Redesign techniques for nonlinear sampled–data control

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(joint work with Dragan Nešić, Jürgen Pannek)

Nowadays, modern controllers are typically implemented digitally and this fact strongly motivates investigation of sampled–data systems that consist of a continuous time plant controlled by a discrete time (digital) controller. While tools for analysis and design of linear sampled–data systems are well developed (see, e.g., [1], [2]), similar results for nonlinear systems still need development.

In this talk we consider the problem of static state feedback stabilization of the origin of a finite dimensional control system

$$\dot{x} = f(x, u)$$

with $x \in \mathbb{R}^n$ and $u \in U \subseteq \mathbb{R}^m$, i.e., we are looking for a map $u : \mathbb{R}^n \to U$ such that for the closed loop system

$$\dot{x}(t) = f(x(t), u(x(t))) \tag{1}$$

the origin $x^* = 0$ is a globally asymptotically stable equilibrium. In order to model a sampled–data implementation of this problem with a zero order hold device we consider the corresponding sampled–data system with constant sampling rate $T > 0$ given by

$$\dot{x}(t) = f(x(t), u_T(x(iT))), \quad t \in [iT, (i+1)T),\ i = 0, 1, \dots \tag{2}$$

and construct a controller u_T for this model. Assuming that a suitable controller u for the continuous time system (1) has been designed, a possible approach for sampled-data controller design is to first design a continuous-time controller for the continuous-time plant ignoring sampling and then discretize the obtained controller for digital implementation, i.e., set $u_T = u$, an approach which is often termed *emulation design*. This approach was shown in [5] to recover the performance of the continuous-time system in a semi-global practical sense. However, due to hardware limitations on the minimum achievable T there may exists critical regions, where this approach yields bad performance as in Figure 1, below, where sampling introduces overshoot, or even instability as in Figure 2, below.

Our goal is hence to design a discrete time controller which improves upon the performance of the emulated continuous time controller $u_T = u$, using, however, the available continuous time controller u, i.e., we want to redesign u.

In our first approach, the *Lyapunov redesign technique* developed in [6], we consider control affine single input systems, i.e., $f(x, u) = f_0(x) + g(x)u$ and assume that there exists a Lyapunov function V corresponding to the continuous time system (1) for which the $\mathcal{KL}$ function β obtained from integrating the Lyapunov inequality $L_f(x, u(x))V(x) \leq -\alpha(V(x))$ yields a good reference estimate $\|x(t, x_0)\| \leq \beta(\|x_0\|, t)$ for the trajectories $x(t, x_0)$ of (1). On the discrete time level this estimate is induced by the Lyapunov difference

$$\Delta V(x) := V(x(T, x)) - V(x).$$

Denoting the trajectories of the sampled data system (2) by $x_T(t, x_0, u_T)$ we can define the sampled–data Lyapunov difference by

$$\Delta V_T(x, u_T) := V(x_T(T, x, u_T)) - V(x).$$

and design u_T in such a way that this difference ΔV_T assumes "good" values. In this context, a "good value" can have several meanings which depend on the redesign objective. For example, one can perform a model reference type redesing by matching the continuous time behavior as close as possible by minimizing $\|\Delta V - \Delta V_T\|$. As an alternative, one can increase the convergence speed by minimizing ΔV_T under suitable gain constraints on the controller u_T.

In order to design u_T in practice we need a computationally feasible approximation of the sampled–data Lyapunov difference ΔV_T. Using the Fliess expansion and neglecting the higher order terms yields such an approximation.

As an example, consider the Moore–Greitzer jet engine model given by

$$\begin{aligned} \dot{x_1} &= -x_2 - \frac{3}{2}x_1^2 - \frac{1}{2}x_1^3 - 3x_3x_1 - 3x_3 \\ \dot{x_2} &= -u \\ \dot{x_3} &= -\sigma x_3(x_3 + 2x_1 + x_1^2) \end{aligned}$$

We have applied the Lyapunov redesign technique to the simplified 2d version obtained by setting $x_3 = 0$ using the stabilizing backstepping controller

$$u(x) = -7x_1 + 5x_2,$$

see [4] for details on the model and the controller design.

Using the Lyapunov function $V(x) = x^2/2$ (which is a Lyapunov function outside a neighborhood of the origin, cf. [6]), we obtain the results shown in Fig. 1.

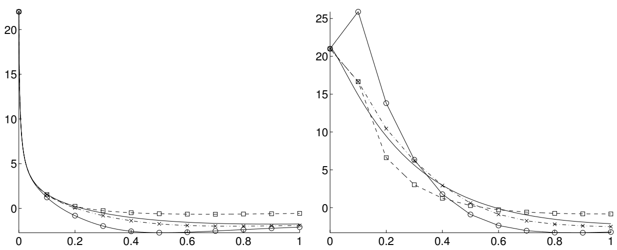

Figure 1. Lyapunov based redesign

In this figure, the unmarked curves show the continuous time system, the curves marked with circles show the emulated continuous time controller $u_T = u$. The Lyapunov difference minimizing redesign is marked with squares while the model reference type redesign is marked with crosses.

In our second approach, the *model predictive redesign* presented in [3, 7], we solve an optimal control problem in order to minimize the distance between x and x_T. While the natural optimal control approach to this problem would be an infinite horizon optimization criterion, this approach is computationally not feasible. Instead we chose a model predictive (or receding horizon) approach by on line solving the finite horizon problem for piecewise constant open loop control $\tilde{u}_T$

$$\min_{\tilde{u}_T} \int_0^{NT} l(x_T(t, x_T^i, \tilde{u}_T) - x(t, x^i), \tilde{u}_T(t))dt + F(x_T(NT, x_T^i, \tilde{u}_T), x(NT, x^i, u))$$

at each sampling instance iT with $x_T^i = x_T(iT, x_0, u_T)$, $x^i = x(iT, x_0)$ and using the sampled–data feedback $u_T(x_T^i) := \tilde{u}_T(0)$. We obtain stability of the closed loop system under mild conditions on l and F and infinite horizon inverse optimality under a local Lyapunov function like condition on the terminal cost F.

We illustrate this method by the 3d Moore–Greitzer model with backstepping stabilizing controller

$$\begin{aligned} u \;=\; & -(c_1 - 3x_1)\left(-x_2 - \frac{3}{2}x_1^2 - \frac{1}{2}x_1^3 - 3x_1x_3 - 3x_3\right) \\ & + c_2\left(x_2 - c_1x_1 + \frac{3}{2}x_1^2 + 3x_3\right) - x_1 - 3\sigma x_3\left(x_3 + 2x_1 + x_1^2\right) \end{aligned}$$

using the parameters $\sigma = 2$, $c_1 = 1$ and $c_2 = 50$. The result is shown in Figure 2.

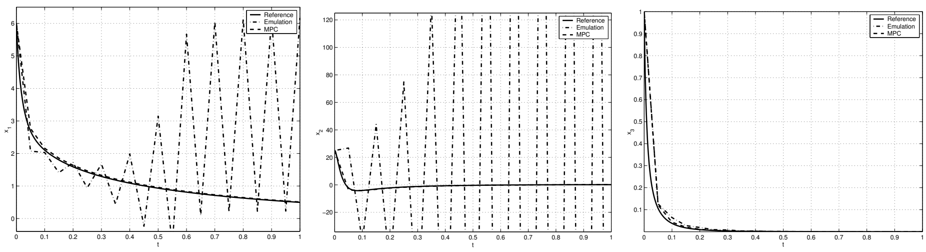

FIGURE 2. Model predictive (MPC) redesign

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