	21.8
13	17.33	17.33	20	18	28	28	1.11	0.93	1.00	0.93	1.08	1.20	1.00	1.11	1.00	17.33	18.1	28
14	17.33	17.33	20	20	28	28	1.00	0.93	1.00	0.93	1.08	1.08	1.00	1.00	1.00	17.33	20	28
15	19	19	25	20	28	28	1.10	1.02	1.14	1.16	0.98	1.08	1.00	1.25	1.00	19	21.2	28
16	21	19	26	20	31	28	1.21	1.13	1.19	1.21	1.09	1.19	1.11	1.30	1.11	19.1	22.3	29.5
17	21.33	19	26	26	36	31	0.95	1.03	1.19	1.09	1.26	1.07	1.12	1.00	1.16	20.23	26	31
18	25.33	19	26	26	36	31	1.13	1.23	1.19	1.09	1.26	1.07	1.33	1.00	1.16	21.23	26	31.8
19	25.33	21.33	29	26	37	31	1.13	1.23	1.18	1.22	1.16	1.10	1.19	1.12	1.19	21.83	26	34.7
20	28	28	32	26	43	36	1.24	1.17	0.99	1.15	1.02	1.27	1.00	1.23	1.19	28	26.6	36

TABLE I
SENSOR DENSITIES

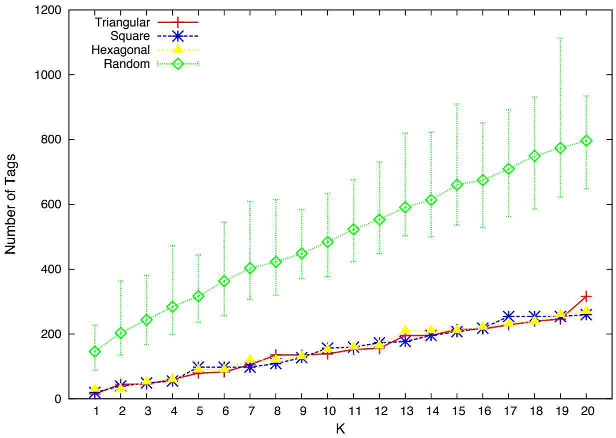


Fig. 6. Comparison of regular and random deployments in terms of the number of sensors

ACKNOWLEDGMENT

This work was partly funded by National ICT Australia (NICTA).

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