Evaluation of IPDA Type Filters with a Low Elevation Sea-Surface Target Tracking*

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Abstract – *The Integrated Probabilistic Data Association (IPDA) type filters provide estimates of the underlying target probability of existence /perceivability/visibility as well as track state maintenance. These quantities are conveniently used as track quality measures and can be used for track confirmation and termination. In this work five filters of this type are investigated and evaluated in a low elevation sea-surface target tracking scenario. The sea-surface induced multipath fading reduces the detection probability of the target at certain ranges which can lead to track loss. In this paper we evaluate the IPDA type filters in such scenario.*

Keywords: PDA, IPDA, IMMPDA, EB-PDA, VM-IPDA, Markov chain 2, Target tracking, Multipath fading.

1 Introduction

A class of target tracking algorithms based on the probabilistic data association (PDA) [1] can provide probabilistic measure on target existence /perceivability/visibility as well as target state estimates. We refer to this class of algorithms as IPDA type filters and the additional statistic measure as track quality measure. Automatic track initiation in clutter will initialize true tracks which follow targets as well as false tracks which do not. We want to confirm true tracks and terminate false tracks. With on-line track quality measure, the IPDA type filters can be used for track confirmation and termination, as well as the state estimation of tracks.

When tracking a single target in the presence of clutter, more than one measurements may be received at each scan after gating (measurement validation) process which eliminates measurements that fall outside a specified confidence region [2]. In general, track maintenance using false measurements can lead to serious filter divergence problem. Therefore, a data association technique is required to differentiate target originated measurement from clutter. The standard PDA technique which incorporates Kalman filter for tracking in clutter has a moderate computational load and a reasonable performance [3]. PDA uses all validated measurements weighted with the posterior probability that the measurement is the target measurement. It also approximates estimation probability density function (PDF) with a Gaussian PDF. However, it is unable to provide the information on track quality measure directly because it assumes that the underlying target always exists and is visible with the probability of detection P_D.

IPDA type filters are based on the PDA technique and they incorporate various models for the underlying track quality measure. The track quality measure is calculated in a recursive manner.

The Interacting Multiple Model Probabilistic Data Association (IMMPDA) algorithm [4, 5, 6], proposed by Bar-Shalom *et. al*, incorporates the interacting multiple model (IMM) estimation algorithm with PDA technique and uses two PDA filters (models). One model assumes that the target is visible with a known probability of detection. The second model assumes that the target is not visible and is modelled with probability of detection equal to zero. The posterior probabilities of each model are calculated in a recursive manner and the probability of the visible model is then used as the track quality measure. The IPDA, proposed by Mušicki *et. al* in [7, 8], has two options on choice of Markov chain models of target existence propagation. Markov Chain One, the default, recognizes two possibilities: the target either does not exist, or it exists and is visible with a probability of detection. Markov Chain Two, denoted with IPDA as IPDA-M2, also recognizes the possibility of target existing but not being visible. A Variable Markov Chain IPDA (VM-IPDA) is presented in this paper. The VM-IPDA comprises both Markov Chain models. It uses IPDA for unconfirmed and IPDA-M2 for confirmed track maintenance. A new formulation of IPDA, the Existence-based PDA (EB-PDA), presented by X. Rong-Li *et. al* in [9, 10], replaces target existence hypotheses with that of target perceivability. Target is perceivable if it is both existent and visible. All algorithms mentioned in this paragraph provide both data association formulae and probability of target existence which serves as track quality measure.

The problem of multipath propagation of the signal received by the radar from a low elevation sea-surface target has been studied in the literature [11, 12, 13, 14]. Sea surface acts like an imperfect mirror for radar signals. Because of this, the radar signal can reach target and return to the receiver using four different paths. Received signal, being the complex sum of the signals over different paths can be either amplified or attenuated. This is known as multipath fading. The probability of detection can be severely affected by multipath fading. Study on the impact of multipath fading to target tracking is subsequently of practical importance. An example of such case was described in [11] and we will adopt it for evaluating the IPDA type filters.

In this paper we compare five algorithms, IPDA, IPDA-M2, VM-IPDA, EB-PDA and IMMPDA using the scenario where tar-

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get detection probability varies due to multipath fading caused by sea-surface signal reflections and in the presence of clutter. When the target is in the low detection probability region, its track may diverge and become a false track, or its track quality measure may fall below the termination threshold resulting in the track termination. Unfortunately, very few references in such situation can be found in the literature.

Following the introduction section, algorithms to be evaluated are described in Section 2, the scenario for algorithm test, computer simulation and result discussions are presented in Section 3 followed by concluding remarks.

2 Tracking with IPDA Type Filters

A target trajectory is described by

$$x_{k+1} = Fx_k + w_k \tag{1}$$

with equivalent (after liberalization) measurement equation

$$y_k = Hx_k + v_k \tag{2}$$

where x_k represents target kinematic state (position, velocity, etc.), F and H are known matrices. w and v are white and independent system and measurement noise respectively and[1]

$$\begin{aligned} w_k &\sim \mathcal{N}\big(w_k;\ 0,\ Q_k\big) \\ v_k &\sim \mathcal{N}\big(v_k;\ 0,\ R_k\big) \end{aligned} \tag{3}$$

Target tracking problem is to find the posterior conditional PDF of the target state $p(x_k|Y^k)$ according to a measurement sequence received up to time k. Since clutter is involved, the measurement sequence in this paper is denoted as $Y^k = \{Y_1, \cdots, Y_k\}$, where $Y_k = \{y_k^1, \cdots, y_k^{m_k}\},\ m_k \geq 0$ signifies the set of validated measurements at time k.

All algorithms compared in this paper can also be used for automatic track confirmation and termination as they recursively calculate track quality measures. These are followed by a Markov chain model on target existence/visibility/perceivability propagation. The nature of the IPDA and IPDA-M2 lead to a non-reversible Markov state (i.e., a "non-existent target" cannot become an existent target). On the other hand, IMMPDA and EB-PDA implicitly assume that the target always exists and the events related to target visibility/perceivability are reversible. In general, they are all extension of PDAF and derived based on following common assumptions:

- clutter is uniform/Possion distributed within the validation gate. The clutter measurement density is not known a-priori-non parametric model is used.
- estimation PDF is approximated with a single Gaussian PDF.
- at most one target exists.
- at most one validated measurement is the target detection.
- track has been initiated using, for example, two point difference method [3].

[1]A standard notation $\mathcal{N}\big(x;\ \bar{x},\ \sigma\big)$ is used in this paper to denote a Gaussian random variable x with mean $\bar{x}$ and variance σ.

2.1 IPDA with Markov Chain One

The IPDA proposed in [7] is derived based on PDAF [3] by introducing the concept of target existence. Two mutually exclusive and exhaustive events associated with target existence were assumed, and modelled as a random variable E_k,

E_k the target exists and is visible at time k,
$\bar{E}_k$ the target does not exist at time k.

The occurrence of these two events is modelled as a two states Markov Chain with transition probability matrix

$$\Pi = \begin{bmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{bmatrix} \tag{4}$$

where

$$\begin{aligned} &\pi_{ij} \triangleq P\{E_k = j | E_{k-1} = i\}\,,\ i, j \in \{1, 2\} \\ &\pi_{11} + \pi_{12} = \pi_{21} + \pi_{22} = 1 \end{aligned}$$

is the transition probability for (i, j)th entries and $P\{\cdot\}$ denotes probability.

Data association events[2] are mutually exclusive and exhaustive:

θ_0 : all validated measurements are false measurements;
θ_i : the ith validated measurement is the target measurement, and others are false measurements.

The mean number of clutter measurements inside the validation gate of volume V_k is given by

$$\hat{m}_k = \begin{cases} 0\,, & m_k = 0 \\ m_k - P_D P_G P\{E_k|Y^{k-1}\}\,, & m_k > 0 \end{cases} \tag{5}$$

where P_D and P_G denote detection probability and gate (data validation) probability respectively and $P\{E_k|Y^{k-1}\}$ is the predicted (a-priori) probability of target existence, which can be obtained from the previous scan probability of target existence $P\{E_{k-1}|Y^{k-1}\}$, i.e.,

$$P\{E_k|Y^{k-1}\} = \pi_{11} P\{E_{k-1}|Y^{k-1}\} + \pi_{21}(1 - P\{E_{k-1}|Y^{k-1}\}) \tag{6}$$

As in the standard PDAF [3], the target state estimate conditioned on target existence and its associated covariance are obtained as

$$\begin{aligned} \hat{x}_{k|k} &= \sum_{i=0}^{m_k} \beta_i(k) \hat{x}_{k|k}^i \\ P_{k|k} &= \sum_{i=0}^{m_k} \beta_i(k) (P_{k|k}^i + \hat{x}_{k|k}^i \hat{x}_{k|k}^{i^T}) - \hat{x}_{k|k}^i \hat{x}_{k|k}^{i^T} \end{aligned} \tag{7}$$

where $\hat{x}_{k|k}^0 = \hat{x}_{k|k-1},\ P_{k|k}^0 = P_{k|k-1}^*$, K is the Kalman gain, $P_{k|k-1}^* = (1 + q_0 KH) P_{k|k-1}$ is the corrected predicted error covariance ($1 \geq q_0 \geq 0$) discussed in [15], which takes gating error into account. In our simulation q_0 was zero for all algorithms with minimal error considering our choice of P_G, $\hat{x}_{k|k}^i$ and $P_{k|k}^i$ are the state estimate and the state estimate covariance conditioned

[2]$\theta_0,\ \theta_i$ always signify the events occurring at time k, unless specified otherwise.

on the event θ_i. The data association probabilities are calculated as below,

$$\begin{aligned}\beta_0(k) &\triangleq P\{\theta_0|E_k, Y^k\} \\ &= \frac{1-P_DP_G}{1-\delta_k} \quad (8)\\ \beta_i(k) &\triangleq P\{\theta_i|E_k, Y^k\} \\ &= \frac{P_DP_G\frac{V_k}{\hat{m}_k}\Lambda_k^i}{1-\delta_k} \quad (9)\end{aligned}$$

where

$$\delta_k = \begin{cases} P_DP_G[1-\frac{V_k}{\hat{m}_k}\sum_{i=1}^{m_k}\Lambda_k^i] & m_k \neq 0 \\ P_DP_G & m_k = 0 \end{cases} \quad (10)$$

where

$$\Lambda_k^i \triangleq p(y_k^i|E_k, Y^{k-1}, \theta_i) = P_G^{-1}\mathcal{N}\big(y_k^i; \hat{y}_k, S_k\big) \quad (11)$$

S_k is the covariance of the predicted measurement $\hat{y}_k$.

The posterior probability of track existence is given by

$$P\{E_k|Y^k\} = \frac{(1-\delta_k)P\{E_k|Y^{k-1}\}}{1-\delta_kP\{E_k|Y^{k-1}\}} \quad (12)$$

which is the IPDA measure of track quality.

2.2 IPDA with Markov Chain Two

By considering that an existing target may or may not be visible by a sensor, the events of the target existence at time k are [7]:

$E_k \cap E_k^v = E_k^v$	the target exists and is visible,
$E_k \cap E_k^n = E_k^n$	the target exists and is not visible,
$\bar{E}_k$	the target does not exist.

Note that for IPDA $E_k = E_k^v$ whereas for IPDA-M2 $E_k = E_k^v \cup E_k^n$.

As a consequence, the occurrence of the above events is modelled as a three state Markov chain with transition probability matrix given by

$$\Pi = \begin{bmatrix} \pi_{11} & \pi_{12} & \pi_{13} \\ \pi_{21} & \pi_{22} & \pi_{23} \\ \pi_{31} & \pi_{32} & \pi_{33} \end{bmatrix} \quad (13)$$

where

$$\sum_{j=1}^{3}\pi_{1j} = \sum_{j=1}^{3}\pi_{2j} = \sum_{j=1}^{3}\pi_{3j} = 1$$

Markov Chain Two propagation is

$$\begin{bmatrix} P\{E_k^v|Y^{k-1}\} \\ P\{E_k^n|Y^{k-1}\} \\ P\{\bar{E}_k|Y^{k-1}\} \end{bmatrix} = \Pi^T \begin{bmatrix} P\{E_{k-1}^v|Y^{k-1}\} \\ P\{E_{k-1}^n|Y^{k-1}\} \\ P\{\bar{E}_{k-1}|Y^{k-1}\} \end{bmatrix} \quad (14)$$

The target state estimate and its covariance are conditioned on target existence and are also given by (7). The posterior conditional probabilities of the track quality measure are given by:

$$\begin{aligned} P\{E_k^v|Y^k\} &= \frac{(1-\delta_k)P\{E_k^v|Y^{k-1}\}}{1-\delta_kP\{E_k^v|Y^{k-1}\}} \quad (15)\\ P\{E_k^n|Y^k\} &= \frac{P(E_k^n|Y^{k-1})}{1-\delta_kP\{E_k^v|Y^{k-1}\}} \quad (16)\\ P\{E_k|Y^k\} &= P\{E_k^v|Y^k\} + P\{E_k^n|Y^k\} \quad (17)\end{aligned}$$

where δ_k is given by (10) and

$$\hat{m}_k = m_k - P_DP_GP\{E_k^v|Y^{k-1}\}. \quad (18)$$

The data association probabilities are given by [7]

$$\begin{aligned}\beta_0(k) &= \frac{(1-P_DP_G)P\{E_k^v|Y^{k-1}\} + P\{E_k^n|Y^{k-1}\}}{(1-\delta_k)P\{E_k^v|Y^{k-1}\} + P\{E_k^n|Y^{k-1}\})} \quad (19)\\ \beta_i(k) &= \frac{P_DP_G\frac{V_k}{\hat{m}_k}\Lambda_k^iP\{E_k^v|Y^{k-1}\}}{(1-\delta_k)P\{E_k^v|Y^{k-1}\} + P\{E_k^n|Y^{k-1}\}} \quad (20)\end{aligned}$$

2.3 VM-IPDA

The implementation of VM-IPDA is inspired by the idea in [8], where the IPDA-M2 is designated for maintaining confirmed tracks only. Markov Chain Two with $\pi_{21} = 1$ and $\pi_{22} = \pi_{23} = \pi_{12} = \pi_{32} = 0$ becomes Markov Chain One model for target existence propagation. Thus, changing the value of (13), IPDA-M2 may behave like IPDA. VM-IPDA is implemented as IPDA-M2 with variable transition probability matrix of (13).

IPDA-M2 causes slower drop in the probability of target existence than others mentioned in this paper when no state estimate update. However, the IPDA-M2 tends to tolerate more false tracks in the track confirmation process while IPDA does not. Thus we want design a filter which possesses the advantages of both IPDA and IPDA-M2. In particular,

- we use IPDA until the track is confirmed.
- and use IPDA-M2 for maintaining confirmed tracks.

Since the above functions can be achieved with an IPDA-M2 by changing its Markov Chain model, we call the new algorithm as Variable Markov Chain IPDA, i.e. VM-IPDA.

2.4 EB-PDA

The EB-PDA approach in [9] incorporates the data association events with target perceivability events rather than target existence events. The process of the target perceivability is modeled as a Markov chain with two states:

O_k	a target is perceivable from a sensor ,
$\bar{O}_k$	a target is not perceivable from a sensor.

Similar to IPDA, the target perceivability transition probability matrix is given by (4). EB-PDA implicitly assumes that the underlying target always exists.

From Markov chain property, the predicted probability that a target is perceivable is given by

$$P\{O_k|Y^{k-1}\} = \pi_{11}P\{O_{k-1}|Y^{k-1}\}+\pi_{21}(1-P\{O_{k-1}|Y^{k-1}\}) \quad (21)$$

and the posterior probability of target perceivability can be calculated using

$$\begin{aligned} P\{O_k|Y^k\} &= \beta_0(k) + \sum_{i=1}^{m_k}\beta_i(k) \\ &= \frac{(1-\delta_k)P\{O_k|Y^{k-1}\}}{1-\delta_kP\{O_k|Y^{k-1}\}} \quad (22)\end{aligned}$$

which is the measure of track quality. The posterior probability that the target is not perceivable is given by

$$P\{\bar{O}_k|Y^k\} = 1 - P\{O_k|Y^k\} = \beta_{\bar{0}}(k) \quad (23)$$

The data association probabilities are given by

$$\beta_{\bar{0}}(k) = \frac{1}{c}\frac{m_k}{V_k}(1+\frac{1-P_DP_G}{P_DP_G}\xi_k)\frac{1-P(O_k|Y^{k-1})}{1-\varepsilon_k P(O_k|Y^{k-1})}$$
$$\beta_0(k) = \frac{1}{c}\frac{m_k}{V_k}\frac{1-P_DP_G}{P_DP_G}\xi_k\frac{(1-\varepsilon_k)P\{O_k|Y^{k-1}\}}{1-\varepsilon_k P\{O_k|Y^{k-1}\}}$$
$$\beta_i(k) = \frac{1}{c}\frac{\Lambda_i(k)}{P_G}\frac{(1-\varepsilon_k)P\{O_k|Y^{k-1}\}}{1-\varepsilon_k P\{O_k|Y^{k-1}\}} \quad (24)$$

where c is a normalization factor which can be obtained from the relation

$$\beta_{\bar{0}}(k)+\beta_0(k)+\sum_{i=1}^{m_k}\beta_i(k)=1$$

and

$$\xi_k = \frac{\lambda}{V_k}$$
$$\varepsilon_k = \begin{cases} P_DP_G & m_k=0 \\ P_DP_G(1-\frac{1}{\lambda}\sum_{i=1}^{m_k}\Lambda_i(k)) & m_k\neq 0\end{cases}$$

and the clutter density λ is given by

$$\lambda = \begin{cases} 0\,, & m_k=0 \\ \frac{1}{V_k}[m_k - P_DP_G\frac{(1-\varepsilon_k)P\{O_k|Y^{k-1}\}}{1-\varepsilon_k P\{O_k|Y^{k-1}\}}]\,, & m_k>0\end{cases}$$

Similar to (7), the target state estimate and covariance are given by [9]:

$$\hat{x}_{k|k} = (\beta_0(k)+\beta_{\bar{0}}(k))\hat{x}_{k|k-1}+\sum_{i=1}^{m_k}\beta_i(k)\hat{x}^i_{k|k}$$
$$P_{k|k} = \sum_{i=\bar{0}}^{m_k}\beta_i(k)(P^i_{k|k}+\hat{x}^i_{k|k}\hat{x}^{i^T}_{k|k})-\hat{x}^i_{k|k}\hat{x}^{i^T}_{k|k} \quad (25)$$

where $P^{\bar{0}}_{k|k} = P_{k|k-1}$, $P^0_{k|k} = P^*_{k|k-1}$ and $\hat{x}^{\bar{0}}_{k|k} = \hat{x}^0_{k|k} = \hat{x}_{k|k-1}$.

2.5 IMMPDA

The Interacting Multiple Model PDAF, proposed in [5, 4], is an application of IMM algorithm with two models. It is assumed that a target always exists. At time k, two mutually exclusive and exhaustive models, $\{M(k)=M_1\}\triangleq M^1_k$ and $\{M(k)=M_2\}\triangleq M^2_k$ are assumed, i.e.,

M^1_k : the target is visible with a detection probability $P^1_D = P_D$.
M^2_k : target is not visible, or the detection probability $P^2_D = 0$.

Such process is modelled as a Markov chain associated with above two models. The (i,j)th entry of its transition probability matrix is defined as

$$P\{M_k=j|M_{k-1}=i\}=\pi_{ij} \qquad i,j\in 1,2. \quad (26)$$

Each recursion starts at time k with the mixed initial state estimate and covariance of Gaussian mixture form

$$\hat{x}^{0i}_{k-1|k-1} = \sum_{j=1}^{r}\hat{x}^j_{k-1|k-1}\mu_{j|i}(k-1|k-1)$$

$$P^{0i}_{k-1|k-1} = \sum_{j=1}^{2}\mu_{j|i}(k-1|k-1)\{P^j_{k-1|k-1} + [\hat{x}^j_{k-1|k-1}-\hat{x}^{0i}_{k-1|k-1}][\hat{x}^j_{k-1|k-1}-\hat{x}^{0i}_{k-1|k-1}]^T\} \quad (27)$$

where the mixing probabilities are determined by

$$\mu_{j|i}(k-1|k-1) = \frac{\pi_{ij}\mu_j(k-1)}{\sum_{l=1}^2\pi_{il}\mu_l(k-1)} = \frac{\pi_{ij}\mu_j(k-1)}{\mu_i(k|k-1)} \quad (28)$$

where $\mu_i(k-1) = P\{M^i_{k-1}|Y^{k-1}\}$ and $\mu_i(k|k-1) = P\{M^i_k|Y^{k-1}\}$ are the initial and predicted model probabilities of model i respectively.

A standard PDA procedure is then applied to each model with validated measurements received at time k and the i model based state estimate is

$$\hat{x}^i_{k|k} = \beta^i_0(k)\hat{x}^i_{k|k-1}+\sum_{j=1}^{m_k}\beta^i_j(k)\hat{x}^i_j(k|k) \quad (29)$$

with covariance $P^i_{k|k}$, where it can be shown that

$$\beta^i_0(k) = \frac{(1-P^i_DP_G)}{1-\delta^i_k}$$
$$\beta^i_j(k) = \frac{P^i_DP_G\frac{V_k}{m_k}\Lambda^i_j}{1-\delta^i_k}$$
$$i=1,2;\ j=1,2,\cdots,m_k \quad (30)$$

where

$$\delta^i_k = \begin{cases} P^i_DP_G(1-\frac{V_k}{m_k}\sum_{h=1}^{m_k}\Lambda^h_j) & m_k>0 \\ P^i_DP_G & m_k=0\end{cases}$$

$$\Lambda^i_j = P_G^{-1}\mathcal{N}\big(y_j(k);\hat{y}^i(k),S^i(k)\big) \quad (31)$$

The updated model probability is

$$P\{M^i_k|Y^k\} = \frac{(1-\delta^i_k)\mu_i(k|k-1)}{\sum_{l=1}^2(1-\delta^l_k)\mu_l(k|k-1)} \quad (32)$$

and $P\{M^1_k|Y^k\}$ is the IMMPDA measure of track quality.

The output state estimate and covariance are

$$\hat{x}_{k|k} = \sum_{j=1}^{2}\hat{x}^j_{k|k}\mu_j(k)$$
$$P_{k|k} = \sum_{j=1}^{2}\mu_j(k)\{P^j_{k|k}+[\hat{x}^j_{k|k}-\hat{x}_{k|k}][\hat{x}^j_{k|k}-\hat{x}_{k|k}]^T\} \quad (33)$$

3 Evaluation Study

The IPDA type algorithms are evaluated using simulation of a low elevation sea-surface target tracking. Target detection probabilities model variable target detection probabilities due to complex sum of multipath signals. The track quality measures of the IPDA type algorithms are used for automatic track confirmation and track termination against pre-determined (separated) thresholds.

3.1 Detection model for the low elevation sea-surface target

The target travels from a range of 30 Km to a range of 5 Km at an altitude 50 meters and relative speed 312.5 m/s. The expected values of the observed signal to noise ratio (SNR) for a S-band (frequency of 4 GHz) phase array radar 20 meters above sea-surface are simulated based on the signal model developed in [11], and shown in Figure 1, where it is assumed that signal from

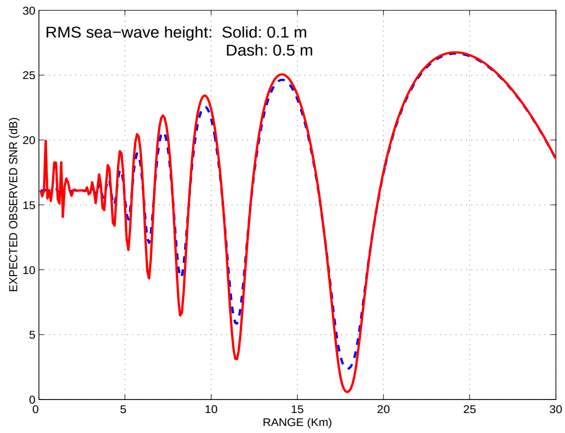

Figure 1: Expected observed SNR by a S-band phased array Radar due to sea-surface induced multipath propagation.

the target has a fixed-amplitude with an expected SNR of 16 dB in the absence of multipath propagation. The radar was modeled as vertically polarized with one-way 3 dB beamwidth 2^o and the antenna beam elevation angle is 1.05^o.

It is observed from Figure 1 that, for a fixed detection threshold of radar receiver, target may not be reliably detected at certain scans and it becomes even worse if sea-surface reflection condition is better (e.g., the average Root-Mean-Square (RMS) sea-wave height is about 0.1 m which corresponds to the solid line plot). Taking these factors into account, we may model the target detection probability as a time function as shown in Figure 2 where we assume the detection probability to be $P_D = 0.9$ in

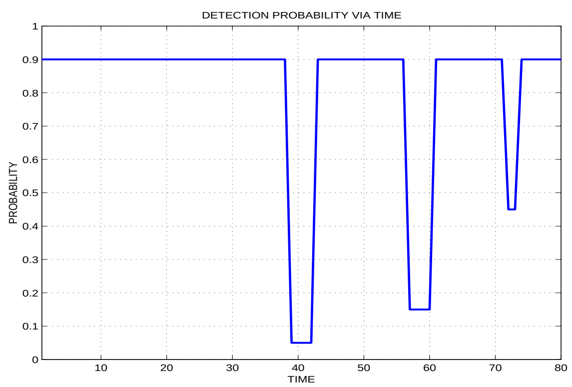

Figure 2: Detection probability for data generation model

the absence of multipath propagation, $P_D = 0.05$ for the first, $P_D = 0.15$ for the second and $P_D = 0.45$ for the third time slots respectively. Clutter was uniformly distributed with density $D_c = 0.12/Km^2$.

3.2 Track formation and maintenance

Track formation is implemented using the method described in [4]. A tentative track is started for any measurement that does not associated with any existing tracks. An acquisition gate is then set up. It centered about the measurement and its volume is determined by the expected maximum velocity of the underlying target. An initial value of track quality measure is assigned for each tentative track. All IPDA type algorithms discussed in this paper have self-contained probabilistic statistics for track quality measure. We use these statistics to confirm and terminate tracks.

Let χ_k denote the event that a track is true at time k.

A track is confirmed if

$$P\{\chi_k|Y^k\} \geq c, \qquad 0 < c < 1 \tag{34}$$

A track is deleted or terminated if

$$P\{\chi_k|Y^k\} < \epsilon, \qquad 0 < \epsilon < c < 1 \tag{35}$$

where c is the track confirmation threshold and ϵ is the track termination threshold. Both of them are assumed to be constants.

The probability $P\{\chi_k|Y^k\}$ is equivalent to

$P\{E_k\|Y^k\}$	for IPDA,
$P\{E_k^v\|Y^k\} + P\{E_k^n\|Y^k\}$	for IPDA-M2 and VM-IPDA,
$P\{O_k\|Y^k\}$	for EB-PDA and,
$P\{M_k^1\|Y^k\}$	for IMMPDA.

In general, both c and ϵ take different values for different algorithms at a given clutter density D_c. Moreover, the performance of an IPDA type filter is influenced by choices of the initial value $P\{\chi_0|y(0)\}$, c and ϵ.

Two sets of c and $P\{\chi_0|y(0)\}$ were considered for each algorithm. As shown in Table 1[3], the Set1 assigns identical $P\{\chi_0|y(0)\}$ and c for all algorithms. The Set2 is chosen via limited number of experiments in the effort to get a better compromise between the number of confirmed false tracks and the percentage of track loss can be achieved. As indicated in [8], obtaining Set2 should use an optimization programming procedure. However, as you can see from the simulation results, the use of an optimized set of parameters in the simulation for the comparison of multipath signal fading effect is not essential.

Parameter Set		IPDA	IPDA-M2	EB-PDA	IMMPDA
Set1	$P\{\chi_0\|y(0)\}$	0.5	0.5	0.5	0.5
Set1	c	0.98	0.98	0.98	0.98
Set2	$P\{\chi_0\|y(0)\}$	0.07	0.07	0.07	0.2
Set2	c	0.99	0.99	0.99	0.99

Table 1: Parameter Set1 and Set2

For each algorithm the threshold ϵ is determined according to the following procedure:
Assume that a true track becomes a false track at time $k = t_1$ within surveillance region. This is simulated by removing target detection from scan $t_1 = 40s$. The track is then terminated at time $k = t_2$ with a time delay $\tau = t_2 - t_1$ for a given threshold

[3]Table 1 IPDA parameters are also applicable for VM-IPDA.

ϵ. The values of ϵ for each algorithm are obtained such that all algorithms will have the same average termination time delay τ over N successful[4] Monte Carlo runs:

$$\tau = \frac{1}{N}\sum_{i=1}^{N}(t_2^i - t_1) \tag{36}$$

Figure 3 shows a plot of average false track termination delay τ versus track termination threshold ϵ, where $N = 1000$. Sensor noise is assumed to be Gaussian zero-mean with standard deviations 300 meters in range and 0.75^o in azimuth respectively. The radar measurements (range & azimuth) are converted to local Cartesian coordinates via method suggested in [16]. Target state is described with position and velocity in orthogonal coordinate system:

$$\boldsymbol{x} = [x, \dot{x}, y, \dot{y}]$$

and state transition matrix F is

$$F = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where $T = 1$ second is the data sampling period. The covariance of system process noise is given by

$$Q = \begin{bmatrix} T^3/3 & T^2/2 & 0 & 0 \\ T^2/2 & T & 0 & 0 \\ 0 & 0 & T^3/3 & T^2/2 \\ 0 & 0 & T^2/2 & T \end{bmatrix} q$$

where q = 0.1 is chosen for a nearly constant velocity model [16]. Selection windows were used with the gating probability $P_G = 0.999$.

The transition probability matrices used in the experiment for the Markov chain model on target existent/observable are

$$\Pi = \begin{bmatrix} 0.98 & 0.02 \\ 0 & 1 \end{bmatrix} \tag{37}$$

for IPDA, EB-PDA and IMMPDA and

$$\Pi = \begin{bmatrix} 0.93 & 0.05 & 0.02 \\ 0.23 & 0.75 & 0.02 \\ 0 & 0 & 1 \end{bmatrix} \tag{38}$$

for IPDA-M2[5].

The choice of the time delay τ is such that it minimizes the chance of deleting true tracks and keeping false tracks. In this multipath fading scenario, a tracker could get no validated measurement within several consecutive scans. Subsequently, it is desirable that track termination time delay is greater than 4. In our simulation, the average time delays $\tau = 5$ is selected and the corresponding track termination thresholds are given in Table 2 which are obtained from Figure 3.

Figure 3 shows that full range of τ for track termination threshold selection is available for IPDA-M2, while the choice of the τ for other three algorithms is limited considerably.

[4]A run where $t_2 < t_1$, i.e., track termination occurs before the true track is terminated is not counted.

[5]IPDA-M2 values in Table 2 and Figure 3 are also valid for VM-IPDA for confirmed tracks. IPDA value of τ is used for VM-IPDA for unconfirmed tracks.

$\tau =$	IPDA	IPDA-M2	EB-PDA	IMMPDA
5	0.0067	0.028	0.0095	0.0024

Table 2: Track Termination Threshold

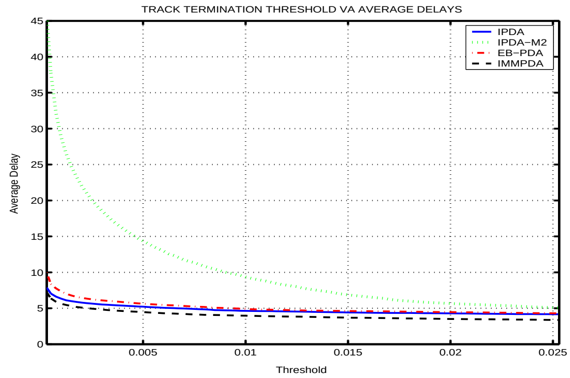

Figure 3: Average false track termination delay.

3.3 Simulation and results

Two cases are considered.

Case 1 Tracker receives measurements observed *in the absence of signal multipath fading*, i.e., the target is observed with detection probability $P_D = 0.9$ for all time.

Case 2 Tracker receives measurements *in the presence of signal multipath fading*, i.e., the target is observed with detection model shown in Figure 2.

Since the performance of the IPDA type algorithms is parameter dependent, two sets of parameters, i.e., Set1= $\{P_1, c_1\}$ and Set2= $\{P_2, c_2\}$ (see Table 1), are used for each case. Monte Carlo simulation of 100 runs are performed for each set of parameters in each case.

A tentative track is started for every detection which has not been associated with any existing tracks. A rectangle gate determined using the method in [4] is then set up to collect possible measurement in the next scan, where we assume that the maximum target speed is 424 m/s. If the $P\{\chi_k|Y^k\}$ of a track is less than ϵ provided in Table 2, it will be deleted. A track is confirmed if the track $P\{\chi_k|Y^k\}$ is great or equal to c provided in Table 1. A track-to-track association test [3] is adopted to eliminate redundant tracks.

The percentages of track loss are given in Table 3, the average number of confirmed false tracks via time is presented in Figure 4 and 6 where all algorithms start to confirm tracks from the 5th scan, and the RMS position error performance comparison is shown in Figures 5 and 7 respectively. The RMS position error was averaged over all successful runs.

Observations and Discussions:

- The average number of confirmed false tracks shown in Figures 4 and 6 depends on the clutter density and parameter set used by the filer. It does not depend on whether or not the multipath signal fading is present. The use of parameter

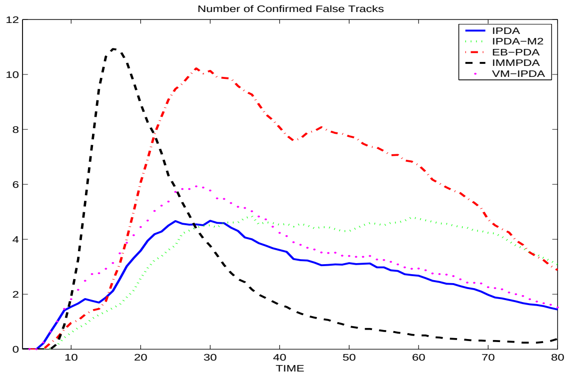

Figure 4: Number of confirmed false tracks via time when using parameter Set1.

Para. Set	IPDA	IPDA-M2	VM-IPDA	EB-PDA	IMMPDA
Set1 for Case 1	18%	15%	16%	18%	26%
Set1 for Case 2	65%	32%	32%	55%	68%
Set2 for Case 1	19%	20%	19%	38%	27%
Set2 for Case 2	76%	43%	40%	66%	69%
Av. dif.	52 %	20 %	18.5 %	32.5 %	42 %

Table 3: Track Loss Comparison ($D_c = 0.12/Km^2$)

Set2 leads to the overall number of confirmed false tracks reduced considerably, but the overall percentage of track loss increases (see Table 3).

- Table 3 indicates all algorithms suffer from more track loss when multipath signal fading is present. The Av. dif. averaged on both parameter set also gives such an indication of the significance. Both IPDA-M2 and VM-IPDA have significantly less track loss than others and the VM-IPDA is slightly better than IPDA-M2.
- The RMS position error comparison shown in Figures 5 and 7 provide an average measure of tracking accuracy of the underlying target. These results indicate that the major impact for multipath signal fading is the increase of track loss rather than the RMS error performance.
- In the absence of multipath fading (Case 1), i.e., $P_D = 0.9$, no significant performance difference between algorithms can be observed with the selected track termination thresholds in Table 2 and parameter set in Table 1. An averaged computational complexity comparison (in terms of computer CPU time) is given in Table 4. A higher computational load respect to the parameter set used indicates a higher confirmed false track rate.

Set	IPDA	IPDA-M2	VM-IPDA	EB-PDA	IMMPDA
Set1	1	1.4226	1.0151	1.1136	1.3574
Set2	1	1.0083	1.0348	1.6944	2.0719

Table 4: Averaged computational load comparison which are calculated in terms of CPU time and normalized based on the CPU time of the IPDA

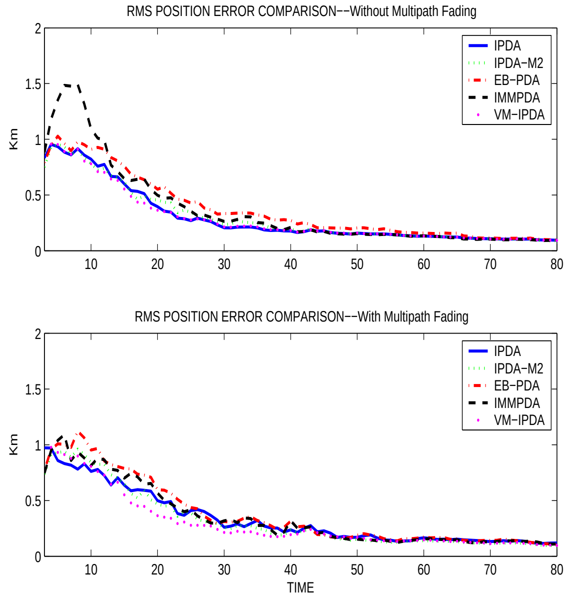

Figure 5: RMS position error comparison using parameter Set1.

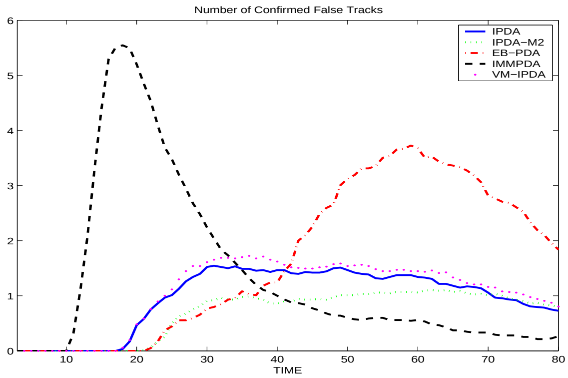

Figure 6: Number of confirmed false tracks using parameter Set2.

- The overall percentage of track loss for all algorithms is too high in this simulation. This is because of heavy clutter density which results the average number of expected measurements in gate over 8.

4 Conclusion

This paper presents a comparative study of IPDA type filters under the scenario where the underlying target detection probability varies due to multipath signal fading. A new IPDA type filter – VM-IPDA is developed. Filter on-line information about track quality is utilized for track maintenance. Preliminary results show that

- When the track quality measures of the IPDA type filters are used in track formation and maintenance, the initial probability of track existence, the track termination and confirmation thresholds need to be optimized to reach a compromise

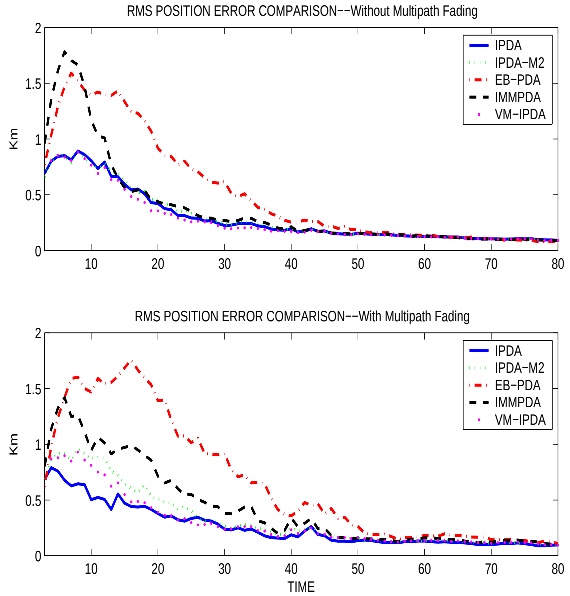

Figure 7: RMS position error comparison using parameter Set2.

between the percentage of (true) track loss and the confirmed false track rate.

- The values of the track termination threshold for IPDA type filters may be obtained statistically using Monte Carlo runs by considering same time delay from the time when a true track becomes a false track to the time when this false track is terminated.
- Very high percentage of track loss is expected for all algorithms except IPDA-M2 and VM-IPDA when multipath signal fading is present in heavy clutter scenario.
- Both IPDA-M2 and VM-IPDA promise a consistently better performance than other filters both in the absence and in the presence of multipath signal fading environments.

Tracking a target with time-varying detection probability has posed new challenges. Further study on tracking in clutter of low elevation sea-surface targets is underway.

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