AUGMENTED STATE IMM-PDA FOR OOSM SOLUTION TO MANEUVERING TARGET TRACKING IN CLUTTER

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ABSTRACT

Out-of-Sequence Measurement (OOSM) problem is one of the most important issues for target tracking in a multi-sensor fusion network. Optimal solution is to augment the state vector to include the past states which correspond to those delayed measurements in the filtering process. In this paper, the maneuvering target tracking in clutter with OOSM problem is considered. An AS-IMM-PDA algorithm that uses OOSM to improve tracking performance is presented. The benefit of the AS-IMM-PDA is demonstrated via a maneuvering target tracking example.

1. INTRODUCTION

Multi-sensor tracking using delayed, out-of-sequence measurement (OOSM) is a problem of growing importance due to an increased reliance on networked sensors interconnected via complex communication network architectures. The approaches to the OOSM problem may classified as either normal state or augmented state approaches. In the normal state approaches, representatively, the optimal solution proposed by Bar-Shalom in [1] for single measurement delay and Mallick *et. al.* who have extended this approach to include multiple delayed measurements [2] by approximation, one need to compute the cross correlation between the current state estimate and the state estimate to which the OOSM corresponds and remove the effect due to past process noise. It becomes very difficult to grasp the above process when multiple measurement delays occur frequently. In addition, the correction can only be made to the current state estimate. The augmented state approaches, initiated by Challa *et. al.* in [3] and further developed in [4, 5], handle the OOSM problem using a fixed lag smoothing framework. It has been shown in [4], that for tracking a straight line target, Kalman filter can be directly fitted into a fixed lag smoothing framework with a modified measurement equation and the resulting filter is known as the augmented state Kalman filter (AS-KF)[1], where maximum measurement delay is characterized by the maximum number of lags. Issues for computational efficiency are discussed and several algorithms like variable dimension AS-KF (VDAS-KF) and augmented state probabilistic data association (AS-PDA) filter were proposed in [4]. However, all algorithms are developed only for tracking non-maneuvering targets as they are only dealing with *single model based* tracking problem. For maneuvering target tracking problem, it is popular to consider *multiple model based* approaches as the maneuver behavior of a target can be more precisely described by using multiple models. The standard interacting multiple model (IMM) algorithm, proposed in [6] and further described in several articles [7], is the most effective suboptimal multiple model algorithm and has been adopted to many applications to maneuvering target tracking [8]. When clutter is presented in the measurement origin, one need to apply a data association technique to eliminate the uncertainties and find the measurement originated from target. Among the data association techniques available in literature [9], the probabilistic data association (PDA) has been popularly used for target tracking problem. In this paper, IMM is referred to as a maneuvering target tracking algorithm and PDA technique is chosen for dealing with uncertainties in the measurement origin.

In the literature, the fixed lag smoothing for an IMM was implemented in [10] and it is extended to an IMM-PDA later on in [11]. However, using the IMM smoothing structure to incorporate the past measurements has not yet been explored. In this paper, the problem of tracking a maneuvering target in clutter using OOSM is considered. An AS-IMM-PDA algorithm that incorporate the PDA technique into an augmented state IMM (AS-IMM) for maneuvering target tracking in clutter using OOSM is presented.

The paper is organized as follows. Following the introduction section, the OOSM problem for multiple model

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[1]We frequently use the prefix AS- in symbolizing an augmented state estimation algorithm to distinguish it from the fixed lag smoothing algorithm. Both algorithms are equivalent only when there is no OOSM involved.

estimation is described in Section 2, the model probability calculation for an AS-IMM is derived in Section 3 and the AS-IMM-PDA is presented in Section 4. Section 5 presents a simulated example to demonstrate performance of the proposed algorithms for OOSM solution, the conclusion is drawn in Section 6 and one cycle recursion of the AS-IMM is attached in the Appendix.

2. PROBLEM STATEMENT

A simple multiple model system is described by a set of N difference equations

$$x_{k+1} = F_k^j x_k + G_k^j w_k^j \tag{1}$$

with corresponding measurement equations

$$y_k = H_k^j x_k + v_k^j \qquad j = 1, 2, \cdots, N \tag{2}$$

where equation parameters depend on a set of N models, i.e., $\mathcal{M} = \{M^1, M^2, \cdots, M^N\}$. The system at any time can only be switched to one model and the model transition from M^i to M^j is governed by a Markov chain with known transition probability

$$\pi_{ij} = P(M_k^j | M_{k-1}^i)$$

the multiple model estimation problem is to find the posterior density of the state x_k given the set of models $\mathcal{M}$ and all the measurements up to time k. In the IMM this density is given by a model probability weighted sum of model based posterior densities, i.e.,

$$p(x_k|Y^k) = \sum_{j=1}^{N} p(x_k|M_k^j, Y^k) P(M_k^j|Y^k) \tag{3}$$

where $Y^k = \{y_1, y_2, \cdots, y_k\}$ is the measurement sequence up to k.

As we know that data delays may randomly occur in a constraint network where some sensors are connected to, if we assume that the maximum delay is of time index d, the measurement sequence up to time k is of the form

$$Y^k = \{\boldsymbol{Y}_k, Y^{k-1}\} = \{y_k, y_{k-1}, \cdots, y_{k-d}, Y^{k-1}\} \tag{4}$$

that is, at any time k, current measurement y_k may not be received due to a network delay and the received measurements may include some past (delayed) measurements[2] up to d. Then, the question of interest is how to update the posterior density of the state (3) at time k using the measurements (4). As shown in [4], the solution to such a problem is to augmenting state vector to include all past states to which all possible delayed measurements correspond. It turns out that in order to use OOSM in the filtering process an augmented state filter with a maximum number of d lags is needed.

Let $\boldsymbol{X}_k$ and $\boldsymbol{Y}_k$ denote the augmented state and the corresponding measurements received at time k respectively[3], i.e.,

$$\boldsymbol{X}_k = \begin{bmatrix} x_k \\ x_{k-1} \\ \vdots \\ x_{k-d} \end{bmatrix} \quad \text{and} \quad \boldsymbol{Y}_k = \begin{bmatrix} y_k \\ y_{k-1} \\ \vdots \\ y_{k-d} \end{bmatrix} \tag{5}$$

equation (1) and (2) are then replaced by following augmented forms.

$$\begin{bmatrix} x_k \\ x_{k-1} \\ \vdots \\ x_{k-d} \end{bmatrix} = \begin{bmatrix} F^j & 0 & \cdots & 0 \\ I & 0 & \cdots & 0 \\ 0 & \ddots & 0 & \vdots \\ 0 & \cdots & I & 0 \end{bmatrix} \begin{bmatrix} x_{k-1} \\ x_{k-2} \\ \vdots \\ x_{k-d-1} \end{bmatrix} + \begin{bmatrix} G_k^j \\ 0 \\ 0 \\ 0 \end{bmatrix} w_k^j \tag{6}$$

with measurement

$$\begin{bmatrix} y_k \\ y_{k-1} \\ \vdots \\ y_{k-d} \end{bmatrix} = \begin{bmatrix} H_k^j & 0 & \cdots & 0 \\ 0 & H_{k-1}^j & 0 & \vdots \\ \vdots & \cdots & \ddots & 0 \\ 0 & \cdots & 0 & H_{k-d}^j \end{bmatrix} \begin{bmatrix} x_k \\ x_{k-1} \\ \vdots \\ x_{k-d} \end{bmatrix} + V_k^j \tag{7}$$

where $H_i^j \equiv 0$ if there were no measurement y_i received at time k. Above equations can be simply written as the following compact form.

$$\begin{aligned} \boldsymbol{X}_k &= \boldsymbol{F}_k^j \boldsymbol{X}_{k-1} + \boldsymbol{G}_k^j \boldsymbol{W}_k^j \\ \boldsymbol{Y}_k &= \boldsymbol{H}_k^j \boldsymbol{X}_k + \boldsymbol{V}_k^j \qquad j = 1, 2, \cdots, N \end{aligned} \tag{8}$$

Then, the underlying problem is to obtain the posterior density of the augmented state based on measurement set (4).

$$p(\boldsymbol{X}_k|Y^k) = \sum_{j=1}^{N} p(\boldsymbol{X}_k|\boldsymbol{M}_k^j, Y^k) P(\boldsymbol{M}_k^j|Y^k) \tag{9}$$

where following notations are used[4]:

$$\boldsymbol{M}_k^j = \begin{bmatrix} M_k^{0,j} & M_k^{1,j} & \cdots & M_k^{d,j} \end{bmatrix}' \tag{10}$$

Note that once the solution of (9) is obtained, the solution of (3) can be extracted from the first lag of (9) and the solution of (3) which utilize the benefit of the smoothing in d lags can also be obtained for the d lags of (9) with the later has a fixed d lags delay.

It has been shown in [10] that the standard IMM can be fitted into a fixed lag smoothing framework more or less straightforward, where all model based Kalman filters (KF)

[2]All sensor measurements considered in this paper are synchronized. However, the algorithms developed in this paper can also be used for asynchronous sensor measurements through engineering approximation mentioned in [4].

[3]In this paper, we use bold face letter to denote either the vector, matrix or a set of parameters for their augmented state expression.

[4]$M_k^{i,j}$ signifies that the model M^j that corresponds to the ith lag of the state augmented system is in effect at time k, $\boldsymbol{\mu}_j(k)$ is the model probability vector corresponding to $\boldsymbol{M}_k^j$ at time k.

are extended to AS-KFs. The major challenge is how to compute model probabilities for all lags of the augmented state since the model probability update cannot be carried out as in the standard IMM, which is to be discussed next.

3. THE MODEL PROBABILITIES OF THE AS-IMM

Provided measurements with different time indices are independent with each other, the model probability update for model $M_k^{i,j}$ at time k using current measurement $Y_k = \{y_k,\ y_{k-s}\}$ is given by

$$P(M_k^{i,j}|Y^k) = \frac{1}{\delta}p(Y_k|M_k^{i,j}, Y^{k-1})P(M_k^{i,j}|Y^{k-1}) \qquad (11)$$
$$= \frac{1}{\delta}p(y_k|M_k^{i,j}, Y^{k-1})p(y_{k-i}|M_k^{i,j}, Y^{k-1})P(M_k^{i,j}|Y^{k-1})$$

where the normalization factor $\delta = \sum_{j=1}^{N} P(M_k^{i,j}|Y^k)$ is available once the right hand of (11) for each individual model $M_k^{i,j},\ j = 1, \cdots, N$ is evaluated.

1. $p(y_k|M_k^{i,j}, Y^{k-1})$ — the model based *in sequence measurement likelihood* for the ith lag:

$$\begin{aligned}
&p(y_k|M_k^{i,j}, Y^{k-1}) = \\
&\quad \sum_{h=1}^{N} p(y_k|M_k^{i+1,h}, M_k^{i,j}, Y^{k-1})P(M_k^{i+1,h}|M_k^{i,j}) \\
&\quad \times \sum_{l=1}^{N} P(M_k^{i,j}|M_k^{i-1,l}) \\
&= \sum_{h=1}^{N} \cdots \sum_{s=1}^{N} \sum_{w=1}^{N} \\
&\quad \times p(y_k|M_k^{0,w}, M_k^{1,s}, \cdots, M_k^{i+1,h}, M_k^{i,j}, Y^{k-1}) \\
&\quad \times P(M_k^{0,w}|M_k^{1,s}), \cdots, P(M_k^{i+1,h}|M_k^{i,j}) \\
&\quad \times \sum_{l=1}^{N} P(M_k^{i,j}|M_k^{i-1,l}) \\
&= \frac{1}{\delta} \sum_{h=1}^{N} \cdots \sum_{s=1}^{N} \sum_{w=1}^{N} \\
&\quad \times \mathcal{N}\left(y_k;\ \hat{y}_{k|k-1}^{w},\ S_k^w\right) \pi_{sw} \cdots \pi_{jh} \sum_{l=1}^{N} \pi_{lj} \qquad (12)
\end{aligned}$$

where following relations are identified according to the structure of the augmented state filter.

$$\begin{aligned}
&p(y_k|M_k^{0,w}, M_k^{1,s}, \cdots, M_k^{i+1,h}, M_k^{i,j}, Y^{k-1}) = \\
&\quad p(y_k|M_k^{0,w}, Y^{k-1}) = \mathcal{N}\left(y_k;\ \hat{y}_{k|k-1}^{w},\ S_k^w\right) \\
&\quad P(M_k^{i+1,h}|M_k^{i,j}) = P(M_{k-i+1}^{0,h}|M_{k-i}^{0,j}) = \pi_{jh} \\
&\quad P(M_k^{i-1,l}|Y^{k-1}) = P(M_{k-1}^{i,l}|Y^{k-1}) \\
&\quad = \mu_l^i(k-1)
\end{aligned}$$

2. $p(y_{k-i}|M_k^{i,j}, Y^{k-1})$ — the model based *out sequence measurement likelihood* for the ith lag:

For $0 \le h \le i$, using total probability theory, we have

$$\begin{aligned}
&p(y_{k-i}|M_k^{h,j}, Y^{k-1}) = \\
&\quad \sum_{l=1}^{N} \cdots \sum_{w=1}^{N} \sum_{g=1}^{N} \\
&\quad \times p(y_{k-i}|M_k^{h,j}, M_k^{h+1,l}, \cdots, M_k^{i-1,w}, M_k^{i,g}, Y^{k-1}) \\
&\quad \times P(M_k^{h,l}|M_k^{h+1,j}) \cdots P(M_k^{i-1,g}|M_k^{i,w}) \\
&= \sum_{l=1}^{N} \cdots \sum_{w=1}^{N} \sum_{g=1}^{N} \\
&\quad \times p(y_{k-i}|M_{k+h}^{0,j}, M_{k+h+1}^{0,l}, \cdots, M_{k-i-1}^{0,w}, M_{k-i}^{0,g}, Y^{k-1}) \\
&\quad \times P(M_{k+h}^{0,l}|M_{k+h+1}^{0,j}) \cdots P(M_{k-i-1}^{0,g}|M_{k-i}^{0,w}) \\
&= \sum_{l=1}^{N} \cdots \sum_{w=1}^{N} \sum_{g=1}^{N} p(y_{k-i}|M_{k-i}^{0,g}, Y^{k-1}) \\
&\quad \times P(M_{k+h}^{0,l}|M_{k+h+1}^{0,j}) \cdots P(M_{k-i-1}^{0,g}|M_{k-i}^{0,w}) \\
&= \sum_{l=1}^{N} \cdots \sum_{w=1}^{N} \sum_{g=1}^{N} \\
&\quad \times \mathcal{N}\left(y_{k-i};\ \hat{y}_{k-i|k-i-1}^{g},\ S_{k-i}^{g}\right) \bar{\pi}_{jl}^{k+h} \cdots \bar{\pi}_{wg}^{k-i} \qquad (13)
\end{aligned}$$

where $\bar{\pi}_{jl}^{k}$ denotes the inverse Markov transition probability from jth model at k to lth model at $k-1$.

$$\begin{aligned}
\bar{\pi}_{jl}^{k} &= P(M_{k-1}^{0,l}|M_k^{0,j}, Y^{k-1}) \\
&= \frac{P(M_k^{0,j}|M_{k-1}^{0,l}, Y^{k-1})P(M_{k-1}^{0,l}|Y^{k-1})}{\sum_{l=1}^{N} P(M_k^{0,j}|M_{k-1}^{0,l}, Y^{k-1})P(M_{k-1}^{0,l}|Y^{k-1})} \\
&= \frac{\pi_{lj} P(M_{k-1}^{0,l}|Y^{k-1})}{P(M_k^{0,j}|Y^{k-1})} = \frac{\pi_{lj}\mu_l^0(k-1)}{\mu_j^0(k|k-1)} \qquad (14)
\end{aligned}$$

3. $P(M_k^{i,j}|Y^{k-1})$ — the predicted model probability for the ith lag, which is calculated as in standard IMM.

The fixed-lag smoothing IMM algorithm was first proposed by Chen & Tugnait in [10]. However, approximations is made for model probabilities for lags corresponding to past states, i.e.,

$$P(M_k^{i,j}|Y^k) \approx P(M_k^{i,j}|\hat{x}_{k-i|k}, Y^{k-i}) \qquad (15)$$

Such an approximation has also been used in [11, 12, 13]. In our simulation we observed that using (12) can perform better when observation noise is smaller though the difference is trivial.

Based on the derivation in this section, we provide the AS-IMM algorithm in the Appendix.

4. AS-IMM-PDA

When clutter is presented in the measurement origin and OOSM is involved in the received data at time k, the measurement vector will have the form

$$\begin{bmatrix} y_k \\ \vdots \\ y_{k-d} \end{bmatrix} = \begin{Bmatrix} y_k^1, & y_k^2, & \cdots, & y_k^m \\ \vdots & \vdots & \cdots & \vdots \\ y_{k-d}^1, & y_{k-d}^2, & \cdots, & y_{k-d}^n \end{Bmatrix} \qquad (16)$$

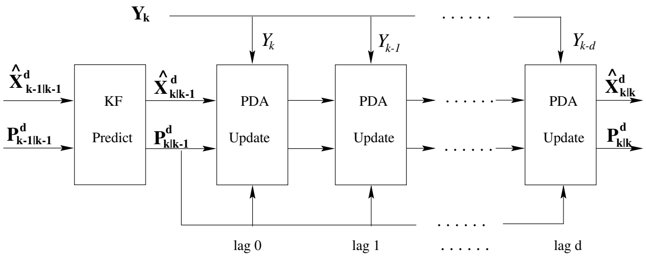
All possible combinations of the current and delayed measurements form the "measurements" in the augmented measurement space. For example, at time k, let us assume that the following four measurements are received - two for each time index

$$\left[\begin{array}{c} y_k \\ y_{k-d} \end{array}\right] = \left[\begin{array}{cc} y_k^1, & y_{k-d}^1 \\ y_k^2, & y_{k-d}^2 \end{array}\right].$$

Then the total measurement set formed by exploring all possible combinations is

$$\begin{aligned} \mathbf{Y}_k &= \{\mathbf{Y}^1, \mathbf{Y}^2, \mathbf{Y}^3, \mathbf{Y}^4\} \\ &= \left\{ \left[\begin{array}{c} y_k^1 \\ y_{k-d}^1 \end{array}\right], \left[\begin{array}{c} y_k^2 \\ y_{k-d}^2 \end{array}\right], \left[\begin{array}{c} y_k^1 \\ y_{k-d}^2 \end{array}\right], \left[\begin{array}{c} y_k^2 \\ y_{k-d}^1 \end{array}\right] \right\} \end{aligned}$$

which should be used for computing the combined innovations and the data association probabilities for the augmented state vector. It is clear that if OOSM involved, as the number of delayed measurements and the number of in-gate measurements increase an straightforward PDA becomes unpractical. One way to get around this problem is to use the iterative augmented state probabilistic data association (Iterative AS-PDA) propose in [4]. In this paper,



One Cycle of AS-IMM-PDA: Iterative PDA lag by lag for each of N models

Fig. 1. Computation Structure for AS-IMM-PDA

the technique of the iterative AS-PDA is adopted for implementing AS-IMM-PDA. Figure 1 illustrates the computation structure involved for each of N models in one cycle of the AS-IMM-PDA recursion. One cycle of AS-IMM-PDA recursion involves following major steps:

Step 1 Filter initialization.

Step 2 Start at time $\boldsymbol{k}$, calculate mixing probabilities and initial estimate and covariance mixing. This is same as AS-IMM.

Step 3 According to all measurements received at current time configure system observation matrix H to accommodate measurements to corresponding lags for update.

Step 4 Iterative IMM-PDA for each lag. This includes

- IMM-PDA Gating: If there is a set of measurements corresponding to current lag, validating measurements using model probability weighted gating (MPWG) technique presented in [14].
- IMM-PDA model based PDAF for each of N models.
- For those lags without measurement, use predicted value instead.

Step 5 Likelihood and model probability update.

Step 6 Estimates combination and output. $\boldsymbol{k} = k + 1$ Go to Step 2 for next time index.

5. SIMULATION STUDY

To demonstrate the performance of the proposed algorithms, a random statistical test scenario is designed as follows.

A two dimensional maneuvering target motion is described by

$$x_{k+1} = Fx_k + Gu_k + w_k \tag{17}$$

with measurement equation

$$y_k = Hx_k + v_k \tag{18}$$

where the matrices F, G and H are standard as given in [4]. The system process noise w_k and measurement noise v_k are assumed to be white Gaussian zero-mean with covariance matrices $Q = GG'q^2$ and $R = I_n\sigma_v$, where $q = 0.01$, I_n is a two dimensional unit matrix (for position only measurement) and $\sigma_v = 0.5km$ is the standard deviation of the measurement noise. Target acceleration is described by the input term u_k which is assumed to take one of a finite set of values at any time k, i.e., $u_k \in \{m_1, m_2, m_3\}$, where $m_1 = [0,0]', m_2 = [a,0]', m_3 = [0,a]'$, the factor $a = 0.2Km/s^2$. The process of u_k is governed by a Markov chain with transition probability matrix

$$\Pi = \left[\begin{array}{ccc} 0.9 & 0.05 & 0.05 \\ 0.2 & 0.8 & 0 \\ 0.2 & 0 & 0.8 \end{array}\right]$$

At each of 100 Monte Carlo runs, (100 sampling periods) data is randomly generated based on the Markov chain process with a target starting point

$$x0 = [200\ km,\ 0.5\ km/s,\ 100\ km,\ -0.08\ km/s]'$$

It is assumed that data is received via a randomly delayed network with a sampling rate $T = 1$ second. The data at any time k has a probability $P_r = 0.25$ to be delayed. Data delay is uniformly distributed with a maximum time delay less than or equal to $4 \times T$. Generated data is corrupted with Gaussian noise and uniform scattered clutter with density D_m. The detection probability is assumed to be $P_D = 0.98$.

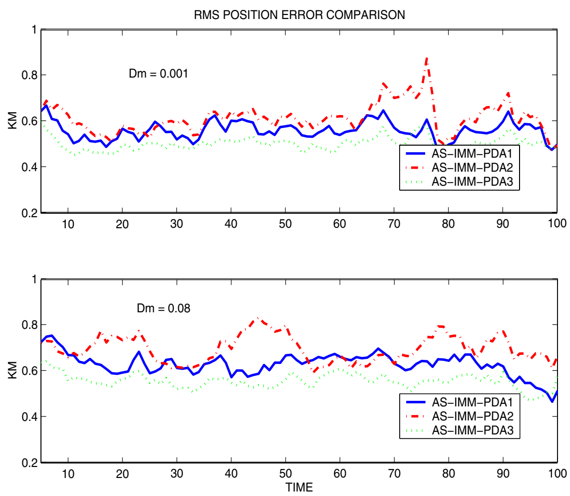


Fig. 2. RMS error comparison between AS-IMM-PDA1 AS-IMM-PDA2 and AS-IMM-PDA3 in two clutter densities

An AS-IMM-PDA with 4 lags is implemented based on the above basic system in the way similar to (6)–(7), where the augmented $\boldsymbol{R}$ and $\boldsymbol{Q}$ matrices are all assumed to be diagonal blocked matrices.

The performance of three tracks are compared. The first track, denoted as AS-IMM-PDA1, uses OOSM; the second track, denoted as AS-IMM-PDA2, treats the OOSM as missing measurements and do not use them; the third track, denoted as AS-IMM-PDA3, is assumed to use full measurement sequence without OOSM problem. The root-mean-squared (RMS) error of the estimated target position, the percentage of track loss (TL) and the expected number of measurements ($\bar{m}_k$) in each gate are used for the performance comparison. A track loss is confirmed when its RMS position error exceeds a threshold E_{max} within at least 10 consecutive sampling periods. In this example, we chose $E_{max} = 30 \times \sigma_v$.

Test results are shown in Figure 2 and Table 1. The RMS position error is obtained based on the average of all survival tracks. We observed that

Table 1. Algorithm Performance Comparison

D_m $point/Km^2$	AS-IMM-PDA1		AS-IMM-PDA2		AS-IMM-PDA3	
	TL	$\bar{m}_k$	TL	$\bar{m}_k$	TL	$\bar{m}_k$
0.001	4%	1.2013	9%	0.9329	2%	1.1940
0.080	35%	3.4150	46%	3.0108	31.5%	3.2518

From the simulation, we observed that

–with random data delay, the AS-IMM-PDA1 outperforms AS-IMM-PDA2 in both clutter densities (light and heavy) as the AS-IMM-PDA1 uses OOSM and thus has improved tracking performance (in terms of RMS error, percentage of track loss) while the AS-IMM-PDA2 ignores OOSM.

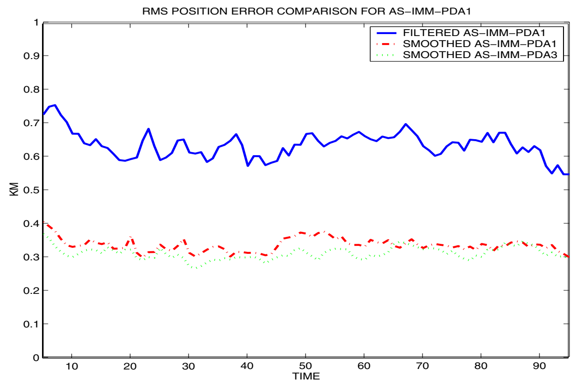


Fig. 3. RMS error comparison between 1). filtered AS-IMM-PDA1 output and smoothed AS-IMM-PDA1 output; 2). smoothed output AS-IMM-PDA1 and AS-IMM-PDA3.

– increasing P_r the AS-IMM-PDA2 will contribute to more track loss.

–as shown in Figure 3, the smoothed output of the AS-IMM-PDA1 is superior to its filtered output and is similar to that of the AS-IMM-PDA3 which uses the full (no delay) measurement sequence.

–the computational load of the proposed AS-IMM and AS-IMM-PDA are roughly d times of an IMM and an IMM-PDA respectively. It is possible to make further computation reduction on augmented state filters, such as the method as mentioned in [11] and we will address this issue in our future research work.

6. CONCLUSION

In this paper, an augmented state solution to OOSM problem for maneuvering target tracking in clutter is studied and an AS-IMM-PDA is presented. The proposed algorithm is able to use OOSM to improve tracking performance and obtain smoothed performance if a fixed lag delay is allowed. An simulated example of maneuvering target tracking in clutter with random data delay has demonstrated that the proposed algorithm has an improved tracking performance.

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8. APPENDIX: ONE CYCLE RECURSION OF AS-IMM ALGORITHM

Following formula are derived based on Markovian switching system (8) described in Section 2 under linear Gaussian assumptions. The one cycle of the AS-IMM algorithm has the following 4 steps.

1. Initialization:

- Predicted model probability vector

$$\boldsymbol{\mu}_j(k|k-1) \triangleq P(\boldsymbol{M}_k^j|Y^{k-1}) = \sum_{i=1}^{N} \pi_{ij}\boldsymbol{\mu}_i(k-1) \quad (19)$$

- Mixing probability

$$\begin{aligned}\boldsymbol{\mu}_{i|j}(k-1|k-1) &\triangleq P(\boldsymbol{M}_{k-1}^i|\boldsymbol{M}_k^j, Y^{k-1}) \\ &= \begin{bmatrix} \mu_{i|j}^0(k-1|k-1) \\ \mu_{i|j}^1(k-1|k-1) \\ \vdots \\ \mu_{i|j}^d(k-1|k-1) \end{bmatrix}\end{aligned} \quad (20)$$

where

$$\mu_{i|j}^h(k-1|k-1) = \frac{\pi_{ij}\mu_i^h(k-1)}{\mu_j^h(k|k-1)} \qquad h = 0, 1, \cdots, d$$

- Mixing state and covariance

$$\begin{aligned}\hat{\boldsymbol{X}}_{k-1|k-1}^{0j} &\triangleq E(\boldsymbol{X}_{k-1}|\boldsymbol{M}_k^j, Y^{k-1}) \\ &= \sum_{i=1}^{N} \text{diag}\{\boldsymbol{\mu}_{i|j}(k-1|k-1)\}\hat{\boldsymbol{X}}_{k-1|k-1}^i\end{aligned} \quad (21)$$

$$\begin{aligned}\boldsymbol{P}_{k-1|k-1}^{oj} &\triangleq E\{[\boldsymbol{X}_{k-1} - \hat{\boldsymbol{X}}_{k-1|k-1}^{0j}][\boldsymbol{X}_{k-1} - \hat{\boldsymbol{X}}_{k-1|k-1}^{0j}]'|\boldsymbol{M}_k^j, Y^{k-1}\} \\ &= \sum_{i=1}^{N} \Big\{\boldsymbol{P}_{k-1|k-1}^i + [\boldsymbol{X}_{k-1} - \hat{\boldsymbol{X}}_{k-1|k-1}^{0j}] \\ &\quad \times[\boldsymbol{X}_{k-1} - \hat{\boldsymbol{X}}_{k-1|k-1}^{0j}]'\Big\}\, \text{diag}\{\boldsymbol{\mu}_{i|j}(k-1|k-1)\}\end{aligned} \quad (22)$$

where

$$\text{diag}\{\boldsymbol{\mu}_{i|j}(k-1|k-1)\} = \begin{bmatrix} \mu_{i|j}^0(k-1|k-1)\boldsymbol{I}_n, & \cdots, & \mu_{i|j}^d(k-1|k-1)\boldsymbol{I}_n \end{bmatrix}$$

and $\boldsymbol{I}_n$ is an unit matrix of dimension identical to single state x.

2. Model based filtering

- Prediction

$$\begin{aligned}\hat{\boldsymbol{X}}_{k|k-1}^j &\triangleq E\left(\hat{\boldsymbol{X}}_k|\boldsymbol{M}_k^j, Y^{k-1}\right) \\ &= \boldsymbol{F}^j\hat{\boldsymbol{X}}_{k-1|k-1}^{0j} + \boldsymbol{G}_k^j\boldsymbol{W}_k^j \\ \boldsymbol{P}_{k|k-1}^j &\triangleq E\left[(\boldsymbol{X}_k - \hat{\boldsymbol{X}}_{k|k-1}^j)(\boldsymbol{X}_k - \hat{\boldsymbol{X}}_{k|k-1}^j)'|\boldsymbol{M}_k^j, Y^{k-1}\right] \\ &= \boldsymbol{F}^j\boldsymbol{P}_{k-1|k-1}^{0j}\boldsymbol{F}^{j'} + \boldsymbol{Q}^j \\ \hat{\boldsymbol{Y}}_{k|k-1}^j &= \boldsymbol{H}^j\hat{\boldsymbol{X}}_{k|k-1}^j\end{aligned} \quad (23)$$

- Innovation

$$\begin{aligned}\tilde{\boldsymbol{Y}}_k^j &= \boldsymbol{Y}_k - \hat{\boldsymbol{Y}}_{k|k-1}^j \\ \boldsymbol{S}_k^j &= \boldsymbol{H}^j \boldsymbol{P}_{k|k-1}^j \boldsymbol{H}^{j'} + \boldsymbol{R} \qquad (24)\end{aligned}$$

- Gain

$$\boldsymbol{K}_k^j = \boldsymbol{P}_{k|k-1}^j \boldsymbol{H}^{j'} \boldsymbol{S}_k^{j^{-1}} \qquad (25)$$

- Update

$$\begin{aligned}\hat{\boldsymbol{X}}_{k|k}^j &\triangleq E\left(\boldsymbol{X}_k|\boldsymbol{M}_k^j, Y^{k-1}\right) \\ &= \hat{\boldsymbol{X}}_{k|k-1}^j + \boldsymbol{K}_k^j \tilde{\boldsymbol{Y}}_k^j \\ \boldsymbol{P}_{k|k}^j &\triangleq E\left\{\left[\boldsymbol{X}_k - \hat{\boldsymbol{X}}_{k|k}^j\right]\left[\boldsymbol{X}_k - \hat{\boldsymbol{X}}_{k|k}^j\right]'|\boldsymbol{M}_k^j, Y^k\right\} \\ &= \boldsymbol{P}_{k|k-1}^j - \boldsymbol{K}_k^j \boldsymbol{S}_k^j \boldsymbol{K}_k^{j'} \qquad (26)\end{aligned}$$

3. Model probability update

- the likelihood for in-sequence measurement $y_k = y_k^0$ and the ith lag of model $\boldsymbol{M}_k^j$:

$$\begin{aligned}&\Lambda_j^{In,i}(k) \triangleq p(y_k|M_k^{i,j}, Y^{k-1}) \qquad (27) \\ &= \sum_{h=1}^{N} \cdots \sum_{w=1}^{N} \sum_{s=1}^{N} \mathcal{N}\left(y_k;\ \hat{y}_{k|k-1}^w,\ S_k^w\right) \pi_{sw} \cdots \pi_{jh}\end{aligned}$$

- the likelihood for out-sequence measurement $y_{k-h} = y_k^i$ and the ith lag of model $\boldsymbol{M}_k^j$, where $0 \leq i \leq h \leq d$:

$$\begin{aligned}&\bar{\Lambda}_j^{Out,i}(k) \triangleq p(y_{k-h}|M_k^{i,j}, Y^{k-1}) \qquad (28) \\ &= \sum_{l=1}^{N} \cdots \sum_{w=1}^{N} \sum_{g=1}^{N} \\ &\quad \times \mathcal{N}\left(y_{k-h};\ \hat{y}_{k-h|k-h-1}^g,\ S_{k-h}^g\right) \bar{\pi}_{jl}^{k+i} \cdots \bar{\pi}_{wg}^{k-h}\end{aligned}$$

- the overall likelihood for the ith lag with measurement $Y_k = [y_k,\ y_{k-h}]$ for model $\boldsymbol{M}_k^j$:

$$\Lambda_j^i(k) \triangleq p(y_k, y_{k-h}|M_k^{i,j}, Y^{k-1}) = \Lambda_j^{In,i}(k)\bar{\Lambda}_j^{Out,i}(k) \qquad (29)$$

- model probability update for for model $\boldsymbol{M}_j(k)$:

$$\boldsymbol{\mu}_j(k|k) \triangleq p(\boldsymbol{M}_j(k)|Y^k) = \begin{bmatrix} \mu_j^0(k) \\ \mu_j^1(k) \\ \vdots \\ \mu_j^d(k) \end{bmatrix} \qquad (30)$$

where

$$\mu_j^i(k) = \frac{\mu_j^i(k|k-1)\Lambda_j^i(k)}{\sum_{l=1}^{N} \mu_l^i(k|k-1)\Lambda_l^i(k)}$$

4. Output combination

$$\begin{aligned}\hat{\boldsymbol{X}}_{k|k} &\triangleq E\left(\boldsymbol{X}_k|Y^k\right) \\ &= \sum_{j=1}^{N} \text{diag}\left\{\boldsymbol{\mu}_j(k|k)\right\} \hat{\boldsymbol{X}}_{k|k}^j \qquad (31)\end{aligned}$$

$$\begin{aligned}\boldsymbol{P}_{k|k} &\triangleq E\left\{[\boldsymbol{X}_k - \hat{\boldsymbol{X}}_{k|k}][\boldsymbol{X}_k - \hat{\boldsymbol{X}}_{k|k}]'|Y^k\right\} \qquad (32) \\ &= \sum_{j=1}^{N} \left\{\boldsymbol{P}_{k|k}^j + [\hat{\boldsymbol{X}}_{k|k} - \hat{\boldsymbol{X}}_{k|k}^j][\hat{\boldsymbol{X}}_{k|k} - \hat{\boldsymbol{X}}_{k|k}^j]'\right\} \text{diag}\left\{\boldsymbol{\mu}_j(k|k)\right\}\end{aligned}$$

where

$$\text{diag}\left\{\boldsymbol{\mu}_j(k|k)\right\} = \text{diag}\left[\mu_j^0(k)\boldsymbol{I}_n, \mu_j^1(k)\boldsymbol{I}_n, \cdots, \mu_j^d(k)\boldsymbol{I}_n\right]$$