Low Elevation Sea-Surface Target Tracking Using IPDA Type Filters *

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Abstract – *The Integrated Probabilistic Data Association (IPDA) type filters provide estimates of the underlying target probability of existence /perceivability/visibility apart from track state maintenance. These quantities are conveniently used as a track quality measure and can be used for track confirmation and termination. The sea-surface induced multipath fading reduces the detection probability of the target at certain ranges which can lead to track loss. In this paper we use IPDA type filters to tracking a target in such scenario. The primary results presented in this paper is encouraging for a further study in the future.*

Keywords: PDA, IPDA,IMMPDA, N-IPDA, Markov chain 2, Target tracking, Multipath fading.

1 Introduction

A class of target tracking algorithms based on the probabilistic data association (PDA) [1] can provide probabilistic measure on track existence /perceivability/visibility as well as target state estimates. We refer to this class of algorithms as IPDA type filters and the additional statistic measure as track quality measure. With on-line track quality measure, the IPDA type filters can be used for track confirmation and termination, apart from the state estimation of tracks.

When tracking a single target in the presence of clutter, more than one measurements may be received at each scan after gating (measurement validation) process which eliminates the measurement uncertainties using a confidence region [2]. In general, track maintenance using false measurements can lead to serious filter divergence problem. Therefore, a data association technique is required to differentiate target originated measurement from clutter. The standard PDA technique which incorporates Kalman filter for tracking in clutter has a moderate computational load and a reasonable performance [3]. PDA uses all validated measurements weighted with the posterior probability that the measurement is the target measurement. It also approximates estimation probability density function (PDF) with Gaussian PDF. However, it is unable to provide the information on track quality measure directly because it assumes that the underlying target always exists and is observable.

IPDA type filters are based on the PDA technique and they incorporate various models for the underlying track quality measure. The track quality measure is calculated in a recursive manner. In this paper, four filters are investigated in a low elevation sea-surface target tracking scenario with multipath fading.

The original IPDA, proposed by Musicki *et. al* in [4, 5], has two options on choice of Markov chain models of target existence propagation. Markov Chain One, known as IPDA, recognizes two possibilities: the target either does not exist, or it exists and is visible with a probability of detection. Markov Chain Two, denoted as IPDA-M2, also recognizes the possibility of target existing but not being visible. A new formulation of IPDA (N-IPDA), presented by X. Rong-Li *et. al* in [6], replaces target existence hypotheses with that of target perceivability. Target is perceivable if it is both existent and visible. The above three IPDA type filters are formulated on one PDA filter. The interacting multiple model probabilistic data association (IMMPDA) algorithm, proposed by Bar-Shalom *et. al* in [7], incorporates the interacting multiple model (IMM) estimation algorithm with PDA technique and uses two PDA filters (models). One model assumes that the target is visible with a known probability of detection. The second model assumes that the target is not visible and is modelled with probability of detection equal to zero. The posterior probabilities of each model are calculated and the probability of the visible model is then used as the track quality measure. IMMPDA provides both data association formulae and probability of target existence which serves as track quality measure at the cost of a larger window size and up to twice the computation requirements of the PDA algorithm [8, 7, 9].

The problem of multipath propagation of the signal received by the radar from a low elevation sea-surface target has been studied in the literature [10, 11, Ch.7]. Sea surface acts like an imperfect mirror for radar signals. Because of this, the radar signal can reach target and return to the receiver using four different paths. Received signal, being the complex sum of the signals over different paths [10, 11, 12, 13] can be either amplified or attenuated. This is known as multipath fading. The probability of detection can be severely affected by multipath fading. Study on the impact of multipath fading to target tracking is subsequently of practical importance. An example of such case was described in [10] and we will adopt it for evaluating the IPDA type filters.

In this paper we compare four algorithms, IPDA, IPDA-M2, N-IPDA and IMMPDA using the above scenario where target detection probability varies due to multipath fading caused by sea-surface signal reflections and in the presence of clutter. When the target is in the low detection probability region, its track may di-

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verge and become a false track, or its track quality measure may fall below the termination threshold resulting in the track termination. Unfortunately, very few references in such situation can be found in the literature.

The paper is organized as follows. Following the introduction section, algorithms to be evaluated are described in Section 2, the scenario for algorithm test, computer simulation and result discussions are presented in Section 3 followed by concluding remarks.

2 Tracking with IPDA Type Filters

A target trajectory is often described by

$$x_{k+1} = Fx_k + w_k \tag{1}$$

with measurement equation

$$y_k = Hx_k + v_k \tag{2}$$

where x_k represents target kinematic state (position, velocity, etc.), F and H are known matrices. w and v are uncorrelated system and measurement noise respectively and[1]

$$\begin{aligned} w_k &\sim \mathcal{N}\big(w_k;\ 0,\ Q_k\big) \\ v_k &\sim \mathcal{N}\big(v_k;\ 0,\ R_k\big) \end{aligned} \tag{3}$$

Target tracking problem is to find the posterior conditional PDF of the target state $p(x_k|Y^k)$ according to a measurement sequence received up to time k. Since clutter is involved, the measurement sequence in this paper is denoted as $Y^k = \{Y_1, \cdots, Y_k\}$, where $Y_k = \{y_k^1, \cdots, y_k^{m_k}\}$, $m_k \geq 0$ signifies the set of validated measurements at time k.

All algorithms compared in this paper can also be used for automatic track confirmation and termination as they recursively calculate track quality measures. These are followed by a Markov chain model on target existence/visibility/perceivability propagation. The nature of the IPDA and IPDA-M2 lead to a non-reversible Markov state (i.e., a "non-existent target" cannot become an existent target). On the other hand, IMMPDA and N-IPDA implicitly assumed that the target always exists and the events related to target visibility/perceivability are reversible. In general, they are all extension of PDAF and derived based on following common assumptions:

- clutter is uniform/Possion distributed within the validation gate.
- estimation PDF is approximated with a single Gaussian PDF.
- at most one target exists.
- track has been initiated using, for example, two point difference method [3].

2.1 IPDA with Markov Chain One

The IPDA proposed in [4] is derived based on PDAF [3] by introducing the concept of target existence. Two mutually exclusive and exhaustive events associated with target existence were assumed, and modelled as a random variable E_k,

E_k	the event that a target exists and is visible,
$\bar{E}_k$	the event that the target does not exist.

[1] A standard notation $\mathcal{N}\big(x;\ \bar{x},\ \sigma\big)$ is used in this paper to denote a Gaussian random variable x with mean $\bar{x}$ and variance σ.

The occurrence of these two events is modelled as a two states Markov Chain with transition probability matrix

$$\Pi = \begin{bmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{bmatrix} \tag{4}$$

where

$$\begin{aligned} &\pi_{ij} \triangleq P\{E_k = j | E_{k-1} = i\},\ i, j \in \{1, 2\} \\ &\pi_{11} + \pi_{12} = \pi_{21} + \pi_{22} = 1 \end{aligned}$$

is the transition probability for (i, j)th entries and $P\{\cdot\}$ denotes probability.

Data association events[2]:

$$\begin{aligned} \theta_0 &: \text{ all validated measurements are false measurements;} \\ \theta_i &: \text{ the } i\text{th validated measurement is the target measurement,} \\ &\quad\ \text{and others are false measurements.} \end{aligned} \tag{5}$$

Let

$$\hat{m}_k = \begin{cases} 0, & m_k = 0 \\ m_k - P_D P_G P\{E_k|Y^{k-1}\}, & m_k > 0 \end{cases}$$

denote the number of expected clutter measurements inside the validation gate of volume V_k, m_k is the number of validated measurements at time k, P_D and P_G denote detection probability and gate (data validation) probability respectively and $P\{E_k|Y^{k-1}\}$ is the predicted probability of target existence, which can be obtained from the prior probability of target existence $P\{E_{k-1}|Y^{k-1}\}$ via the properties of Markov chain, i.e.,

$$P\{E_k|Y^{k-1}\} = \pi_{11}P\{E_{k-1}|Y^{k-1}\} + \pi_{21}(1 - P\{E_{k-1}|Y^{k-1}\}) \tag{6}$$

As in the standard PDAF [3], the target state estimate and its associated covariance are obtained as

$$\begin{aligned} \hat{x}_{k|k} &= \sum_{i=0}^{m_k} \beta_i(k)\hat{x}_{k|k}^i \\ P_{k|k} &= \beta_0(k)P_{k|k-1} + (1 - \beta_0(k))[I - KH]P_{k|k-1} \\ &\quad + K\left[\sum_{i=1}^{m_k} \beta_i(k)\tilde{y}_k^i \tilde{y}_k^{i^T} - \tilde{y}_k\tilde{y}_k^T\right]K^T \end{aligned} \tag{7}$$

where K is the Kalman gain, $\hat{x}_{k|k}^i$ is the updated state estimate conditioned on the event θ_i, $\tilde{y}_k^i$ is the innovation of the ith validated measurement. The data association probabilities are calculated as below,

$$\begin{aligned} \beta_0(k) &\triangleq P\{\theta_0|E_k, Y^k\} \\ &= \frac{1 - P_D P_G}{1 - \delta_k} \end{aligned} \tag{8}$$

$$\begin{aligned} \beta_i(k) &\triangleq P\{\theta_i|E_k, Y^k\} \\ &= \frac{P_D P_G \frac{V_k}{\hat{m}_k}\Lambda_k^i}{1 - \delta_k} \end{aligned} \tag{9}$$

where

$$\delta_k = \begin{cases} P_D P_G[1 - \frac{V_k}{\hat{m}_k}\sum_{i=1}^{m_k}\Lambda_k^i] & m_k \neq 0 \\ P_D P_G & m_k = 0 \end{cases} \tag{10}$$

[2] θ_0, θ_i always signify the events occurring at time k, unless specified otherwise.

where

$$\Lambda_k^i \triangleq p(y_k^i|x_k, m_k, E_k, Y^{k-1}, \theta_i) = P_G^{-1}\mathcal{N}\big(y_k^i; \hat{y}_k, S_k\big) \quad (11)$$

S_k is the covariance of the predicted measurement $\hat{y}_k$.

The posterior probability of track existence is given by

$$P\{E_k|Y^k\} = \frac{(1-\delta_k)P\{E_k|Y^{k-1}\}}{1-\delta_k P\{E_k|Y^{k-1}\}} \quad (12)$$

which is the IPDA measure of track quality. In the system recursion, the predicted probability $P\{E_k|Y^{k-1}\}$ can be obtained from (6).

2.2 IPDA with Markov Chain Two

By considering that an existing target may be or not be visible by a sensor, the events of the underlying target existence can be extended as follows:

$E_k \cap E_k^o = E_k^o$	the event that a target exists and is visible,
$E_k \cap E_k^n = E_k^n$	the event that a target exists but is not visible,
$\bar{E}_k$	the event that the target does not exist.

As a consequence, the occurrence of the above events is modelled as a three state Markov chain with transition probability matrix given by

$$\Pi = \begin{bmatrix} \pi_{11} & \pi_{12} & \pi_{13} \\ \pi_{21} & \pi_{22} & \pi_{23} \\ \pi_{31} & \pi_{32} & \pi_{33} \end{bmatrix} \quad (13)$$

where

$$\sum_{j=1}^{3}\pi_{1j} = \sum_{j=1}^{3}\pi_{2j} = \sum_{j=1}^{3}\pi_{3j} = 1$$

Given the prior probabilities $P\{E_{k-1}^o|Y^{k-1}\}$, $P\{E_{k-1}^n|Y^{k-1}\}$ and Π,

$$P\{E_k^o|Y^{k-1}\} = \pi_{11}P\{E_{k-1}^o|Y^{k-1}\} + \pi_{21}P\{E_{k-1}^n|Y^{k-1}\} + \pi_{31}P\{\bar{E}_{k-1}|Y^{k-1}\} \quad (14)$$

$$P\{E_k^n|Y^{k-1}\} = \pi_{12}P\{E_{k-1}^o|Y^{k-1}\} + \pi_{22}P\{E_{k-1}^n|Y^{k-1}\} + \pi_{32}P\{\bar{E}_{k-1}|Y^{k-1}\} \quad (15)$$

$$P\{\bar{E}_{k-1}|Y^{k-1}\} = 1 - P\{E_{k-1}^o|Y^{k-1}\} - P\{E_{k-1}^n|Y^{k-1}\} \quad (16)$$

The target state estimate and its covariance are conditioned on target existence and they are same as (7) of the IPDA Markov Chain One. The posterior conditional probabilities of the track quality measure are given by [4]:

$$P\{E_k^o|Y^k\} = \frac{(1-\delta_k)P\{E_k^o|Y^{k-1}\}}{1-\delta_k P\{E_k^o|Y^{k-1}\}} \quad (17)$$

$$P\{E_k^n|Y^k\} = \frac{P(E_k^n|Y^{k-1})}{1-\delta_k P\{E_k^o|Y^{k-1}\}} \quad (18)$$

$$P\{E_k|Y^k\} = P\{E_k^o|Y^k\} + P\{E_k^n|Y^k\} \quad (19)$$

where δ_k is given by (10). The data association probabilities are given by [4]

$$\beta_0(k) = \frac{(1-P_DP_G)P\{E_k^o|Y^{k-1}\} + P\{E_k^n|Y^{k-1}\}}{(1-\delta_k)P\{E_k^o|Y^{k-1}\} + P\{E_k^n|Y^{k-1}\})} \quad (20)$$

$$\beta_i(k) = \frac{P_DP_G\frac{V_k}{\hat{m}_k}\Lambda_k^i P\{E_k^o|Y^{k-1}\}}{(1-\delta_k)P\{E_k^o|Y^{k-1}\} + P\{E_k^n|Y^{k-1}\}} \quad (21)$$

2.3 NEW IPDA

The N-IPDA approach in [6] incorporates the data association events with target perceivability events rather than target existence events. The process of the target perceivability is modeled as a Markov chain with two states:

O_k	a target is perceivable from a sensor ,
$\bar{O}_k$	a target is not perceivable from a sensor.

Similar to IPDA, the target perceivability transition probability matrix is given by (4). N-IPDA implicitly assumes that the underlying target always exists.

From Markov chain property, the predicted probability that a target is perceivable is given by

$$P\{O_k|Y^{k-1}\} = \pi_{11}P\{O_{k-1}|Y^{k-1}\} + \pi_{21}(1 - P\{O_{k-1}|Y^{k-1}\}) \quad (22)$$

and the posterior probability of target perceivability can be calculated using

$$P\{O_k|Y^k\} = \beta_0(k) + \sum_{i=1}^{m_k}\beta_i(k) = \frac{(1-\delta_k)P\{O_k|Y^{k-1}\}}{1-\delta_k P\{O_k|Y^{k-1}\}} \quad (23)$$

which is the measure of track quality. The posterior probability that target is not perceivable is given by

$$P\{\bar{O}_k|Y^k\} = 1 - P\{O_k|Y^k\} = \beta_{\bar{0}}(k) \quad (24)$$

The data association probabilities are given by

$$\begin{aligned} \beta_{\bar{0}}(k) &= \frac{1}{c}\frac{m_k}{V_k}\Big(1 + \frac{1-P_DP_G}{P_DP_G}\xi_k\Big)\frac{1-P(O_k|Y^{k-1})}{1-\varepsilon_k P(O_k|Y^{k-1})} \\ \beta_0(k) &= \frac{1}{c}\frac{m_k}{V_k}\frac{1-P_DP_G}{P_DP_G}\xi_k\frac{(1-\varepsilon_k)P\{O_k|Y^{k-1}\}}{1-\varepsilon_k P\{O_k|Y^{k-1}\}} \\ \beta_i(k) &= \frac{1}{c}\frac{\Lambda_i(k)}{P_G}\frac{(1-\varepsilon_k)P\{O_k|Y^{k-1}\}}{1-\varepsilon_k P\{O_k|Y^{k-1}\}} \end{aligned} \quad (25)$$

where c is a normalization factor which can be obtained from the relation

$$\beta_{\bar{0}}(k) + \beta_0(k) + \sum_{i=1}^{m_k}\beta_i(k) = 1$$

and

$$\xi_k = \frac{\lambda}{V_k}$$

$$\varepsilon_k = \begin{cases} P_DP_G & m_k = 0 \\ P_DP_G(1 - \frac{1}{\lambda P_G}\sum_{i=1}^{m_k}\Lambda_i(k)) & m_k \neq 0 \end{cases}$$

and the clutter density λ is given by

$$\lambda = \begin{cases} 0\ , & m_k = 0 \\ \frac{1}{V_k}[m_k - P_DP_G\frac{(1-\varepsilon_k)P\{O_k|Y^{k-1}\}}{1-\varepsilon_k P\{O_k|Y^{k-1}\}}]\ , & m_k > 0 \end{cases}$$

Similar to (7), the target state estimate and covariance are given by [6]:

$$\begin{aligned} \hat{x}_{k|k} &= (\beta_0(k) + \beta_{\bar{0}}(k))\hat{x}_{k|k-1} + \sum_{i=1}^{m_k}\beta_i(k)\hat{x}_{k|k}^i \\ P_{k|k} &= \{\beta_{\bar{0}}(k) + \beta_0(k)[I + q_0KH]\}P_{k|k-1} \\ &\quad + [1 - \beta_0(k) - \beta_{\bar{0}}(k)][I - KH]P_{k|k-1} \\ &\quad + K\left[\sum_{i=1}^{m_k}\beta_i(k)\tilde{y}_k^i\tilde{y}_k^{i^T} - \tilde{y}_k\tilde{y}_k^T\right]K^T \end{aligned} \quad (26)$$

where $1 \geq q_0 \geq 0$, discussed in [14], is a diffusion factor[3] of the state error covariance in a standard PDAF.

The data association probabilities of N-IPDA depend on the probability of track quality measure while in IPDA and IPDA-M2 they do not.

2.4 IMMPDA

The Interacting Multiple Model PDAF, proposed in [7, 8], is an application of IMM algorithm with two models. It is assumed that a target always exists. At time k, two mutually exclusive and exhaustive models, $M(k) = M_1 \triangleq M_k^1$ and $M(k) = M_2 \triangleq M_k^2$ are assumed, i.e.,

M_k^1 : target is visible with a detection probability $P_D^1 = P_D$.
M_k^2 : target is not visible, or the detection probability $P_D^2 = 0$

Such process is modelled as a Markov chain associated with above two models. The (i, j)th entry of its transition probability matrix is defined as

$$P\{M_k = j | M_{k-1} = i\} = \pi_{ij} \qquad i, j \in 1, 2. \tag{27}$$

Each recursion starts at time k with the mixed initial state estimate and covariance of Gaussian mixture form

$$\begin{aligned} \hat{x}^{0i}_{k-1|k-1} &= \sum_{j=1}^{r} \hat{x}^j_{k-1|k-1} \mu_{j|i}(k-1|k-1) \\ P^{0i}_{k-1|k-1} &= \sum_{j=1}^{2} \mu_{j|i}(k-1|k-1)\{P^j_{k-1|k-1} \\ &+ [\hat{x}^j_{k-1|k-1} - \hat{x}^{0i}_{k-1|k-1}][\hat{x}^j_{k-1|k-1} - \hat{x}^{0i}_{k-1|k-1}]^T\} \end{aligned} \tag{28}$$

where the mixing probabilities are determined by

$$\mu_{j|i}(k-1|k-1) = \frac{\pi_{ij}\mu_j(k-1)}{\sum_{l=1}^2 \pi_{il}\mu_l(k-1)} = \frac{\pi_{ij}\mu_j(k-1)}{\mu_i(k|k-1)} \tag{29}$$

where $\mu_i(k-1) = P\{M^i_{k-1}|Y^{k-1}\}$ and $\mu_i(k|k-1) = P\{M_k^i|Y^{k-1}\}$ are the initial and predicted model probabilities of model i respectively.

A standard PDA procedure is then applied to each model with validated measurements received at time k and the i model based state estimate is

$$\hat{x}^i_{k|k} = \beta_0^i(k)\hat{x}^i_{k|k-1} + \sum_{j=1}^{m_k} \beta^i_j(k)\hat{x}^i_j(k|k) \tag{30}$$

with covariance $P^i_{k|k}$, where it can be shown that

$$\begin{aligned} \beta_0^i(k) &= \frac{(1 - P_D^i P_G)}{1 - \delta_k^i} \\ \beta_j^i(k) &= \frac{P_D^i P_G \frac{V_k}{m_k} \Lambda_j^i}{1 - \delta_k^i} \\ & i = 1, 2;\ j = 1, 2, \cdots, m_k \end{aligned} \tag{31}$$

[3]The factor q_0 is set to zero in our simulation for all algorithms under consideration.

where

$$\delta_k^i = \begin{cases} P_D^i P_G (1 - \frac{V_k}{m_k}\sum_{h=1}^{m_k} \Lambda_j^h) & m_k > 0 \\ P_D^i P_G & m_k = 0 \end{cases}$$

$$\Lambda_j^i = P_G^{-1}\mathcal{N}\big(y_j(k); \hat{y}^i(k), S^i(k)\big) \tag{32}$$

The updated model probability

$$P\{M_k^i|Y^k\} = \frac{(1-\delta_k^i)\mu_i(k|k-1)}{\sum_{l=1}^2 (1-\delta_k^l)\mu_l(k|k-1)} \tag{33}$$

is the IMMPDA measure of track quality.

The outputs of state estimate and covariance take Gaussian mixture form and weighted by model probabilities.

$$\begin{aligned} \hat{x}_{k|k} &= \sum_{j=1}^{2} \hat{x}^j_{k|k}\mu_j(k) \\ P_{k|k} &= \sum_{j=1}^{2} \mu_j(k)\{P^j_{k|k} + [\hat{x}^j_{k|k} - \hat{x}_{k|k}][\hat{x}^j_{k|k} - \hat{x}_{k|k}]^T\} \end{aligned} \tag{34}$$

3 Evaluation Study

The IPDA type algorithms are evaluated using simulation of a scenario of a low elevation sea-surface target tracking. Target detection probabilities model variable target detection probabilities due to complex sum of multipath signals. The track quality measures of the IPDA type algorithms are used for an automatic false track termination via termination thresholds, which are different for each algorithm.

3.1 Detection model for the low elevation sea-surface target

As shown in Figure 1, a target travels from a range of 30 Km to a range of 5 Km at an altitude 50 meters and relative speed 312.5 m/s. The expected values of the observed signal to noise ratio (SNR) for a S-band (frequency of 4 GHz) phase array radar 20 meters above sea-surface is simulated based on the signal model developed in [10], where it is assumed that signal from the target has a fixed-amplitude with an expected SNR of 16 dB in the absence of multipath propagation. The radar was modeled as vertically polarized with one-way 3 dB beamwidth 2^o and the antenna beam elevation angle is 1.05^o.

It is observed from Figure 1 that, for a fixed detection threshold of radar receiver, target may not be reliably detected at certain scans and it becomes even worse if sea-surface reflection condition is better (e.g., the average RMS sea-wave height is about 0.1 m which corresponds to the plot in solid line). Taking these factors into account, we may reasonably model the target detection probability as a time function as shown in Figure 2 where we assume the detection probability to be $P_D = 0.9$ in the absence of multipath propagation, $P_D = 0.05$ for the first, $P_D = 0.15$ for the second and $P_D = 0.45$ for the third time slots respectively.

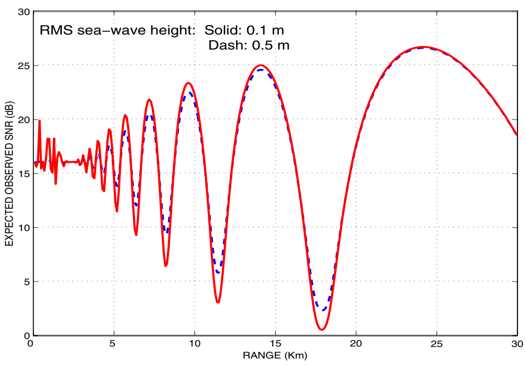

Figure 1: Expected observed SNR by a S-band phased array Radar due to sea-surface induced multipath propagation.

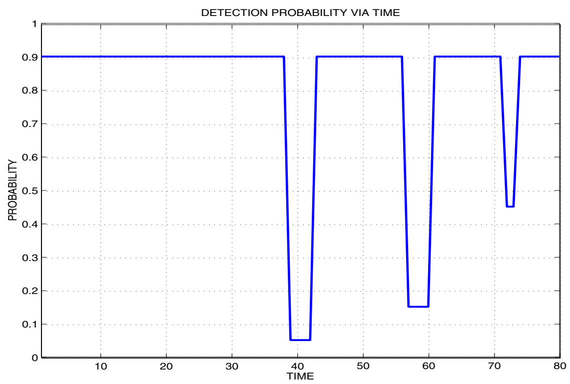

Figure 2: Detection probability for data generation model

3.2 Track termination

A criterion which will lead to a decision on terminating false tracks is necessary. All IPDA type algorithms discussed in this paper have self-contained probabilistic statistics for track quality measure and we will directly use these statistics to differentiate true track from false tracks against a threshold as in [5, 6].

Let χ_k denote the event that a track is true at time k. Then, a track is deleted or terminated if

$$P\{\chi_k|Y^k\} < \epsilon \tag{35}$$

where ϵ is the threshold of track termination.

The probability $P\{\chi_k|Y^k\}$ is equivalent to

$P\{E_k|Y^k\}$	for IPDA,
$P\{E_k^o|Y^k\} + P\{E_k^n|Y^k\}$	for IPDA-M2,
$P\{O_k|Y^k\}$	for N-IPDA and
$P\{M_k^1|Y^k\}$	for IMMPDA.

In general, the threshold ϵ takes different values for different algorithms at a given clutter density D_c, and is determined according to the following procedure.

Assume that a true track turns to be a false track at time $k = t_1$ within surveillance region. The track is then terminated at time $k = t_2$ with a time delay $\tau = t_2 - t_1$ for a given threshold ϵ. The values of ϵ for each algorithm are obtained such that all algorithms will have the same average termination time delay τ over N successful[4] Monte Carlo runs:

$$\tau = \frac{1}{N}\sum_{i=1}^{N}(t_2^i - t_1) \tag{36}$$

Figure 3 shows a plot of average false track termination delay τ versus track termination threshold ϵ. Clutter was uniformly distributed with density $D_c = 0.12/Km^2$, target detection probability was $P_D = 0.9$ and $N = 1000$. Sensor noise is assumed to be Gaussian zero-mean with standard deviations 300 meters in range and 0.75^o in azimuth respectively. The radar measurements (range & azimuth) are converted to local Cartesian coordinates via method suggested in [15]. Target state $\boldsymbol{x}$ is described with position and velocity in orthogonal coordinate system, and state transition matrix F is

$$F = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where $T = 1$ second is the data sampling period. The covariance of system process noise is given by

$$Q = \begin{bmatrix} T^3/3 & T^2/2 & 0 & 0 \\ T^2/2 & T & 0 & 0 \\ 0 & 0 & T^3/3 & T^2/2 \\ 0 & 0 & T^2/2 & T \end{bmatrix} q$$

where q = 0.1 is chosen for a nearly constant velocity model [15]. All true tracks are assumed to have been initiated and confirmed prior to the experiment and no tracks will be initiated during the experiment. Selection windows were used with the gating probability $P_G = 0.999$.

The transition probability matrices used in the experiment for the Markov chain model on target existent/observable are

$$\Pi = \begin{bmatrix} 0.98 & 0.02 \\ 0 & 1 \end{bmatrix} \tag{37}$$

for IPDA, N-IPDA and IMMPDA and

$$\Pi = \begin{bmatrix} 0.98 & 0.015 & 0.005 \\ 0.1 & 0.88 & 0.02 \\ 0 & 0 & 1 \end{bmatrix} \tag{38}$$

for IPDA-M2.

The choice of the time delay τ is such that it minimizes the chance of deleting true tracks and keeping false tracks. In this multipath fading scenario, a tracker could get no validated measurement within several consecutive scans. Subsequently, it is desirable that track termination time delay is great than 4. In our simulation, the average time delays $\tau = 5$ and $\tau = 6$ are selected and the corresponding track termination thresholds are given in Table 1 which are obtained from Figure 3.

Figure 3 shows that full range of τ for track termination threshold selection is available for IPDA-M2, while the choice of the τ for other three algorithms is limited considerably.

[4] A run where $t_2 < t_1$, i.e., track termination occurs before the true track is terminated is not counted.

$\tau =$	IPDA	IPDA-M2	N-IPDA	IMMPDA
5	0.0067	0.028	0.0095	0.0024
6	0.0015	0.0182	0.0035	0.0008

Table 1: Track Termination Threshold

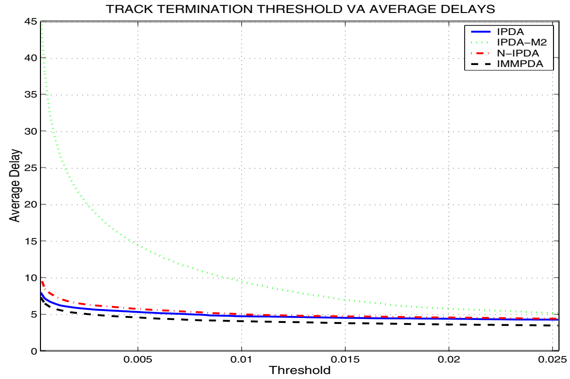

Figure 3: Average false track termination delay via threshold for clutter density $0.12/Km^2$.

3.3 Comparison results

Each experiment consists of 1000 Monte Carlo runs for all algorithms under consideration using the same data. The following cases were considered:

Case 1 Tracker receives measurements observed in the absence of signal multipath fading, i.e., $P_D = 0.9$ for all time. The track termination thresholds for average time delay of 6 scans are used.

Case 2 Tracker receives measurements in the presence of signal multipath fading, i.e., the target detection model shown in Figure 2 is considered and track termination thresholds for average time delay of 6 scans are used.

Case 3 Same as Case 1, but track termination thresholds for average time delay of 5 scans are used.

Case 4 Same as Case 2, but track termination thresholds for average time delay of 5 scans are used.

The percentages of track loss are give in Table 2, the average number of expected measurements in gates are presented in Table 3 and the RMS error performance comparison is shown in Figures 4 and 5 respectively.

Case	IPDA	IPDA-M2	N-IPDA	IMMPDA
Case 1	6.9%	5.9%	7.3%	6.6%
Case 2	52.2%	22.0%	40.6%	52.1%
Case 3	9.9%	7.2%	9.3%	9.3%
Case 4	69.6%	25.2%	56.5%	67.8%

Table 2: Track Loss Comparison ($D_c = 0.12/Km^2$)

Figure 6 shows the number of retained true tracks via time in Case 2.

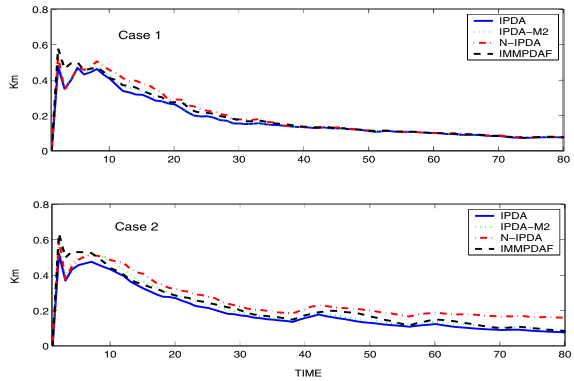

Figure 4: RMS position estimation error comparison for Case 1 and 2.

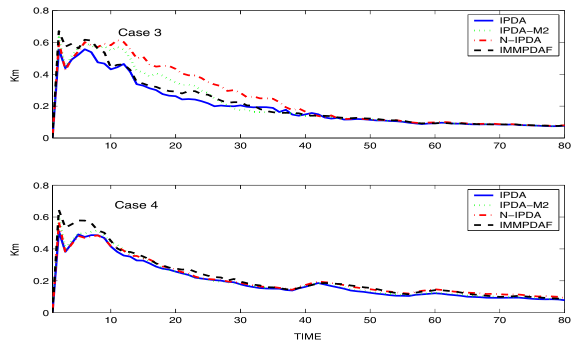

Figure 5: RMS position estimation error comparison for Case 3 and 4.

Observations and Discussions:

- Signal Multipath fading when tracking a low elevation sea-surface target causes higher percentage of track loss. Figure 7 shows the RMS position estimation error comparison without false track termination and Figures 4 and 5 shows the RMS position estimation errors with track termination.
- In view of Figure 1 and 6, before the first drop of the target detection probability all algorithms have similar number of retained true tracks. However, IPDA-M2 will have a higher number of retained true tracks when there is multipath fading problem (i.e., after 40 scans).
- IPDA-M2 has significantly lower percentage of track loss compared to the others when multipath fading (Case 2 and 4) is present. An obvious reason for this is finer modelling of target existence and visibility state in the case of IPDA-M2.
- In the case of absence of multipath fading (Case 1 and 3), i.e., $P_D = 0.9$, there is no significant performance difference between all algorithms with the selected track termination thresholds. IMMPDA has a higher number of expected validated measurements and slightly higher computational load.
- Results in Table 1 need to be augmented with the average success of establishing and confirming the true track for each

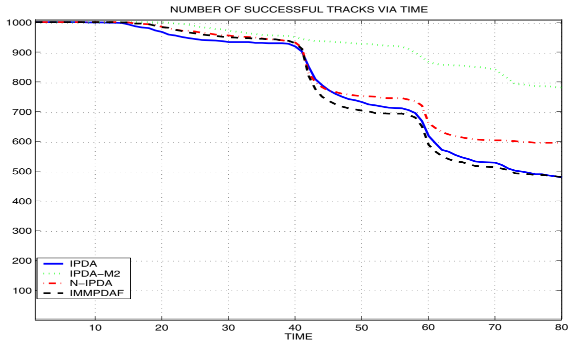

Figure 6: Number of retained true tracks for Case 2.

Case	IPDA	IPDA-M2	N-IPDA	IMMPDA
Case 1	1.6381	1.7977	1.8224	2.7197
Case 2	1.5746	1.7062	1.8768	2.7039
Case 3	1.6541	1.8574	1.9897	2.6962
Case 4	1.5584	1.6524	1.6593	2.5860

Table 3: Expected Number of Validated Measurements $\bar{m}_k$

algorithm under consideration. The corresponding research is being carried out.

4 Conclusion

This paper presents a comparison study of IPDA type target tracking filters. The filters are compared when tracking in clutter a target whose probability of detection varies due to multipath signal fading. Preliminary results show that

- The values of the track termination threshold for IPDA type filters may be obtained statistically using Monte Carlo runs.
- Higher percentage of track loss is observed when multipath fading is present.
- The IPDA-M2 appears to perform better than other filters in this environment.
- Further study which considers also track initiation and confirmation must be carried out before final conclusions can be drawn.

Tracking a target with time-varying detection probability has posed new challenges. Further study on tracking in clutter of low elevation sea-surface targets is being conducted.

References

[1] Y. Bar-Shalom, and E. Tse. "Tracking in a Cluttered Environment With Probabilistic Data Association", *Automatica*, vol. 11, pp. 451–460, 1975.

[2] X. Wang, S. Challa, and R. J. Evans. "Gating Techniques for Maneuvering Target Tracking in Clutter", *IEEE AES*, Vol. 38, No. 3, pp. 1087–1097, July 2002.

[3] Y. Bar-Shalom, and T. E. Fortmann. *Tracking and Data Association*, Academic Press, 1988.

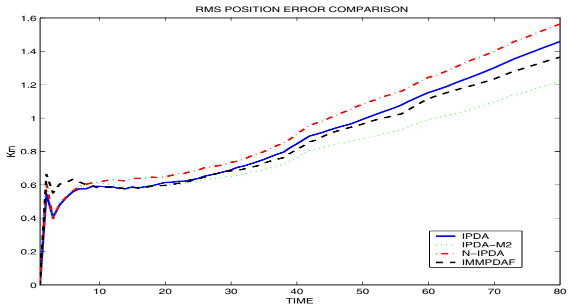

Figure 7: RMS position estimation error comparison without deleting false tracks

[4] D. Mušicki, R. J. Evans, and S. Stanković. "Integrated Probabilistic Data Association", *IEEE Trans. Auto. Control*, Vol. 39, no. 6, pp. 1237–1241, June 1994.

[5] D. Mušicki. "Automatic tracking of maneuvering targets in clutter using IPDA", PhD theses, (Sept 1994) University of Newcastle, New South Wales, Australia.

[6] X. R. Li, and N. Li. "Intelligent PDAF: Refinement of IPDAF for Tracking in Clutter", *Proc. 29th SSST*, pp. 133–137, 1997.

[7] Y. Bar-Shalom, K. C. Chang, and H. A. P. Blom. "Automatic Track Formation in Clutter with a Recursive Algorithm", *Proc. 28th Conf. Dec. Control*, Florida, pp. 1402–1408, Dec. 1989.

[8] Y. Bar-Shalom, K. C. Chang, and H. A. P. Blom. "Automatic Track Formation in Clutter with a Recursive Algorithm", in *Multitarget-Multisensor Tracking: Advanced Applications*, pp. 25–42, Artech House, MA., 1990.

[9] A. Houles, Y. Bar-Shalom. "Multisensor Tracking of a Maneuvring Target in Clutter", *IEEE Trans. AES*, vol. AES-25, no. 2, pp. 176–189, Mar. 1989.

[10] W.D. Blair and B. M. Keel. Radar systems modeling for tracking. In Yaakov Bar-Shalom and William Dale Blair, editors, *Multitarget-Multisensor Tracking Applications and Advances*, volume III, chapter 7. Artech House, Boston, MA, 2000.

[11] W. D. Blair, G. W. Groves, Y. Bar-Shalom, and E. Daeipour. "Frequency Agility and Fusion for Tracking Targets in the Presence of Multipath Propagation", *Proc. IEEE Nat. Radar Conf.*, Atlanta, GA, pp. 166–170, Mar. 1994.

[12] Y. Bar-Shalom and W. D. Blair. " Tracking low elevation targets in the presence of multipath propagation", *IEEE Trans. AES* , Vol. 30, No. 3, July 1994, pp. 973–979.

[13] E. Daeipour, W. D. Blair and Y. Bar-Shalom. "Bias Compensation and Tracking with Monopulse Radars in the Presence of Multipath," *IEEE Trans. AES*, vol. 33, no. 3, pp. 863–882, July 1997.

[14] X. R. Li. "Track in clutter with strongest neighbot measurements - Part 1: Theoretical analysis," *IEEE Trans. AC,*, Vol. 43, No. 11, pp. 1560–1578, Nov. 1998.

[15] Y. Bar-Shalom, and X.-R. Li. *Estimation and Tracking: Principles, Techniques, and Software*, Artech House, MA, 1993.