Available online at www.sciencedirect.com
ScienceDirect
ELSEVIER
Automatica 44 (2008) 2171–2178
automatica
www.elsevier.com/locate/automatica

Brief paper

Decentralized control design of interconnected chains of integrators: A case study[☆]

Guangyu Liu[a,b,*], Iven Mareels[b], Dragan Nešić[b]

[a] Victoria Research Lab, National ICT, Australia
[b] Department of Electrical and Electronic Engineering, The University of Melbourne, Parkville, 3010, Victoria, Australia

Received 2 November 2006; received in revised form 27 September 2007; accepted 31 December 2007
Available online 7 March 2008

Abstract

We develop a constructive decentralized control design procedure for a class of systems that may be loosely described as chained integrators which are dynamically coupled. The design method is inspired by nested saturation control ideas and formulated by applying the singular perturbation theory. We demonstrate that the proposed design provides a Lyapunov function for an associated closed loop system from which semi-global stability may be deduced. Using the proposed idea, we design a semi-globally stabilizing control law for a four degree of freedom spherical inverted pendulum.

Keywords: A spherical inverted pendulum; Decentralized control; Semi-global stability; Nonlinear

1. Introduction

Nested saturation control, useful in a nonlinear system with a forwarding structure, is associated with chains of integrators (Arcak, Teel, & Kokotović, 2001; Grognard, Sepulchre, & Bastin, 1999; Kaliora & Astofi, 2004, 2005; Marconi & Isidori, 2000; Teel, 1996). This often leads to a slow closed loop response. Indeed, its transients exhibit a time-scale separation between various "nested" controllers, which is not inherent in the nonlinear system itself. It appears that some structure, without necessarily emulating the conservativeness of these nested saturating controllers, can be achieved using linear control ideas combined with singular perturbation tools that exploit natural time scales.

Here, we show how linear control ideas with time scaling recover many properties inherent in the nested saturation design. Our design is constructive and comes with a Lyapunov function for formally stating stability and robustness. In this regard, our paper extends the work by Grognard, Sepulchre, Bastin, and Praly (1998) and Mazenc (1997) studying a single input single chain of integrators. In our approach, time scales are selected "per block of states" and not for each state component on succession. A spherical inverted pendulum is a beam attached to a horizontal plane via a universal joint that is free to move in the plane under the influence of a planar force (see Fig. 1). The pendulum in the upper space is assumed. Its modelling was given in Liu (2006); its non-local stabilization and output tracking were first explicitly solved in Liu, Nešić, and Mareels (2008a,b) respectively. We design a stabilizing controller using the proposed idea for this pendulum that achieves an arbitrarily large domain of attraction in the upper space by tuning a scaling parameter. Beside recovering many features in the nested saturating controller (Liu et al., 2008a), it exploits natural time scaling and hence is less conservative. The case study is a representative from a large class of mechanical systems that can be viewed as dynamically coupled chains of integrators. It is for this family that we propose a decentralized control strategy.

2. Notation

Let a vector $v \triangleq (v_1^T, v_2^T, \ldots, v_n^T)^T \in R^{n_1} \times R^{n_2} \times \cdots \times R^{n_N}$. For a vector $v_j \in R^{n_j}$, $v_{i,j}$, $i = 1, \ldots, n_j$, denotes ith element

[☆] This paper was not presented at any IFAC meeting. This paper was recommended for publication in revised form by Associate Editor Zongli Lin under the direction of Editor Hassan Khalil.
* Corresponding author at: Department of Electrical and Electronic Engineering, The University of Melbourne, Parkville, 3010, Victoria, Australia.
E-mail address: lgyyfe@yahoo.com (G. Liu).

0005-1098/$ - see front matter © 2008 Elsevier Ltd. All rights reserved.
doi:10.1016/j.automatica.2007.12.011

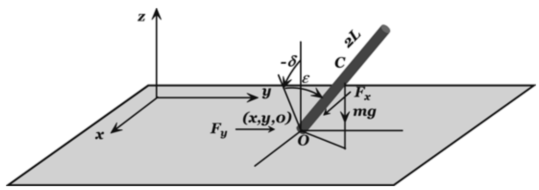


Fig. 1. A spherical inverted pendulum.

of v_j. With a polynomial $s^n + a_n s^{n-1} + \cdots + a_2 s + a_1$, we associate a companion matrix:

$$A = \begin{pmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & & \cdots & 1 \\ -a_1 & -a_2 & \cdots & -a_n \end{pmatrix}.$$

$s(\cdot)$ and $c(\cdot)$ denote $\sin(\cdot)$ and $\cos(\cdot)$ respectively. The methodology used here is based on standard singular perturbation tools (see Kokotović, Khalil, and O'Reilly (1986) for details) to an autonomous singularly perturbed system

$$\dot{x} = f(x, z, \varepsilon), \qquad \varepsilon \dot{z} = g(x, z, \varepsilon), \quad \text{for } \forall \varepsilon > 0, \tag{1}$$

where $x \in D_x \subset R^n$, $z \in D_z \subset R^m$, which has an isolated equilibrium at its origin $x = 0$, $z = 0$.

3. The general result

3.1. Problem statement

We consider a sequence of N interconnected chains of integrators where each subsystem $j \in \{1, 2, \ldots, N\}$ consists of two blocks as follows

$$\Sigma_{x,j} \; : \; \dot{x}_{1,j} = x_{2,j} + \varphi_{1,j}(y), \ldots, \\ \dot{x}_{n_j,j} = y_{1,j} + \varphi_{n_j,j}(y), \tag{2}$$

$$\Sigma_{y,j} : \dot{y}_{1,j} = y_{2,j}, \ldots, \dot{y}_{m_j-1,j} = y_{m_j,j}, \; \dot{y}_{m_j,j} = u_j. \tag{3}$$

Let state vectors be $x \triangleq (x_1, \ldots, x_N) = (x_{1,1}, \ldots, x_{n_1,1}, \ldots, x_{1,N}, \ldots, x_{n_N,N}) \in R^{n_1} \times \cdots \times R^{n_N}$ and $y \triangleq (y_1, \ldots, y_N) = (y_{1,1}, \ldots, y_{m_1,1}, \ldots, y_{1,N}, \ldots, y_{m_N,N}) \in R^{m_1} \times \cdots \times R^{m_N}$ and an input vector be $u \triangleq (u_1, \ldots, u_N) \in R^N$. $\varphi_{i,j}(\cdot)$, $i \in \{1, 2, \ldots, n_j\}$ and $j \in \{1, 2, \ldots, N\}$, are zero at $y = 0$, analytic and higher order terms with respect to y in a neighborhood of the origin which is denoted by $o(y)$. We consider a control law u_j, $j \in \{1, \ldots, N\}$ given by

$$u_j = -\boldsymbol{\varepsilon} L_{1,j} \left(\sum_{i=1}^{n_j} \boldsymbol{\varepsilon}^{n_j - i} K_{i,j} x_{i,j} \right) - \sum_{i=1}^{m_j} L_{i,j} y_{i,j}, \tag{4}$$

where $\boldsymbol{\varepsilon}$ is a small positive parameter.

Remark 1. One could use a different $\boldsymbol{\varepsilon}$ for each u_j. For simplicity, a common $\boldsymbol{\varepsilon}$ is used. With $\boldsymbol{\varepsilon}$ small, the control law introduces a time scale separation property. We present our result for two coupled subsystems in the sequel, but it can be easily generalized.

We use the following shorthand notation for the system (2) and (3) with the controller (4) for $N = 2$,

$$\begin{aligned} &\Sigma_{x,j} : \dot{x}_j = f_{x,j}(x_j, y) \\ &\Sigma_{y,j} : \dot{y}_j = f_{y,j}(x_j, y_j, \boldsymbol{\varepsilon}), \end{aligned} \qquad \text{for } j = 1, 2, \tag{5}$$

where $x = (x_1, x_2) = (x_{1,1}, \ldots, x_{n_1,1}, x_{1,2}, \ldots, x_{n_2,2}) \in R^{n_1} \times R^{n_2}$, $y = (y_1, y_2) = (y_{1,1}, \ldots, y_{n_1,1}, y_{1,2}, \ldots, y_{n_2,2}) \in R^{m_1} \times R^{m_2}$.

Assumption 1. There exist analytic functions $\psi_1(y)$ and $\psi_2(y)$ that solve the following PDE:

$$\frac{\partial \psi_j(y)}{\partial y} \left[\begin{pmatrix} A_{y,1} & 0 \\ 0 & A_{y,2} \end{pmatrix} y \right] = -\varphi_{n_j,j}(y), \tag{6}$$

for $j = 1, 2$, where $A_{y,j}$ is the linearization of $f_{y,j}$ at the origin, subject to boundary conditions

$$\begin{aligned} &\left.\frac{\partial \psi_1}{\partial y_{n_1,1}}\right|_{y=0} = 0, \qquad \left.\frac{\partial \psi_1}{\partial y_{n_2,2}}\right|_{y=0} = 0, \\ &\psi_1(0) = 0, \\ &\left.\frac{\partial \psi_2}{\partial y_{n_1,1}}\right|_{y=0} = 0, \qquad \left.\frac{\partial \psi_2}{\partial y_{n_2,2}}\right|_{y=0} = 0, \\ &\psi_2(0) = 0. \end{aligned} \tag{7}$$

Remark 2. The existence of solutions to (6) is assumed here. In PDEs (6), $A_{y,j}$ is the companion form with the characteristic polynomial, $\det(sI - A_{y,j}) = s^{m_j} + L_{m_j,i} s^{m_j - 1} + \cdots + L_{2,j} s + L_{1,j}$, $j = 1, 2$.

3.2. Standard singular perturbation form

Define a state vector $z = (z_1, z_2) \in R^{n_1} \times R^{n_2}$ as follows

$$z_j = \Bigg(\varepsilon^{n_j - 1} x_{1,j}, \ldots, \varepsilon x_{n_j - 1,j}, x_{n_j,j} \\ + \frac{1}{L_{1,j}} \left(\sum_{i=1}^{m_j - 1} L_{i+1,j} y_{i,j} + y_{m_j,j} \right) + \psi_j(y) \Bigg), \tag{8}$$

for $j = 1, 2$, where $\psi_j(y)$, $j = 1, 2$, satisfy Assumption 1.

Lemma 3.1. *Consider the closed loop system (5) under Assumption 1. Then, y is the fast variable and z is the slow variable. Define a new time scale $\tau = \boldsymbol{\varepsilon} t$. In the time scale τ, the system (5) in (z, y) takes on the form (see also Box I):*

$$\begin{aligned} &\Sigma_{z,j} : \frac{\mathrm{d}z_j}{\mathrm{d}\tau} = f'_{z,j}(z_j, y, \boldsymbol{\varepsilon}) \\ &\Sigma_{y,j} : \boldsymbol{\varepsilon} \frac{\mathrm{d}y_j}{\mathrm{d}\tau} = f'_{y,j}(z_j, y, \boldsymbol{\varepsilon}), \end{aligned} \qquad \textit{for } j = 1, 2, \tag{9}$$

where $\Sigma_{z,j}$ are slow dynamics, $\Sigma_{y,j}$ are fast dynamics.

Expressions for equations (9) *become* :

$$f'_{z,j} = \left(\begin{array}{c} z_{2,j} + \varepsilon^{n_j-2}\varphi_{1,j}(y) \\ z_{3,j} + \varepsilon^{n_j-3}\varphi_{2,j}(y) \\ \vdots \\ z_{n_j,j} - \frac{1}{L_{1,j}}\left(\sum_{i=1}^{m_j-1} L_{i+1,j}y_{i,j} + y_{m_j,j}\right) - \psi_j(y) + \varphi_{n_j-1,j}(y) \\ \hline \left| \begin{array}{c} -\left(1+\frac{\partial\psi_j}{\partial y_{m_j,j}}\right)\left(\sum_{i=1}^{n_j-1} K_{i,j}z_{i,j} + K_{n_j,j}z_{n_j,j} - \frac{K_{n_j,j}}{L_{1,j}}\left(\sum_{i=1}^{m_j-1} L_{i+1,j}y_{i,j}\right.\right. \\ \left.\left. +y_{m_j,j}\right) - K_{n_j,j}\psi_j(y)\right) - \frac{\partial\psi_j}{\partial y_{m_k,k}}\left(\sum_{i=1}^{n_k-1} K_{i,k}z_{i,k} + K_{n_k,k}z_{n_k,k}\right. \\ \left. -\frac{K_{n_k,k}}{L_{1,k}}\left(\sum_{i=1}^{m_k-1} L_{i+1,k}y_{i,k} + y_{m_k,k}\right) - K_{n_k,k}\psi_k(y)\right) \end{array}\right| \\ \hline \end{array}\right),$$

for $j \neq k$, $k, j \in \{1, 2\}$

$$f'_{y,j} = \left(\begin{array}{c} y_{2,j} \\ \vdots \\ y_{m_j,j} \\ \hline \left| \begin{array}{c} -\varepsilon\left(\sum_{i=1}^{m_j-1} K_{i,j}z_{i,j} + K_{n_j,j}z_{n_j,j} - \frac{K_{n_j,j}}{L_{1,j}}\left(\sum_{i=1}^{m_j-1} L_{i+1,j}y_{i,j}\right.\right. \\ \left.\left. +y_{m_j,j}\right) - K_{n_j,j}\psi_j(y)\right) - \sum_{i=1}^{m_j} L_{i,j}y_{i,j} \end{array}\right| \\ \hline \end{array}\right), \quad \text{for } j \in \{1, 2\}$$

Box I.

Proof. The proof is solely based on a method in Kokotović et al. (1986, Page 31) that converts a non-standard singularly perturbed form to a standard form as is (1). By some technical computation, it can be shown that, given that Assumption 1 is satisfied, (5) is a non-standard singularly perturbed form that can be converted to the standard one (9). For space reason, the details are omitted (see Liu (2006, Chapter 6)). □

3.3. Stability analysis

Lemma 3.1 implies the result.

Corollary 3.2. *Consider the standard singularly perturbed system* (9)*. Then, its quasi-steady state model (see Kokotović et al. (1986) for definition) is described by $y = 0$ by letting $\varepsilon \equiv 0$. Its reduced system in the slow time scale τ is given by*

$$\frac{dz_j}{d\tau} = A_{z,j}z_j, \quad \textit{for } j = 1, 2, \tag{10}$$

where $A_{z,j}$ is the linearization of $f'_{z,j}$. The boundary-layer system, in fast time scale t, is given by

$$\dot{y}_j = A_{y,j}y_j, \quad \textit{for } j = 1, 2, \tag{11}$$

where $A_{y,j}$ is the linearization of $f'_{z,j}$.

Remark 3. $A_{z,j}$ becomes a companion form with a characteristic polynomial: $\det(sI - A_{z,j}) = s^{n_j} + K_{n_j,i}s^{n_j-1} + \cdots + K_{2,j}s + K_{1,j}$, $j = 1, 2$. $A_{y,j}$ becomes a companion form with the characteristic polynomial: $\det(sI - A_{y,j}) = s^{m_j} + L_{m_j,i}s^{m_j-1} + \cdots + L_{2,j}s + L_{1,j}$, $j = 1, 2$.

Proof. Let $\boldsymbol{\varepsilon} = 0$ for the system (9) which gives the quasi-steady state model $y = 0$. Substituting $y = 0$ into Eq. (9), we obtain the reduced system (10) where we use the properties $\left.\frac{\partial\psi_i}{\partial y_{n_j,j}}\right|_{y=0} = 0$, $\psi_i(0) = 0$, $i = 1, 2$ and $j = 1, 2$, in Assumption 1 and the property $\varphi_{n_j,j}(0) = 0$. The boundary layer system (11) is trivially obtained by letting $\boldsymbol{\varepsilon} = 0$. □

Next, we use a standard result from the singular perturbation theory (see Kokotović et al. (1986)) to conclude that the trivial solution of the system (9) is semi-globally stable.

As $K_{i,j}$ and $L_{k,j}$, for $i \in \{1, \ldots, n_j\}$ and $k \in \{1, \ldots, m_j\}$ can be chosen so that $A_{z,j}$ and $A_{y,j}$, $j = 1, 2$, are Hurwitz matrices, there always exist some positive numbers $\alpha_{z,j}$, $\alpha_{y,j}$ and positive symmetric matrices $P_{z,j}$, $P_{y,j}$, $j = 1, 2$ such that for the quadratic Lyapunov functions $V_{z,j} = \frac{1}{2}z_j^{\mathrm{T}}P_{z,j}z_j$, $V_{y,j} = \frac{1}{2}y_j^{\mathrm{T}}P_{y,j}y_j$, $j = 1, 2$, their derivatives along the reduced system (10) in the slow time scale τ and the boundary layer system (11) in the fast time scale t satisfy $\frac{\partial V_{z,j}}{\partial z_j}\frac{dz_j}{d\tau} \leq -\alpha_{z,j}\|z_j\|_2^2$, $\frac{\partial V_{y,j}}{\partial y_j}\frac{dy_j}{dt} \leq -\alpha_{y,j}\|y_j\|_2^2$. Let $V = \sum_{j=1}^{2} V_{z,j}$ and $W = \sum_{j=1}^{2} W_{y,j}$. Then, we have $\frac{\partial V}{\partial z}\frac{dz}{d\tau} \leq -\alpha_z\|z\|_2^2$, $\frac{\partial W}{\partial y}\frac{dy_j}{dt} \leq -\alpha_y\|y\|_2^2$, where $\alpha_z \triangleq \min_{j\in\{1,2\}}\{\alpha_{z,j}\}$ and $\alpha_y \triangleq \min_{j\in\{1,2\}}\{\alpha_{y,j}\}$. Consider now the composite Lyapunov function (see Kokotović et al. (1986))

$$v_d(z, y) = (1 - d)V(z) + dW(y), \quad 0 < d < 1. \tag{12}$$

One can show that, for any $(z, y) \in S' \subset R^{n_1} \times R^{n_2} \times R^{m_1} \times R^{m_2}$, the following conditions hold

$$\begin{aligned}&\left\|\frac{\partial V}{\partial z}\right\| \le \kappa_1\|z\|_2, \qquad \left\|\frac{\partial W}{\partial y}\right\| \le \kappa_2\|y\|_2,\\ &\|f_z'(z,y,\varepsilon)-f_z'(z,0,0)\| \le \kappa_3\|y\|_2+\varepsilon\kappa_4\|z\|_2,\\ &\|f_y'(z,y,\varepsilon)-f_y'(z,y,0)\| \le \varepsilon\kappa_5\|z\|_2+\varepsilon\kappa_6\|y\|_2.\end{aligned} \tag{13}$$

The first and second inequalities hold simply because V and W are quadratic Lyapunov functions. The last two inequalities of (13) arise from the analytical functions $f_z'(z,y,\boldsymbol{\varepsilon}) = f_z'(z,0,0) + (f_z'(z,y,0) - f_z'(z,0,0)) + \boldsymbol{\varepsilon} f_z''(z,y)$, $f_y'(z,y,\boldsymbol{\varepsilon}) = f_y'(z,y,0) + \boldsymbol{\varepsilon} f_y''(z,y)$ in the context of system (9). This implies that, for any compact set S' (13), there exists scalars κ_i, $i = 1,\ldots,6$. Then, the derivative of (12) along the trajectory of the full system (9) in the time scale τ is given by

$$\begin{pmatrix}\dfrac{\partial v_d}{\partial z}\\ \dfrac{\partial v}{\partial y}\end{pmatrix}^{\mathrm T}\begin{pmatrix}f_z(z,y,\varepsilon)\\ f_y(z,y,\varepsilon)\end{pmatrix} \le -\begin{pmatrix}\|z\|_2\\ \|y\|_2\end{pmatrix}^{\mathrm T} \times \begin{pmatrix}(1-d)(\alpha_z-\varepsilon\kappa_1\kappa_4) & -\dfrac{1-d}{2}\kappa_1\kappa_3-\dfrac{d}{2}\kappa_2\kappa_5\\ -\dfrac{1-d}{2}\kappa_1\kappa_3-\dfrac{d}{2}\kappa_2\kappa_5 & \dfrac{d}{\varepsilon}(\alpha_y-\varepsilon\kappa_2\kappa_6)\end{pmatrix} \times \begin{pmatrix}\|z\|_2\\ \|y\|_2\end{pmatrix}. \tag{14}$$

The right hand side of the inequality (14) is negative definite if we choose $\varepsilon \in (0, \varepsilon_d^*)$ and

$$\varepsilon_d^* \triangleq \frac{\alpha_z\alpha_y}{\alpha_z\kappa_1\kappa_4+\alpha_y\kappa_2\kappa_6+\frac{((1-d)\kappa_1\kappa_3+d\kappa_2\kappa_5)^2}{4d(1-d)}}. \tag{15}$$

Proposition 3.3. *Consider the singularly perturbed system (9) and suppose that Assumption 1 holds. Assume $A_{z,j}$ in (10) and $A_{y,j}$ in (11) are Hurwitz and Lyapunov functions $V(z)$ and $W(y)$ satisfying (13). Let ε_d^*, $0 < d < 1$, defined by (15). Select a compact set $S' \ni 0$. There exists a $\boldsymbol{\varepsilon}_d^* > 0$, such that, for all $\varepsilon \in (0, \varepsilon_d^*)$, S' is contained in the domain of attraction of the trivial solution and the origin of (9) is asymptotically stable. Moreover, the corresponding $v_d(z, y)$, defined by (12), is a Lyapunov function for (9).*

Proof. The proof is standard in terms of the singular perturbation theory (Kokotović et al., 1986, Page 314). □

Proposition 3.3 implies the following result.

Corollary 3.4. *Consider the system (5). Select a compact set $S \ni 0$ in the state space $R^{n_1} \times R^{n_2} \times R^{m_1} \times R^{m_2}$. There exists a $\boldsymbol{\varepsilon}_d^* > 0$, such that for all $\boldsymbol{\varepsilon} \in (0, \boldsymbol{\varepsilon}_d^*)$, S is contained in the domain of attraction of the trivial solution.*

Proof. We show that as $\boldsymbol{\varepsilon}$ decreases, the domain of attraction S' (respectively S) enlarges. Because all functions considered here are analytic by assumption, for any $(z, y) \in S'$, we can find κ_i, $i = 1,\ldots,5$ to satisfy the boundedness conditions (13). So, the derivative (14) of the composite Lyapunov function (12) along the trajectory of the full system (9) in the original time scale t rather than $\tau = \boldsymbol{\varepsilon} t$ is rewritten as follows

$$\begin{pmatrix}\dfrac{\partial v_d}{\partial z}\\ \dfrac{\partial v}{\partial y}\end{pmatrix}^{\mathrm T}\begin{pmatrix}f_z(z,y,\varepsilon)\\ f_y(z,y,\varepsilon)\end{pmatrix} \le -\varepsilon\begin{pmatrix}\|z\|_2\\ \|y\|_2\end{pmatrix} \times \begin{pmatrix}(1-d)(\alpha_z-\varepsilon\kappa_1\kappa_4) & -\dfrac{1-d}{2}\kappa_1\kappa_3-\dfrac{d}{2}\kappa_2\kappa_5\\ -\dfrac{1-d}{2}\kappa_1\kappa_3-\dfrac{d}{2}\kappa_2\kappa_5 & \dfrac{d}{\varepsilon}(\alpha_y-\varepsilon\kappa_2\kappa_6)\end{pmatrix} \times \begin{pmatrix}\|z\|_2\\ \|y\|_2\end{pmatrix}. \tag{16}$$

To guarantee that the right hand side of (16) is negative definite, we choose $\boldsymbol{\varepsilon}$ satisfying (15). If we enlarge the set S', it is clear that κ_i, $i = 1, 2, 3, 4$ increase and hence $\boldsymbol{\varepsilon}_d^*$ must decrease for $\boldsymbol{\varepsilon} \in (0, \boldsymbol{\varepsilon}_d^*)$. Moreover, by definition

$$z_j = \left(\boldsymbol{\varepsilon}^{n_j-1}x_{1,j}, \ldots, \boldsymbol{\varepsilon}x_{n_j-1,j},\ x_{n_j,j} + \frac{1}{L_{1,j}}\left(\sum_{i=1}^{m_j-1} L_{i+1,j}y_{i,j} + y_{m_j,j}\right) + \psi_j(y)\right)$$

we see that for any fixed (z, y), $|x|$ does not increase with the decreasing $\boldsymbol{\varepsilon}$. □

Remark 4. Consider Corollary 3.4. One concern is that, when $\boldsymbol{\varepsilon}_d^*$ become too small, the negative definiteness of (16) is weak in the natural time scale t. This is true when other parameters are fixed. Still, as seen from (15), we can either increase α_z, α_y or suppress κ_i, $i = 1,\ldots,6$ such that $\boldsymbol{\varepsilon}_d^*$ is increased. Suppressing κ_i, $i = 1,\ldots,6$ means that the domain of attraction S' (or S) shrinks.

Remark 5. Suppose that, in the controller (4), the nested low gains with the scaling parameter ε are replaced by nested saturation functions without using ε. Then, a decentralized controller with saturations is obtained for (2) and (3), which makes classical nested saturation designs (e.g., Teel (1996)) extendable to multiple "forwarding" systems possessing locally vanishing higher order nonlinear "interconnected" terms. In the context, the latter is a global stabilizing controller. From this point of view, our (semi-global) nested low gains emulate (global) nested saturation functions. However, the linear control functions generated by directly removing saturation levels are not parameterized to achieve an arbitrarily large domain of attraction as the nested low gains in (4) do. We are not aware of anywhere this kind of parameterization has been done in the literature. Actually, our effort so far is to provide a tool as Corollary 3.4 that can find some parameterized linear controller as (4) achieving an arbitrarily large stability region. Our approach is based on the Lyapunov stability analysis.

4. The case study

4.1. The model

Refer to Fig. 1. Consider a spherical inverted pendulum denoted by a set of generalized coordinates $q \triangleq (x, y, \delta, \epsilon)$ in a configuration space $U \triangleq R \times R \times (-\pi/2, \pi/2) \times (-\pi/2, \pi/2)$. $F = (F_x\ F_y)^{\mathrm{T}}$ is a planar control signal applied to a pivot attached to the bottom of the pendulum. Unknown exogenous inputs are collected by $v_f \in R^4$. We review the model in Liu et al. (2008a) as follows,

$$\mathbf{D}(q) \cdot \ddot{q} + \mathbf{C}(q, \dot{q}) \cdot \dot{q} + \mathbf{G}(q) = \mathbf{Q}, \tag{17}$$

where $\mathbf{D}(q)$, $\mathbf{C}(q, \dot{q})$, $\mathbf{G}(q)$ and $\mathbf{Q}$ are given in Appendix.

4.2. Decentralized control design

The nominal dynamics of (17) with the exogenous input $v_f = 0$ can be rewritten as follows

$$\begin{pmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{\delta} \\ \ddot{\epsilon} \end{pmatrix} = \begin{pmatrix} H_{11}(\delta, \epsilon) \\ H_{21}(\delta, \epsilon) \end{pmatrix} F + \begin{pmatrix} H_{12}(\delta, \dot{\delta}, \epsilon, \dot{\epsilon}) \\ H_{22}(\delta, \dot{\delta}, \epsilon, \dot{\epsilon}) \end{pmatrix}, \tag{18}$$

where $H_{21}(\delta, \epsilon)$ is invertible on U and the explicit expressions for $H_{11}(\delta, \epsilon)$, $H_{21}(\delta, \epsilon)$, $H_{12}(\delta, \dot{\delta}, \epsilon, \dot{\epsilon})$, $H_{22}(\delta, \dot{\delta}, \epsilon, \dot{\epsilon})$ are omitted (see Liu et al. (2008a) for details).

4.2.1. Finding the chains of integrators

The dynamics (18) are not in a form of chains of integrators but we shall find some coordinate and control transformations to convert the dynamics (18) to two interconnected chains of integrators in a form such that the decentralized idea in Section 3 applies. This is stated in Lemma 4.1. To this end, let $q_e \triangleq (x, y)$ and $q_s \triangleq (\delta, \epsilon)$. Let $p_s = \dot{q}_s$, $p_e = \dot{q}_e$ and define a state vector $\xi \triangleq (q_e^{\mathrm{T}}\ p_e^{\mathrm{T}}\ q_s^{\mathrm{T}}\ p_s^{\mathrm{T}})^{\mathrm{T}}$.

Lemma 4.1. *There exists a mapping $T : U \to R^8$ such that using a state transformation*

$$\mathrm{X} = T(\xi) \tag{19}$$

and a feedback transformation

$$F = H_{21}^{-1}(q_s) \left(H_{31}^{-1}(q_s)(u - H_{32}(q_s, p_s)) - H_{22}(q_s, p_s) \right) \tag{20}$$

where $H_{31}(q_s) \triangleq \begin{pmatrix} 1 + \tan^2(q_{s1}) & 0 \\ 0 & 1 + \tan^2(q_{s2}) \end{pmatrix}$, $H_{32}(q_s, p_s) \triangleq \begin{pmatrix} 2p_{s1}^2 q_{s1}(1 + \tan^2(q_{s1})) \\ 2p_{s2}^2 q_{s2}(1 + \tan^2(q_{s2})) \end{pmatrix}$, *and u is the new control, the nominal system (18) converts to:*

$$\begin{aligned} \dot{X}_1 &= X_3 + \varphi_1(X_5, \ldots, X_8), & \dot{X}_5 &= X_7, \\ \dot{X}_2 &= X_4 + \varphi_2(X_5, \ldots, X_8), & \dot{X}_6 &= X_8, \\ \dot{X}_3 &= X_5 + \varphi_3(X_5, \ldots, X_8), & \dot{X}_7 &= u_1, \\ \dot{X}_4 &= X_6 + \varphi_4(X_5, \ldots, X_8), & \dot{X}_8 &= u_2. \end{aligned} \tag{21}$$

where

$$\begin{aligned} \varphi_1(\cdot) &= \frac{L}{g} \left(-\frac{4}{3(1 + X_5^2)} + \frac{\left(1 + \frac{1}{3(1+X_6^2)}\right)}{(1 + X_5^2)^{1/2}} \right) X_7 \\ &\quad - \frac{L}{g} \frac{X_8 X_5 X_6 (1 + X_5^2)^{1/2} \left(\frac{1}{3} + \frac{1}{1+X_5^2}\right)}{1 + X_6^2}, \\ \varphi_2(\cdot) &= \frac{L}{g} \left(\left(\frac{4}{3(1 + X_6^2)} - \frac{\frac{(1+X_5^2)^{1/2}}{3} + \frac{1}{(1+X_5^2)^{1/2}}}{(1 + X_6^2)(1 + X_5^2)} \right) X_8 \right), \\ \varphi_3(\cdot) &= X_5 \left(\sqrt{1 + X_6^2} - 1 \right) + \frac{L}{g} \left(\frac{2 X_5 X_7^2 (2 + X_6^2)}{(1 + (X_5)^2)^{3/2} (1 + X_6^2)} \right. \\ &\quad + \frac{X_5 X_8^2 \left((4 + X_5^2)(1 + X_6^2) - 3\right)}{3(1 + X_6^2)^2 (1 + X_5^2)^{1/2}} \\ &\quad \left. - \frac{X_8 X_7 X_6 (1 + 2X_5^2)(4 + X_5^2)}{3(1 + X_5^2)^{3/2} (1 + X_6^2)} \right), \\ \varphi_4(\cdot) &= \frac{L}{g} \left(\left(\frac{X_8^2 (1 + X_5^2)^{1/2}}{(1 + X_6^2)^{3/2}} \left(\frac{1}{3} + \frac{1}{1 + X_5^2} \right) \right) X_6 \right. \\ &\quad + \left(\frac{1}{3} + \frac{1}{1 + X_5^2} \right) \times \frac{X_7 X_8 X_5}{(1 + X_5^2)^{1/2} (1 + X_6^2)^{1/2}} \\ &\quad \left. - \frac{X_7^2 X_6}{3(1 + X_5^2)^{3/2} (1 + x_6^2)^{1/2}} \right) \end{aligned}$$

and $\lim_{\|s\| \to 0} \frac{\|\varphi_i(s)\|}{\|s\|} = 0$, $i = 1, \ldots, 4$.

Proof. The proof is conducted in two steps: (i) as in Olfati-Saber (2001, Theorem 4.4.2, Proposition 4.4.1), apply a change of coordinates eliminating the control signal in new state variables and take a feedback transformation leading to a partial state feedback linearization; (ii) introducing a further coordinate transformation to map U into R^8.

The inertial matrix, $D(q_s) \triangleq \begin{pmatrix} D_{ee}(q_s) & D_{es}(q_s) \\ D_{se}(q_s) & D_{ss}(q_s) \end{pmatrix}$, is invertible on U. Let $\mu(q_s) = D_{se}^{-1}(q_s) D_{ss}(q_s)$. It is not difficult to check that all conditions in Olfati-Saber (2001, Theorem 4.4.2, Proposition 4.4.1) are satisfied.[1] Applying (Olfati-Saber, 2001, Theorem 4.4.2, Proposition 4.4.1) provides a nonlinear transformation

$$q_r = q_e + \mu(0) q_s, \qquad p_r = p_e + \mu(q_s) p_s, \tag{22}$$

on U, that is, $T_a : U \to U'$, $(q, \dot{q}) \mapsto (q_r, p_r, q_s, p_s)$ and an input transformation

$$F = H_{21}^{-1}(q_s)(v - H_{22}(q_s, p_s)) \tag{23}$$

realizes a partial feedback linearization over $\dot{p}_s$. Next, we perform one further change of control:

$$u \triangleq H_{31}(q_s) v + H_{32}(q_s, p_s). \tag{24}$$

[1] A similar result is obtained for a 3D inverted pendulum with a bob m on a massless pole in Olfati-Saber (2001, Chapter 4).

and map the configuration space U' into R^8 using T_b : $(q_r, p_r, q_s, p_s) \mapsto X$,

$$(X_1, \ldots, X_8)^{\mathrm{T}} = (q_{r1}, q_{r2}, p_{r1}, p_{r2}, \tan(q_{s1}), \tan(q_{s2}), (1+\tan^2(q_{s1}))p_{s1}, (1+\tan^2(q_{s2}))p_{s2})^{\mathrm{T}}. \quad (25)$$

By the coordinate transformation $T(\xi) \triangleq T_b \circ T_a(q)$ and the change of control (23) and (24), we obtain the dynamics (21) which comprises of two interconnected chains of integrators: (X_1, X_3, X_5, X_7) and (X_2, X_4, X_6, X_8) respectively. $\varphi_i(\cdot)$, $i = 1, 2, 3, 4$ are some higher order terms with respect to the origin, that is, $\lim_{\|s\|\to 0} \frac{\|\varphi_i(s)\|}{\|s\|} = 0$. □

4.2.2. Control design

Using our constructive method in Corollary 3.4, we assign a linear control law according to (4),

$$\begin{aligned} u_1 &= -\varepsilon L_{1,1}(\varepsilon K_{1,1}X_1 + K_{2,1}X_3) - (L_{1,1}X_5 + L_{1,2}X_7) \\ u_2 &= -\varepsilon L_{1,2}(\varepsilon K_{1,2}X_2 + K_{2,2}X_4) - (L_{1,2}X_6 + L_{2,2}X_8) \end{aligned} \quad (26)$$

and we identify a slow variable $z = (z_1, z_2) \in R^2 \times R^2$ according to (8),

$$\begin{aligned} z_1 &= \left(\varepsilon X_1,\ X_3 + \frac{1}{L_{1,1}}\left(L_{2,1}X_5 + X_7\right) + \psi_1(y)\right) \\ z_2 &= \left(\varepsilon X_2,\ X_4 + \frac{1}{L_{1,2}}\left(L_{2,2}X_6 + X_8\right) + \psi_2(y)\right) \end{aligned} \quad (27)$$

and a fast variable $\mathbf{y} = (\mathbf{y}_{1,1}, \mathbf{y}_{2,1}, \mathbf{y}_{1,2}, \mathbf{y}_{2,2}) \triangleq (X_5, X_7, X_6, X_8)$. The quantities $\psi_1(y)$ and $\psi_2(y)$ that define the slow variable z are the subject of the following lemma.

Lemma 4.2. *The system (21) with the linear control law (26) satisfies Assumption 1 (or PDEs (6)) in that there exist analytic functions $\psi_1(y)$, $\psi_2(y)$ that solve:*

$$\begin{aligned} &\frac{\partial\psi_1}{\partial X_5}X_7 + \frac{\partial\psi_1}{\partial X_6}X_8 + \frac{\partial\psi_1}{\partial X_7}(-L_{1,1}X_5 - L_{2,1}X_7) \\ &\quad + \frac{\partial\psi_1}{\partial X_8}(-L_{1,2}X_6 - L_{2,2}X_8) = -\varphi_3(X_5, X_6, X_7, X_8) \\ &\frac{\partial\psi_2}{\partial X_5}X_7 + \frac{\partial\psi_2}{\partial X_6}X_8 + \frac{\partial\psi_2}{\partial X_7}(-L_{1,1}X_5 - L_{2,1}X_7) \\ &\quad + \frac{\partial\psi_2}{\partial X_8}(-L_{1,2}X_6 - L_{2,2}X_8) = -\varphi_4(X_5, X_6, X_7, X_8) \end{aligned} \quad (28)$$

subject to the boundary conditions:

$$\begin{aligned} &\left.\frac{\partial\psi_1}{\partial X_7}\right|_{\mathbf{y}=0} = 0, \qquad \left.\frac{\partial\psi_1}{\partial X_8}\right|_{\mathbf{y}=0} = 0, \\ &\psi_1(0) = 0, \\ &\left.\frac{\partial\psi_2}{\partial X_7}\right|_{\mathbf{y}=0} = 0, \qquad \left.\frac{\partial\psi_2}{\partial X_8}\right|_{\mathbf{y}=0} = 0, \\ &\psi_2(0) = 0. \end{aligned} \quad (29)$$

Proof. We solve the linear first order PDEs (28) subject to the boundary conditions (29) using the method of characteristics (refers to Rhee, Aris, and Amundson (1986) for introduction of PDEs). Let s represent an independent variable parameterizing the characteristics. We take $y|_{s=0} = (0, \alpha, \beta, 0)$ as initial conditions with $\alpha \neq 0$ and $\beta \neq 0$. The characteristic equations are,

$$\begin{aligned} &\frac{\mathrm{d}X_5}{\mathrm{d}s} = X_7,\ X_5(0) = 0, \quad \frac{\mathrm{d}X_6}{\mathrm{d}s} = X_8,\ X_6(0) = \beta, \\ &\frac{\mathrm{d}X_7}{\mathrm{d}s} = -L_{1,1}X_5 - L_{2,1}X_7,\ X_7(0) = \alpha, \\ &\frac{\mathrm{d}X_8}{\mathrm{d}s} = -L_{1,2}X_6 - L_{2,2}X_8,\ \ X_8(0) = 0. \end{aligned} \quad (30)$$

Given that $L_{i,j} > 0$, $i = 1, 2$ and $j = 1, 2$, $A_1 = \begin{pmatrix} 0 & 1 \\ -L_{1,1} & -L_{2,1} \end{pmatrix}$ and $A_2 = \begin{pmatrix} 0 & 1 \\ -L_{1,2} & -L_{2,2} \end{pmatrix}$ are Hurwitz. We have solutions to (30) as

$$(X_5, X_7)^{\mathrm{T}} = \mathrm{e}^{A_1 s}(0, \alpha)^{\mathrm{T}}, \qquad (X_6, X_8)^{\mathrm{T}} = \mathrm{e}^{A_2 s}(0, \beta)^{\mathrm{T}} \quad (31)$$

with $\mathrm{e}^{A_1 s} \to 0$ and $\mathrm{e}^{A_2 s} \to 0$ as $s \to \infty$.

Next, we integrate the following ODEs

$$\frac{\mathrm{d}\psi_1}{\mathrm{d}s} = -\varphi_3(X_5, \ldots, X_8), \qquad \frac{\mathrm{d}\psi_2}{\mathrm{d}s} = -\varphi_4(X_5, \ldots, X_8), \quad (32)$$

subject to $\psi_1(0) = 0$, $\varphi_3(0, \alpha, \beta, 0) = 0$, $\psi_2(0) = 0$, $\varphi_4(0, \alpha, \beta, 0) = 0$, where the characteristic curves X_5, X_6, X_7, X_8 are parameterized by s as given in (31). Then, we obtain the integration

$$\begin{aligned} \psi_1 &= -\int_0^s \varphi_3(0, \alpha)\mathrm{e}^{A_1 s'}, ((0, \beta)\mathrm{e}^{A_2 s'})\mathrm{d}s' \\ \psi_2 &= -\int_0^s \varphi_4((0, \alpha)\mathrm{e}^{A_1 s'}, (0, \beta)\mathrm{e}^{A_2 s'})\mathrm{d}s'\,. \end{aligned} \quad (33)$$

As $s \to \infty$, $(X_5(s), X_6(s), X_7(s), X_8(s)) \to 0$ hold (see (31)) and hence

$$\begin{aligned} &\lim_{s\to\infty} \psi_1(X_5(s), X_6(s), X_7(s), X_8(s)) = 0 \\ &\lim_{s\to\infty} \psi_2(X_5(s), X_6(s), X_7(s), X_8(s)) = 0 \end{aligned}$$

also hold. Without loss of generality, we let initial conditions $\psi_1(0, \alpha, \beta, 0) = 0$ and $\psi_2(0, \alpha, \beta, 0) = 0$ because of $\varphi_3(0, \alpha, \beta, 0) = 0$ and $\varphi_4(0, \alpha, \beta, 0) = 0$. Therefore, we obtain $\psi_1(0) = 0$ and $\psi_2(0) = 0$. When integrating (33), we play a trick to replace the upper limit s with respect to (31): whenever the original X_5, X_7 (or X_6, X_7) is integrated, the upper limit is replaced by the corresponding equation in (31). Then, $\psi_1(\mathbf{y})$ and $\psi_2(\mathbf{y})$ are the same order as $\varphi_3(\mathbf{y})$ and $\varphi_4(\mathbf{y})$ with respect to y. Therefore, $\left.\frac{\partial\psi_1}{\partial x_7}\right|_{\mathbf{y}=0} = 0$, $\left.\frac{\partial\psi_1}{\partial x_8}\right|_{\mathbf{y}=0} = 0$, $\left.\frac{\partial\psi_2}{\partial x_7}\right|_{\mathbf{y}=0} = 0$, $\left.\frac{\partial\psi_2}{\partial x_8}\right|_{\mathbf{y}=0} = 0$ also hold.

In conclusion, there exists quantities ψ_1 and ψ_2 that solve the linear PDEs (28). □

4.2.3. Stability and robustness

The transformed closed loop system (21) with the intermediate control law (26) can be brought into the standard singular perturbation form in coordinates (z, y) as required in Proposition 3.3. The robustness of the associated closed loop system is guaranteed because the control design comes with

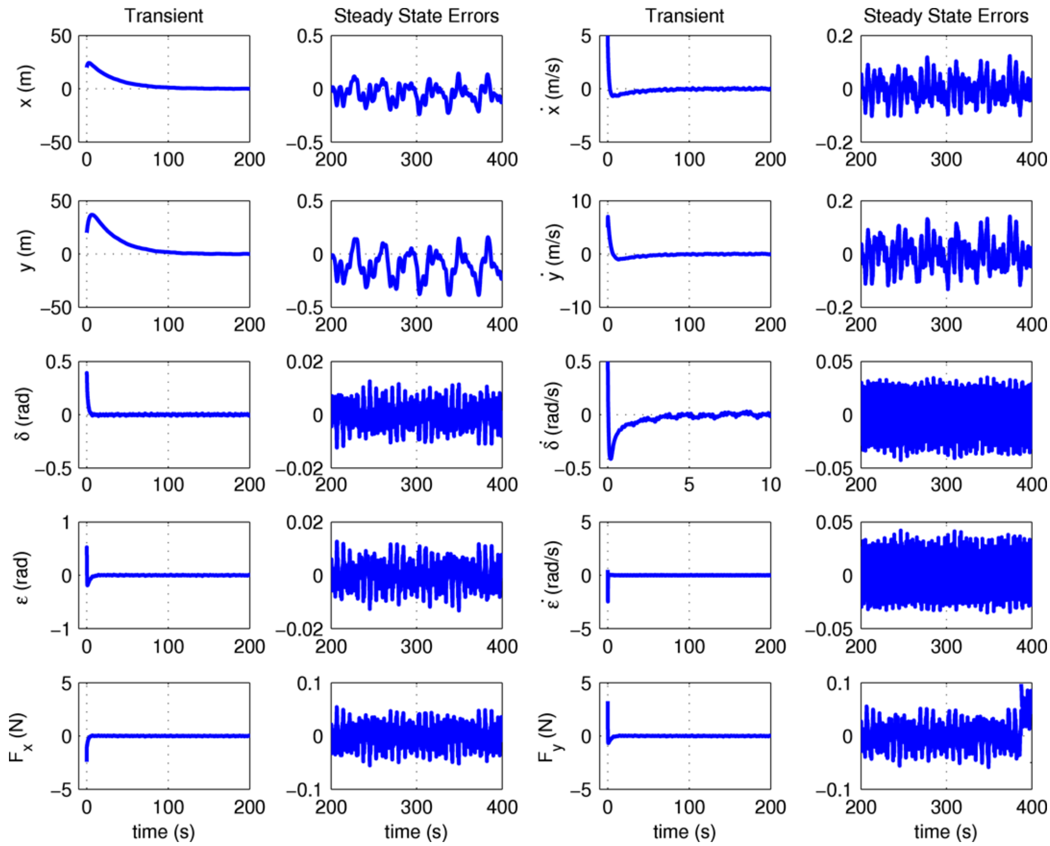


Fig. 2. Simulation results for with $\boldsymbol{\varepsilon} = 0.03$.

its own associated Lyapunov function (12) which yields total stability. Moreover, the control law (26) can be tuned to ensure that the trivial solution of (21) has an arbitrarily large domain of attraction. This implies that the original closed loop system (18) with the control law (20) and (26) yields an arbitrarily large domain of attraction in U by adjusting the positive scalar ε.

4.3. Simulations

The controlled system is evaluated through computer simulation based on a perturbed nonlinear model that is used as the plant. Let $m = 0.35$ (kg), $g = 9.8$ (m/s^2) and $2L = 0.6$ (m). We assign gains $K_{1,1} = K_{1,2} = 20$, $K_{2,1} = K_{2,2} = 10$, $L_{1,1} = L_{1,2} = 100$, $L_{2,1} = L_{2,2} = 20$. The exogenous inputs (see Appendix) are $v_f = ((\Delta_{11} - C_x)\dot{x} + \Delta_{12}, (\Delta_{21} - C_y)\dot{y} + \Delta_{22}, (\Delta_{31} - C_\delta)\dot{\delta}, (\Delta_{41} - C_\epsilon)\dot{\epsilon})^{\mathrm{T}}$, where C_x, C_y, C_δ and C_ϵ are viscous friction coefficients, and $\Delta_{ij} \triangleq \sum_{k=1}^{M} a_{k,ij} \sin(\omega_{k,ij} t + \varphi_{k,ij})$, for $i = 1, 2, 3, 4$ and $j = 1, 2$, with the real numbers $a_{k,ij}$, $\omega_{k,ij}$, $\varphi_{k,ij}$, $k = 1, \ldots, M$, characterize the external disturbances (for example, viscous friction and small quasi-periodic forces). The root mean square value of the exogenous disturbances Δ_{ij} is given by $\mathrm{RMS}_{\Delta_{ij}} = \sqrt{\frac{1}{2}\sum_{k=1}^{M} a_{k,ij}^2}$. For some exogenous disturbances used in simulations, let $C_x = 10^{-4}$(N s/m), $C_y = 10^{-4}$(N s/m), $C_\delta = 10^{-4}$ (N s/rad), $C_\epsilon = 10^{-4}$ (N/rad). Let the RMS for Δ_{i1} with $i = 1, 2$ be 0.01 (N s/m), the RMS for Δ_{i1} with $i = 3, 4$ be 0.01 (N s/rad), and the RMS for Δ_{i2} with $i = 1, 2$ be 0.02 (N). Let $\boldsymbol{\varepsilon} = 0.03$ and choose some large initial state as $(x(0), \dot{x}(0), y(0), \dot{y}(0), \delta(0), \dot{\delta}(0)\epsilon(0), \dot{\epsilon}(0)) = (20, 5, 20, 5, 0.384, 0.5, 0.524, 0.5)$. The simulation results are shown in Fig. 2. The transient responses of (x, δ) and (y, ϵ) parts are almost independent of each other, which results from the intermediate decentralized control law (26). The angular variables and the translational variables are tightly regulated due to the high gain feedback of (26) and slowly regulated due to the low gain feedback of (26) respectively. The closed loop system (18) with the controller (20) and (26) is robust to the given disturbance. We observe that when $\boldsymbol{\varepsilon}$ is tuned to be much larger, say $\boldsymbol{\varepsilon} = 0.8$, the controlled system becomes unstable. This illustrates that the domain of attraction is adjustable by the parameter $\boldsymbol{\varepsilon}$.

Remark 6. In Liu, Mareels, and Nešić (2007c), the proposed controller is compared with a number of other approaches against the same case study (e.g., the controlled Lagrangians (Bloch, Chang, Leonard, & Marsden, 2001) and the forwarding method (Liu et al., 2008a)).

5. Summary

A decentralized linear control scheme is proposed for certain systems possessing interconnected chains of integrators and is applied to a spherical inverted pendulum. The corresponding closed loop systems yield some arbitrarily large domains of attraction by adjusting a design parameter, which is guaranteed by the associated Lyapunov functions.

$$\mathbf{G}(q) = \begin{pmatrix} 0 \\ 0 \\ -mgLs(\delta)c(\epsilon) \\ -mgLc(\delta)s(\epsilon) \end{pmatrix}, \qquad \mathbf{Q} = \begin{pmatrix} F_x + v_{f_1} \\ F_y + v_{f_2} \\ v_{f_3} \\ v_{f_4} \end{pmatrix},$$

$$\mathbf{D}(q) = m \times \begin{pmatrix} 1 & 0 & -Lc(\delta) & 0 \\ 0 & 1 & -Ls(\delta)s(\epsilon) & Lc(\epsilon)c(\delta) \\ -Lc(\delta) & -Ls(\delta)s(\epsilon) & L^2(1 + 1/3c(\epsilon)^2) & 0 \\ 0 & Lc(\epsilon)c(\delta) & 0 & L^2(1/3 + c^2(\delta)) \end{pmatrix}$$

$$\mathbf{C}(q, \dot{q}) = \begin{pmatrix} 0 & 0 & mL\dot{\delta}s(\delta) & 0 \\ 0 & 0 & -mL(\dot{\delta}s(\epsilon)c(\delta) + \dot{\epsilon}c(\epsilon)s(\delta)) & -mL(\dot{\epsilon}s(\epsilon)c(\delta) + \dot{\delta}c(\epsilon)s(\delta)) \\ 0 & 0 & -1/3mL^2\dot{\epsilon}c(\epsilon)s(\epsilon) & -1/3mL^2\dot{\delta}c(\epsilon)s(\epsilon) + mL^2\dot{\epsilon}c(\delta)s(\delta) \\ 0 & 0 & 1/3mL^2\dot{\delta}c(\epsilon)s(\epsilon) - mL^2\dot{\epsilon}c(\delta)s(\delta) & -mL^2\dot{\delta}c(\delta)s(\delta) \end{pmatrix}.$$

Box II.

Appendix. The entries of the model (17)

See Box II.

References

Arcak, M., Teel, A., & Kokotović, P. (2001). Robust nonlinear control of feedforward systems with unmodeled dynamics. *Automatica*, *37*, 265–272.

Bloch, A., Chang, D., Leonard, N., & Marsden, J. (2001). Controlled lagragians and the stabilisation of mechanical systems ii: Potential shaping. *IEEE Transactions on Automatic Control*, *46*, 1556–1571.

Grognard, F., Sepulchre, R., Bastin, G., & Praly, L. (1998). Nested linear low-gain design for semiglobal stabilization of feedforward systems. In *Proceedings of IFAC NOLCOS'98 symposium* (pp. 830–835).

Grognard, F., Sepulchre, R., & Bastin, G. (1999). Global stabilization of feedforward systems with exponentially unstable Jacobian linearization. *Systems & Control Letters*, *37*, 107–115.

Kaliora, G., & Astofi, A. (2004). Nonlinear control of feedforward systems with bounded signals. *IEEE Transactions on Automatic Control*, *49*, 1975–1990.

Kaliora, G., & Astofi, A. (2005). On the stabilization of feedforward systems with bounded control. *Systems & Control Letters*, *54*, 263–270.

Kokotović, P., Khalil, H., & O'Reilly, J. (1986). *Singular perturbation methods in control: Analysis and design*. Academic Press Inc.

Liu, G. (2006). Modelling, stabilising control and trajectory tracking of a spherical inverted pendulum. *Ph.D. thesis*. The University of Melbourne.

Liu, G., Nešić, D., & Mareels, I. (2008a). Non-local stabilization of a spherical inverted pendulum. *International Journal of Control* (in print).

Liu, G., Nešić, D., & Mareels, I. (2008b). Nonlinear stable inversion based control for a spherical inverted pendulum. *International Journal of Control*, *81*, 116–133.

Liu, G., Mareels, I., & Nešić, D. (2007). A note on the control of a spherical inverted pendulum. In *Proceedings of IFAC NOLCOS'07 symposium* (pp. 838–843).

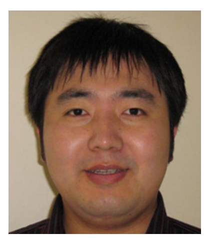
Marconi, L., & Isidori, A. (2000). Robust global stabilization of a class of uncertain feedforward nonlinear systems. *Systems & Control Letters*, *41*, 281–290.

Mazenc, F. (1997). Stabilization of feedforward systems approximated by a non-linear chain of integrators. *Systems & Control Letters*, *32*, 223–229.

Olfati-Saber, R. (2001). Nonlinear control of underactuated mechanical systems with application to robotics and aerospace vehicles. *Ph.D. thesis*. Massachusetts Institute of Technology.

Rhee, H., Aris, R., & Amundson, N. (1986). *First-order partial differential equations: Volume 1 theory and application of single equations*. Prentice Hall.

Teel, A. (1996). A nonlinear small gain theorem for the analysis of control systems with saturation. *IEEE Transactions on Automatic Control*, *41*, 1256–1270.



Dr. Guangyu Liu received his Ph.D. degree in dynamics and control from The University of Melbourne, Australia in 2006. He was a mechanical engineer during 1997–2001 at Dong-feng motor corporation (NISSAN), China and a researcher at a micro-fabrication lab, Australia in 2002. He is presently a researcher at NICTA, Australia. His research interests are dynamics, decision and control and various applications in both engineering and life science.

Dr. Iven Mareels obtained his Ph.D. degree in Systems Engineering from Australian National University, Australia in 1987. He is a Professor and Dean of Faculty of Engineering, The University of Melbourne. Prof. Mareels is a co-editor in chief for Systems & Control Letters, a Fellow of ATSE (Australia), IEEE (USA) and IEAust, a member of SIAM, Vice-Chair of the Asian Control Professors Association, Deputy Chair of the National Committee for Automation, Control and Instrumentation, Chair of the Australian Research Council's panel of experts for Mathematics, Information and Communication sciences, a member of the Board of Governors of the Control Systems Society IEEE and so on. He is registered as a professional engineer. His research interests are in adaptive and learning systems, nonlinear control and modeling.

Dr. Dragan Nešić is a Professor at The University of Melbourne, Australia. He received his Ph.D. degree in Systems Engineering from Australian National University, Australia in 1997. In 1997–1999, he held postdoctoral positions at University of California, Santa Barbara and two other institutions. Since 1999 he has been with The University of Melbourne. His research interests include networked control systems, discrete-time, sampled-data and continuous-time nonlinear control systems, input-to-state stability, extremum seeking control and so on. Prof. Nešić is a Fellow of IEEE, a Fellow of IEAust, a recipient of a Humboldt Research Fellowship (2003) and an Australian Professorial Fellow (2004–2009). He is an Associate Editor for the journals: Automatica, IEEE Transactions on Automatic Control, Systems and Control Letters and European Journal of Control.