Track-to-Track Fusion of Out-of-Sequence Tracks

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Abstract – *Fusing out-of-sequence information is a problem of growing importance due to an increased reliance on networked sensors embedded in complicated network architectures. The problem of fusing out-of-sequence measurements (OOSM) has received some attention in literature; however, most practical fusion systems, owing to compatibility with legacy sensors and limited communication bandwidth, send track information instead of raw measurements to the fusion node. Delays introduced by the network can result in the reception of out-of-sequence tracks (OOST). This paper considers the problem of fusing out-of-sequence measurements in general, and proposes an optimal Bayesian solution involving a joint probability density of current and past target states, referred to as augmented states. By representing tracks using equivalent measurements, the relationship between OOSM and OOST-based fusion is shown. The special case of Gaussian statistics is also addressed.*

Keywords: Out-of-Sequence Information, Out-of-Sequence Measurements, Out-of-Sequence Tracks, Equivalent Measurements, Track Fusion, OOSI, OOSM, OOST.

1 Introduction

It is well known that optimal target state estimates may be found by combining measurements from each of the contributing sensors in a distributed network [1]. However, most such networks communicate tracks rather than measurements owing to the prior existence of embedded trackers, and prohibitive communications bandwidths that favour summaries of measurements, rather than the measurements themselves. Equivalent measurements derived from track information forms such a summary of measurements.

Equivalent measurements, their use and methods of extraction from track estimates are discussed extensively by Blackman and Popoli [2], Frenkel [3] and Drummond [4], however, its Bayesian underpinnings are not considered in their efforts. Frenkel [3] demonstrates that the equivalent measurements remove all cross correlations between sensor level tracks and the central fused tracks, enabling them to be treated as normal sensor measurements with white measurement noise. Drummond [4] uses equivalent measurements to deal with the complex cross-correlations involved in track fusion when global tracks are fed back to sensor level tracks. However, Frenkel and Drummond did not extend the use of equivalent measurements beyond the track fusion problem, nor did they provide any Bayesian foundations for their use. In [5], Okello and Challa present the £rst use of equivalent measurements to the registration problem and show their Bayesian foundation in [6].

Transmissions over a complex network may result in the reception of data out of sequence; that is, there may be no guarantee that data are received in the order they have originated. This out-of-sequence data fusion problem has been addressed in the case of measurements, for example [7, 8, 9]; however, it is poorly understood for out-of-sequence tracks (OOST). This paper addresses this problem with equivalent measurements in an optimal augmented state framework. This implicitly compensates for the correlation between associated tracks. Section 2 formulates the Bayesian solution to the out-of-sequence information (OOSI) problem, Section 3 formulates track-to-track fusion and the equivalent measurement approach, Section 4 describes the augmented state approach with OOST, Section 5 has a simulated example of OOST fusion and conclusions are drawn in Section 6.

2 Bayesian Solution to the OOSI Problem

Let $x(t_k)$ be the target state at time t_k, $Y(\tau)$ be the set of sensor measurements $\{y_1(\tau), \cdots, y_n(\tau)\}$ from n sensors corresponding to time τ, Y_i^k be the measurement sequence up to time k for sensor i and $Y^k = \{Y_1^k, \cdots, Y_n^k\}$ be the set of sensor measurement sequences received from all sensors up to time t_k.

Having processed all the measurements Y^k, the complete information about the target state $x(t_k)$ is described in the probability density function $p(x(t_k)|Y^k)$. The OOSI problem arises as a consequence of receiving information representing a measurement set $Y(\tau)$ at time t_k that corresponds to time $\tau < t_k$. This may be in the form of measurements or tracks. The solution to this OOSI problem seeks to update $p(x(t_k)|Y^k)$ to obtain $p(x(t_k)|Y^k, Y(\tau))$.

One of the recent approaches [7] to solve this problem assumes that the measurement delay is less than one sampling period, i.e., $t_{k-1} \leq \tau < t_k$. This approach, although presented from a non-Bayesian point of view is Firmly rooted in the Bayesian logic. The solution to the OOSM problem requires the conditional density $p(x(t_k)|Y(\tau), Y^k)$. Using Bayes' theorem, we have,

$$p(x(t_k)|Y(\tau), Y^k) = \frac{p(x(t_k), Y(\tau)|Y^k)}{p(Y(\tau)|Y^k)} \quad (1)$$

This approach considers only target originated measurements, thus $Y(\tau)$ can be replaced by $y(\tau)$. Using the results in [10], the Bayesian solution can be shown to be a Gaussian density if $x(t_k)$ and $y(\tau)$ are Gaussian distributed with mean

$$\hat{x}(t_{k|\tau,k}) = \hat{x}(t_{k|k}) + P_{xy}P_{yy}^{-1}(y(\tau) - \hat{y}(\tau)) \quad (2)$$

and covariance

$$P(t_{k|\tau,k}) = P_{xx} - P_{xy}P_{yy}^{-1}P_{yx} \quad (3)$$

where

$$P_{xx} = E[(x(t_k) - \hat{x}(t_{k|k}))(x(t_k) - \hat{x}(t_{k|k}))^T|Y^k] = P_{k|k} \quad (4)$$

$$P_{yy} = E[(y(\tau) - \hat{y}(\tau))(y(\tau) - \hat{y}(\tau))^T|Y^k] = S_{\tau|k} \quad (5)$$

$$P_{xy} = E[(x(t_k) - \hat{x}(t_{k|k}))(y(\tau) - \hat{y}(\tau))^T|Y^k] = P_{yx}^T \quad (6)$$

where the backward predicted measurement is expressed as

$$\hat{y}(\tau) = H_\tau F_{\tau|k}[\hat{x}(t_{k|k}) - Q_k(\tau)H_\tau^T S_{\tau|k}^{-1}(y(t_k) - \hat{y}(t_{k|k-1}))] \quad (7)$$

In this expression, H_τ is the observation matrix at time τ, $F_{\tau|k}$ is the system backward transition matrix[1] from t_k to τ, the last term, which is ignored in [2, 11, 8], accounts for the effect of process noise (with covariance $Q_k(\tau)$) on the estimate $\hat{x}(t_{k|k})$.

The cross covariance P_{xy} in (6) is given by

$$P_{xy} = [P_{k|k} - P_{x\tilde{y}}]F_{\tau|k}^T H_\tau^T \quad (8)$$

where

$$\begin{aligned} P_{x\tilde{y}} &= Cov\{x(t_k), w_k(\tau)|Y^k\} \\ &= Q_k(\tau) - P(t_{k|k-1})H_\tau^T S^{-1}(t_k)H_\tau P(t_{k|k-1}) \end{aligned}$$

[1] Although τ represents time, it also is used to indicate corresponding time index whenever no confusion arises.

The Y-algorithm, as pointed out in [7], requires storage of the last innovation and can be interpreted as a type of non-standard smoothing.

When at time t_k, multiple delayed measurements corresponding to known previous times, are received, the problem of updating the current state using these delayed measurements is called the OOSM problem with multiple delays. Mallick *et. al* [8] addressed this problem as summarized below and is referred to as the M-algorithm. The M-algorithm, proposed in [8, 12], extends the Y-algorithm, in an approximate way, to account for multiple delays. The key idea of this approach is to determine the cross covariance of (6) for each delayed measurement and at each time interval. By expressing the delayed measurement $y(\tau)$ as a function of the current state $x(t_k)$, the multiple lag OOSM problem can be solved by computing the cross covariance P_{xy} in a recursive manner for each time delayed measurement. For example, when the delay time τ is more than n sampling intervals, we have

$$\begin{aligned} P_{xy|n} &= -M_{k-n+1}Q(k-n+1, k; k-n+1, k) \\ &- \sum_{i=1}^{n} M_{k-i+1}Q(k-i+1, k-i; k-i+1, k) \end{aligned} \quad (9)$$

where

$$M_{k-i+1} = \begin{cases} B_k, & i = 1 \\ C_k C_{k-1} \cdots C_{k-i+2} B_{k-i+1}, & i = 2, \cdots, n \end{cases} \quad (10)$$

$$B_i = I - K_i H_i \quad (11)$$

$$C_i = B_i F_{i-1|i} \quad (12)$$

and the covariance of process noise

$$\begin{aligned} &Q(k-i+1, k-i; k-i+1, k) \\ &= E\{w(k-i+1, k-i; k-i+1) \\ &\times w^T(k-i+1, k-i; k-i+1)\} \end{aligned}$$

Clearly, in the calculation of the covariance in (9), one needs to evaluate the process noise from the time when measurement delay occurs to the current time and all time steps in between. One also has to evaluate all corresponding Filter gains.

Alternatively, the Bayes' theorem can be applied in a slightly different manner starting with

$$p(x(t_k)|Y^k, Y(\tau)) = \frac{p(Y(\tau)|x(t_k), Y^k)\,p(x(t_k)|Y^k)}{p(Y(\tau)|Y^k)}. \quad (13)$$

Considering the numerator of (13) and introducing the target state at time τ, $x(\tau)$,

$$\begin{aligned} &p(Y(\tau)|x(t_k), Y^k)\,p(x(t_k)|Y^k) \\ &= \int p(Y(\tau), x(\tau)|x(t_k), Y^k)\,p(x(t_k)|Y^k)\,dx(\tau) \end{aligned}$$

$$= \int p(Y(\tau)|x(\tau), x(t_k), Y^k) p(x(\tau)|x(t_k), Y^k) \times p(x(t_k)|Y^k)\, dx(\tau).$$

Since

$$p(x(\tau), x(t_k)|Y^k) = p(x(\tau)|x(t_k), Y^k)\, p(x(t_k)|Y^k),$$

we have

$$p(Y(\tau)|x(t_k), Y^k) p(x(t_k)|Y^k) = \int p(Y(\tau)|x(\tau), x(t_k), Y^k)\, p(x(\tau), x(t_k)|Y^k)\, dx(\tau). \tag{14}$$

Substituting (14) back into (13) yields

$$p(x(t_k)|Y^k, Y(\tau)) = \frac{\int p(Y(\tau)|x(\tau), x(t_k), Y^k)\, p(x(\tau), x(t_k)|Y^k)\, dx(\tau)}{p(Y(\tau)|Y^k)} = \int \frac{p(Y(\tau)|x(\tau), x(t_k), Y^k)\, p(x(\tau), x(t_k)|Y^k)}{p(Y(\tau)|Y^k)}\, dx(\tau).$$

Using the inverse form of Bayes' rule

$$p(x(t_k)|Y^k, Y(\tau)) = \int p(x(\tau), x(t_k)|Y^k, Y(\tau))\, dx(\tau). \tag{15}$$

It is thus clear that solving the OOST problem involves *consideration of the joint density of the current target state and the target state corresponding to the delayed information.* In contrast, the Y-algorithm and M-algorithm needs to evaluate the past process noise from the time when measurement delay occurs to the current time and all time steps in between and the corresponding Filter gains. Thus, in all delayed data algorithms, some form of past information and its correlation with the current information is needed. In the joint density of current and past states approach proposed in this paper, this information is implicitly present, while it is explicitly calculated in Y-algorithm and M-algorithms. Moreover, joint density approach enjoys signigicant advantages of smoothing that come for free while handling the delayed data and can be easily extended to clutter scenarios while the other techniques cannot.

Generalizing (15), the OOST problem involving multiple delays can be stated as follows: Let the delayed measurements received at time t_k be denoted by $Y(\tau) = \{Y(\tau_1), Y(\tau_2), \cdots, Y(\tau_d)\}$, where $\tau_i < t_k, \forall\ i \in \{1, \cdots, d\}$ and τ_d is the time corresponding to the maximum time delay. Then the solution to the OOSI problem is to determine the density

$$p(x(t_k)|Y^k, Y(\tau))$$

$$= \int_{x(\tau_1)} \int_{x(\tau_2)} \cdots \int_{x(\tau_d)} p(x(t_k), x(\tau_1), x(\tau_2), \cdots, x(\tau_d)|Y^k, Y(\tau))\, dx(\tau_1)\, dx(\tau_2) \cdots dx(\tau_d), \tag{16}$$

which indicates that, in general, the solution involves a Bayes recursion for the joint probability density of an augmented state vector $\mathbf{X}_k = \left[x^T(t_k), x^T(\tau_1), \cdots, x^T(\tau_d)\right]^T$, *i.e.*,

$$p(x(t_k), x(\tau_1), x(\tau_2), \cdots, x(\tau_d)|Y^k, Y(\tau)) = p(\mathbf{X}_k|Y^k, Y(\tau)). \tag{17}$$

Consider a discrete time system where

$$\tau_1 = t_{k-1}, \tau_2 = t_{k-2}, \cdots, \tau_d = t_{k-d}.$$

Then, by Bayes rule,

$$p(x(t_k), x(t_{k-1}), \cdots, x(t_{k-d})|Y^k, Y(\tau)) = \frac{1}{\delta} p(Y(\tau)|x(t_k), x(t_{k-1}), \cdots, x(t_{k-d}), Y^k) \times p(x(t_k), x(t_{k-1}), \cdots, x(t_{k-d})|Y^k) \tag{18}$$

for a normalizing constant δ.

Denoting $[x^T(t_k), \cdots, x^T(t_{k-d})]^T$ as $\mathbf{X}_k^d$, the Bayes recursion for (18) becomes

$$p(\mathbf{X}_k^d|Y^k, Y(\tau)) = \frac{1}{\delta} p(Y(\tau)|\mathbf{X}_k^d, Y^k)\, p(\mathbf{X}_k^d|Y^k). \tag{19}$$

If the two densities on the right hand side of (19) are Gaussian, then the posterior density on the left hand side of (19) is also Gaussian and the solution reduces to a standard Kalman Filter with an augmented state [13]. Equation (19) assumes that the measurement $Y(\tau)$ is available along with a measurement likelihood function. However, as pointed out earlier, tracks rather than measurements are available. In asuch a case, OOST fusion can be addressed using an augmented state approach in conjunction with the consideration of equivalent measurements. Such an approach is based on the standard track-to-track fusion using equivalent measurements and a review of this is considered in the next section.

3 Track-to-Track Fusion and Equivalent Measurements

Let $x(t_k)$ be the target state at time t_k for a target that is visible to two non-collocated sensors 1 and 2. If $y_i(t_k)$ is the sensor measurement from sensor $i = 1, 2$ at time t_k, then let $Y_i^k = \{y_i(j) : j = 1, ..., k\}$ be the set of sensor measurements up to time t_k generated by sensor i. At the central location,

$$p(x(t_k)|Y_1^k, Y_2^k)$$

$$
\begin{aligned}
&= p(x(t_k)|y_1(t_k), y_2(t_k), Y_1^{k-1}, Y_2^{k-1}) \\
&= \frac{1}{\delta_{12}} p(y_1(t_k)|x(t_k), Y_1^{k-1})\, p(y_2(t_k)|x(t_k), Y_2^{k-1}) \\
&\quad \times p(x(t_k)|Y_1^{k-1}, Y_2^{k-1}) \qquad (20) \\
&= \frac{1}{\delta_{12}} p(y_1(t_k)|x(t_k))\, p(y_2(t_k)|x(t_k)) \\
&\quad \times p(x(t_k)|Y_1^{k-1}, Y_2^{k-1}) \qquad (21)
\end{aligned}
$$

where δ_{12} is a normalizing constant and (21) follows from (20) under the assumption of white measurement noise. If the network has limited bandwidth such that only track outputs $p(x(t_k)|Y_i^k)$, $i = 1, 2$, rather that sensor outputs $p(y_i(t_k)|x(t_k))$, $i = 1, 2$, can be transmitted to the fusion center at each processing time t_k, then the fusion algorithm must carry out track-to-track fusion based only on $p(x(t_k)|Y_i^k)$, $i = 1, 2$. At sensor i, we have

$$
\begin{aligned}
& p(x(t_k)|Y_i^k)) \\
&\quad = p(x(t_k)|y_i(t_k), Y_i^{k-1}) \\
&\quad = \frac{1}{\delta_i} p(y_i(t_k)|x(t_k))\, p(x(t_k)|Y_i^{k-1}),\ i = 1, 2, \qquad (22)
\end{aligned}
$$

where δ_i is a normalizing constant. Thus we have

$$
p(y_i(t_k)|x(t_k)) = \delta_i \frac{p(x(t_k)|Y_i^k)}{p(x(t_k)|Y_i^{k-1})} \qquad (23)
$$

This equation together with equation (21) yields

$$
\begin{aligned}
& p(x(t_k)|Y_1^k, Y_2^k) \\
&\quad = \frac{\delta_1 \delta_2}{\delta_{12}} \frac{p(x(t_k)|Y_1^k)}{p(x(t_k)|Y_1^{k-1})} \\
&\qquad \times \frac{p(x(t_k)|Y_2^k)}{p(x(t_k)|Y_2^{k-1})} p(x(t_k)|Y_1^{k-1}, Y_2^{k-1}). \qquad (24)
\end{aligned}
$$

Thus, at each time t_k, the fusion center requires the state estimate and state prediction from each sensor. Assuming Gaussian distributions, $p(x(t_k)|Y_i^{k-1}) = \mathcal{N}[x(t_k); \hat{x}_i(t_{k|k-1}), P_i(t_{k|k-1})]$ and $p(x(t_k)|Y_i^k) = \mathcal{N}[x(t_k); \hat{x}_i(t_{k|k}), P_i(t_{k|k})]$ at sensor $i = 1, 2$. The quotient of probability density functions from sensor i, for $i = 1, 2$, therefore takes the form
$\frac{p(x(t_k)|Y_i^k)}{p(x(t_k)|Y_i^{k-1})}$

$$
\begin{aligned}
&= K\exp\Bigg\{-\frac{1}{2}\Bigg[\Big(x(t_k) - \hat{x}_i(t_{k|k})\Big)^T P_i^{-1}(t_{k|k})\Big(x(t_k) - \hat{x}_i(t_{k|k})\Big) \\
&\qquad - \Big(x(t_k) - \hat{x}_i(k|k-1)\Big)^T P_i^{-1}(k|k-1)\Big(x(t_k) - \hat{x}_i(k|k-1)\Big)\Bigg]\Bigg\} \\
&= K\exp\Bigg\{-\frac{1}{2}\Bigg[x^T(t_k)\Big(P_i^{-1}(t_{k|k}) - P_i^{-1}(k|k-1)\Big)x(t_k) \\
&\qquad - 2x^T(t_k)\Big(P_i^{-1}(t_{k|k})\hat{x}_i(t_{k|k}) - P_i^{-1}(k|k-1)\hat{x}_i(k|k-1)\Big) \\
&\qquad + \hat{x}_i^T(t_{k|k})P_i^{-1}(t_{k|k})\hat{x}_i(t_{k|k}) - \hat{x}_i^T(k|k-1)P_i^{-1}(k|k-1)\hat{x}_i(k|k-1)\Bigg]\Bigg\} \\
&= K\exp\Bigg\{-\frac{1}{2}\Bigg[x^T(t_k)A(t_k)x(t_k) - 2x^T(t_k)b(t_k) + c(t_k)\Bigg]\Bigg\} \\
&= K\exp\Bigg\{-\frac{1}{2}\Bigg[\Big(x(t_k) - A^{-1}(t_k)b(t_k)\Big)^T A(t_k) \\
&\qquad \Big(x(t_k) - A(t_k)^{-1}b(t_k)\Big) - b^T(t_k)A^{-1}(t_k)b(t_k) + c(t_k)\Bigg]\Bigg\} \qquad (25)
\end{aligned}
$$

where $K = \frac{|2\pi P_i(t_{k|k-1})|^{\frac{1}{2}}}{|2\pi P_i(t_{k|k})|^{\frac{1}{2}}}$. Denoting

$$
\begin{aligned}
u_i(t_k) &= A^{-1}(t_k)(t_k)b(t_k) \\
&= \Big[P_i^{-1}(t_{k|k}) - P_i^{-1}(t_{k|k-1})\Big]^{-1} \\
&\quad \times \Big[P_i^{-1}(t_{k|k})\hat{x}_i(t_{k|k}) - P_i^{-1}(t_{k|k-1})\hat{x}_i(t_{k|k-1})\Big] \qquad (26) \\
U_i(t_k) &= A^{-1} = \Big[P_i^{-1}(t_{k|k}) - P_i^{-1}(t_{k|k-1})\Big]^{-1}, \qquad (27)
\end{aligned}
$$

(25) can be rewritten in the form
$\frac{p(x(t_k)|Y_i^k)}{p(x(t_k)|Y_i^{k-1})}$

$$
\begin{aligned}
&= K\exp\Bigg\{-\frac{1}{2}\Bigg[\Big(x(t_k) - u_i(t_k)\Big)^T U_i^{-1}(t_k)\Big(x(t_k) - u_i(t_k)\Big) \\
&\qquad - b^T(t_k)A^{-1}(t_k)b(t_k) + c(t_k)\Bigg]\Bigg\} \\
&= K_1\exp\Bigg\{-\frac{1}{2}\Bigg[\Big(x(t_k) - u_i(t_k)\Big)^T U_i^{-1}(t_k) \\
&\qquad \times\Big(x(t_k) - u_i(t_k)\Big)\Bigg]\Bigg\} \\
&= \mathcal{N}\Big(u_i(t_k); x(t_k), U_i(t_k)\Big) \\
&= p(u_i(t_k)|x(t_k)). \qquad (28)
\end{aligned}
$$

Hence,

$$
p(y_i(t_k)|x(t_k)) \propto \frac{p(x(t_k)|Y_i^k)}{p(x(t_k)|Y_i^{k-1})} \propto p(u_i(t_k)|x(t_k)) \qquad (29)
$$

The variable $u_i(t_k)$ in equation (26) is the equivalent measurement vector from sensor i at time t_k and $U_i(t_k)$ in (27) is its covariance matrix. These expressions are identical to the equivalent measurements derived by inverting the Kalman £lter equations when $H = I$, *i.e.*, when the equivalent measurements are expressed in state space variables [3]. It may be noted that these equivalent measurements are no longer correlated with each other and the target state at the fusion node. Hence they may be treated as conventional measurements in a standard Kalman Filter.

Substituting equation (28) into (24), the probability density function at the fusion center takes the form

$$p(x(t_k)|Y_1^k, Y_2^k) = \frac{\delta_1 \delta_2}{\delta_{12}} \Big[\prod_{1=1}^{2} p(u_i(t_k)|x(t_k))\Big] p(x(t_k)|Y_1^{k-1}, Y_2^{k-1}) \quad (30)$$

This is the same as a centralized Kalman filter with equivalent measurements expressed in state space variables, *i.e.*, $u(t_k) = [u_1^T(t_k), u_2^T(t_k)]^T$ is the combined equivalent measurement vector,

$$U(t_k) = \begin{bmatrix} U_1(t_k) & 0 \\ 0 & U_2(t_k) \end{bmatrix}$$

is the associated covariance matrix, and $H = [I, I]^T$ is the measurement matrix, where I is the identity matrix with dimension equal to that of the state vector. In other words, $u_i(t_k)$ can be expressed in terms of the standard measurements equation

$$u_i(t_k) = Ix(t_k) + \eta_i(t_k) \quad (31)$$

This is equivalent to

$$u_i(t_k) = \begin{bmatrix} I & 0 & \ldots & 0 \end{bmatrix} \mathbf{X}(t_k) + \begin{bmatrix} \eta_i(t_k) & 0 & \ldots & 0 \end{bmatrix}^T \quad (32)$$

Thus

$$p(u_i(t_k)|x(t_k)) = p(u_i(t_k)|\mathbf{X}(t_k)) \quad (33)$$

Equation (32) is of great relevance to OOST problem using equivalent measurements as shown in the next section.

4 Track-to-Track Fusion with OOST

To demonstrate the problem of track-to-track fusion with OOST, we consider a two sensor case where the fusion is done at sensor 1. Suppose the estimated track sequence from sensor 2 has a random delay of r sampling intervals compared to that of sensor 1 as shown in Figure 1. Track to track fusion at the fusion center can be performed using the augmented state approach as follows.

Let the augmented state at time t_k at the fusion center be $\mathbf{X}(t_k) \triangleq [x(t_k), \cdots, x(k-r)]^T$. Thus, the fusion problem reduces to the problem of finding a solution to the joint density $p(x(t_k), \cdots, x(k-r)|Y_1^k, Y_2^{k-r})$. Using Bayes' rule, we have

$$\begin{aligned} p(x(t_k), \cdots, x(k-r)|Y_1^k, Y_2^{k-r}) &= \frac{1}{\delta} p(y_1(t_k), y_2(k-r)|\mathbf{X}(t_k), Y_2^{k-r-1}) \\ &\quad \times p(\mathbf{X}(t_k)|Y_1^{k-1}, Y_2^{k-r-1}) \\ &= \frac{1}{\delta} p(y_1(t_k)|\mathbf{X}(t_k) p(y_2(k-r)|\mathbf{X}(t_k)) \\ &\quad \times p(\mathbf{X}(t_k)|Y_1^{k-1}, Y_2^{k-r-1}) \\ &= \frac{1}{\delta^*} p(u_1(t_k)|\mathbf{X}(t_k)) p(u_2(k-r)|\mathbf{X}(t_k)) \\ &\quad \times p(\mathbf{X}(t_k)|Y_1^{k-1}, Y_2^{k-r-1}) \end{aligned} \quad (34)$$

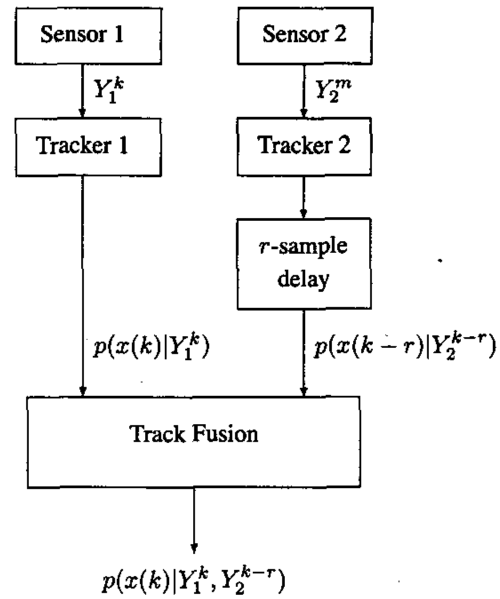

Figure 1: A distributed sensor fusion scenario.

where δ^* and δ are normalizing constants. In obtaining the last equation of (34), we use the result in (29). Under Gaussian assumptions, a solution to (34) can be obtained using the Augmented State Kalman Filter (ASKF) structure, proposed in [9], at the fusion node.

Let $\mathbf{Y}(t_k) = [u_1(t_k), u_2(k-r)]^T$ be the set of equivalent measurements received from the two sensors in the network. The system dynamics involving the augmented past states is given by

$$\begin{aligned} \mathbf{X}(t_{k+1}) &= \mathbf{F}_k \mathbf{X}(t_k) + \mathbf{W}_k \\ \mathbf{Y}(t_k) &= \mathbf{H}_k \mathbf{X}(t_k) + \mathbf{V}_k \end{aligned} \quad (35)$$

where the system transition matrix is obtained as proposed in [14]

$$\mathbf{F}_k = \begin{bmatrix} F & 0 & \cdots & 0 \\ I & 0 & \cdots & 0 \\ 0 & \ddots & 0 & \vdots \\ \vdots & 0 & I & 0 \end{bmatrix} \quad (36)$$

F is the system matrix of the standard (non-agumented) system. The observation matrix is slightly different to that used for the standard smoothing algorithms proposed in

[14] and is given by

$$\mathbf{H}_k = \begin{bmatrix} H_k & 0 & \cdots & 0 \\ 0 & \ddots & 0 & \vdots \\ \vdots & \cdots & \ddots & 0 \\ 0 & \cdots & 0 & H_{k-r} \end{bmatrix} \tag{37}$$

The predicted density and the likelihood are given by

$$p(\mathbf{X}(t_k)|\mathbf{Y}^{k-1}) = \mathcal{N}(\mathbf{X}(t_k); \widehat{\mathbf{X}}(t_{k|k-1}), \mathbf{P}_{k|k-1}) \tag{38}$$

$$p(\mathbf{Y}_k|\mathbf{X}(t_k), \mathbf{Y}^{k-1}) = \mathcal{N}(\mathbf{Y}_k; \mathbf{H}_k\widehat{\mathbf{X}}(t_{k|k-1}), \mathbf{S}_k) \tag{39}$$

where

$$\widehat{\mathbf{X}}(t_{k|k-1}) = \mathbf{F}_k\widehat{\mathbf{X}}(t_{k-1|k-1})$$
$$\mathbf{P}_{k|k-1}) = \mathbf{F}_k\mathbf{P}_{k|k-1})\mathbf{F}_k' + Q(t_k)$$

and the updated density [13] is given by

$$p(\mathbf{X}(t_k)|\mathbf{Y}^k) = \mathcal{N}(\mathbf{X}(t_k); \widehat{\mathbf{X}}(t_{k|k}), \mathbf{P}_{k|k}) \tag{40}$$

with mean and covariance

$$\widehat{\mathbf{X}}(t_{k|k}) = \widehat{\mathbf{X}}(t_{k|k-1}) + \mathbf{K}_k\widetilde{\mathbf{Y}}_k \tag{41}$$

$$\mathbf{P}_{k|k} = (\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)\mathbf{P}_{k|k-1} \tag{42}$$

where the innovation is

$$\widetilde{\mathbf{Y}}_k = \mathbf{Y}_k - \mathbf{H}_k\widehat{\mathbf{X}}(t_{k|k-1}) \tag{43}$$

with covariance

$$\mathbf{S}_k = \mathbf{H}_k\mathbf{P}_{k|k-1}\mathbf{H}_k^T + \mathbf{R}_k \tag{44}$$

and Kalman gain matrix

$$\mathbf{K}_k = \mathbf{P}_{k|k-1}\mathbf{H}_k^T\mathbf{S}_k^{-1} \tag{45}$$

5 A Simulated Example of OOST Fusion Problem

In our simulation, the scenario involves two local track nodes and a central fusion node. The central fusion node always receives track updates from Tracker 1 without delay and from Tracker 2 with a delay of 3 time steps.

The root mean squared (RMS) error is used to compare the fusion performance in the following cases:

1. The fusion node receives tracks from both trackers without delay.

2. The fusion node does not receive any tracks from Tracker 2, i.e., it treats the delayed tracks as missed information.

3. The fusion node receives delayed tracks from Tracker 2 with appropriate time stamps.

The basic target model is described by a discrete time system

$$x(t_k) = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix} x(t_{k-1}) + v(t_k) \tag{46}$$

where $T = 1$ is the sampling interval, $v(t_k)$ is a zero-mean white Gaussian process noise with covariance

$$Cov\{v(t_k)\} = \begin{bmatrix} Q_1(t_k) & 0 \\ 0 & Q_1(t_k) \end{bmatrix} \tag{47}$$

where

$$Q_1(t_k) = \begin{bmatrix} T^3/3 & T^2/2 \\ T^2/2 & T \end{bmatrix} q,$$

with $q = 0.01$.

The observation model for the sensors is given by

$$y_i(t_k) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} x(t_k) + w_i(t_k), \quad i \in 1, 2, \tag{48}$$

where w_i is a zero-mean white Gaussian process with covariance

$$Cov\{w_i(t_k)\} = R_i = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad i \in 1, 2. \tag{49}$$

An initial state (observed by sensor 1) is assumed to be

$$x(0) = [200\,\text{km}, 0.5\,\text{km/s}, 100\,\text{km}, -0.08\,\text{km/s}]. \tag{50}$$

The two sensors are separated by about 100 km.

An ASKF is formed at the central fusion node with r states, where $r = 4$ and allows for a £xed time delay of 3 steps for sensor 2's "equivalent measurements" while it assumes no delay for sensor 1.

The target state estimate $\hat{x}_i(t_{k|k})$ and its error covariance $P_i(t_{k|k})$ are obtained by each tracking node. These are then converted into the equivalent measurements $u_i(t_k)$ with variance $U_i(t_k)$ using (26) and (27) at the local nodes and are sent to the central fusion node (the ASKF).

The equivalent measurement sequences from the tracking nodes consist of both position and velocity components. Thus, the measurement model required by the centralized fusion node (35) is

$$\begin{bmatrix} u_1(t_k) \\ u_2(t_{k-r}) \end{bmatrix} = \begin{bmatrix} \mathcal{H}_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathcal{H}_2 \end{bmatrix} \mathbf{X}(t_k) + \begin{bmatrix} n_1(t_k) \\ n_2(t_{k-r}) \end{bmatrix} \tag{51}$$

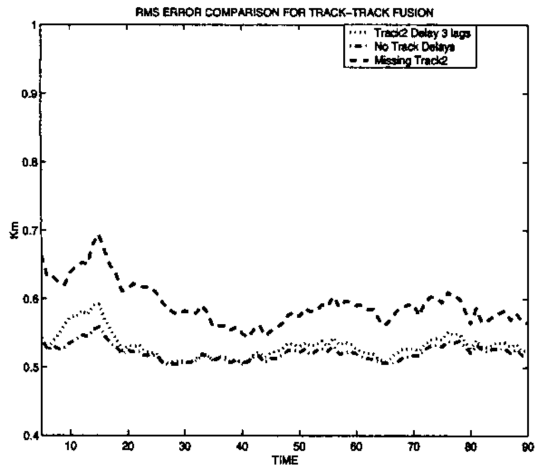

Figure 2: RMS error comparison for an ASKF fusion of two tracks when (1) there are no delays; (2) track 2 is missing; (3) track 2 has a delay of 3 time steps.

where

$$\mathcal{H}_1 = \mathcal{H}_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and n_1 and n_2 are independent, zero mean, white noise processes with covariances $U_1(t_k)$ and $U_2(t_{k-r})$ respectively. The system at fusion node is given by

$$\begin{bmatrix} x(t_{k+1}) \\ x(t_k) \\ x(t_{k-1}) \\ x(t_{k-2}) \end{bmatrix} = \begin{bmatrix} F & 0 & 0 & 0 \\ I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \end{bmatrix} \begin{bmatrix} x(t_k) \\ x(t_{k-1}) \\ x(t_{k-2}) \\ x(t_{k-3}) \end{bmatrix} + \begin{bmatrix} w(t_k) \\ w(t_{k-1}) \\ w(t_{k-2}) \\ w(t_{k-3}) \end{bmatrix}$$

where F is given in (48).

The results from averaging 1000 Monte Carlo runs is shown in Figure 2. Through this simulation, the following observations can be made.

1. There is a signi£cant performance gain when tracks are fused, compared to the one obtained from only one sensor and treating the delayed tracks as lost tracks.

2. The optimal fusion of a track that has been delayed by three time steps with an up-to-date track gives a result that is close to that obtained when there is zero delay.

6 Conclusions

The problem of fusing out-of-sequence or delayed track information is considered in this paper. We have shown that solving the OOST problem involves *consideration of the joint density of the current target state and the target state corresponding to the delayed information.* In contrast, the Y-algorithm and M-algorithm needs to evaluate the past process noise from the time when measurement delay occurs to the current time and all time steps in between and the corresponding Filter gains. Thus, in all delayed data algorithms, some form of past information and its correlation with the current information is needed. In the joint density of current and past states approach proposed in this paper, this information is implicitly present, while it is explicitly calculated in the Y-algorithm and M-algorithms. However, joint density approach enjoys signigicant beni£ts of smoothing that come for free while handling the delayed data and can be easily extended to clutter while the other techniques cannot.

Using Bayesian logic we demonstrate that, in general, track-to-track fusion can be solved using equivalent measurements generated from individual sensor tracks. These equivalent measurements are then used in an augmented state framework to solve the problem. Simulation results for a simple two dimensional problem is presented to demonstrate the results. We conclude that the augmented state approach in conjunction with equivalent measurements provides an effective way to solve the OOST fusion problem.

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