Discussion on: “Stabilizability and stability for explicit and implicit polynomial systems: a symbolic computation approach”

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We agree in principle with both remarks raised by Dr. Diop. Indeed, in our paper some of these comments were already acknowledged. We, however, do not quite share to the same degree the pessimism reflected in the comments made by Dr. Diop.

The first remark concerns the computational complexity.

Dr. Diop notes that “... such software package are far from bing ripe for a usage in real life problems”. As observed in the paper, we agree that the computational complexity is an obstacle in general. However, it is very difficult to speculate on how long it may take before we see quantifier elimination packages being used as standard tool-boxes for solving real engineering problems. It is our opinion that this event will occur rather sooner than later. We base our opinion on the following facts:

1. Depending on the class of problems we consider, the computational complexity of quantifier elimination can be drastically reduced. This has been recognized in the literature and much faster algorithms have been developed that outperform QEPCAD (the package referred to in our paper) in problems for which they were designed for. Examples can be found for cases of linear quantifier elimination [2] and quadratic quantifier elimination in [5] (see also [1, 3]). In some of these special cases, problems with more than 20 symbolic variables can easily be tackled and this is already well within the realm of reasonably big and interesting engineering problems.

2. If we observe what happened with some other symbolic computation software over the last 10 years, we can see that many of the methods have rather quickly earned their place in engineering tool-boxes for addressing real problems. For instance, see [4] where a Mathematica package for polynomial methods was used to tackle a benchmark control problem (the dimension of the system is 25, there are 6 inputs and 4 outputs!).

3. The reference [4] is especially interesting since it shows an engineering problem that standard Matlab packages failed to solve whereas a combination of Matlab and Mathematica solved the problem. This was a rather particular situation where the exact computation performed using symbolic packages was essential to solve the problem. The author combined numerical methods (in Matlab) with symbolic computation methods (in Mathematica) and used advantages of both to obtain a solution. It is our opinion that this approach of combining numerical methods with symbolic ones is at present the way to go and this approach may lessen the burden of computational complexity.

The second remark on “rational versus real fields” indeed leads to the relevant question of robustness of computations with respect to the approximated coefficients. We make the following observations:

1. There is already a large body of recent research which addresses this and related questions concerning the approximation of coefficients in symbolic computation. This area goes by the name of *Approximate algebraic computation* and it is attracting a lot of interest. For instance, there were two sessions dedicated exclusively to this area at a recent IMACS Conference on Applications of Computer Algebra, Prague, 1998 (see, for instance **http://math.unm.edu/ACA/1998/sessions.html**).

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2. We also note that in principle one can analyze the robustness of computations with respect to coefficients using the QEPCAD, by letting the uncertain coefficients be symbolic variables. However, the computational complexity explodes even further and this approach is still impractical.

References

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