Time Geography for Ad-Hoc Shared-Ride Trip Planning

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Abstract

Ad-hoc shared-ride trip planning is a planning task on a non-deterministic transportation network. We propose to solve this task in a mobile geosensor network, which consists of transportation clients and hosts. In a mobile geosensor network the communication costs are a critical factor. Trip planning agents need communication to collect knowledge about the current network, and any way to limit this need reduces the costs of a solution. This paper introduces a theoretical model based on time geography, where clients, as trip planning agents, can actively identify relevant transportation hosts before communication starts, and hosts can identify whether their route is relevant for a specific planning task before responding to any request. This model reduces the communication costs significantly, which is at first derived theoretically, and then confirmed by an example.

1. Introduction

Ad-hoc shared-ride trip planning assigns vehicles with free transportation capacity to passengers (or goods) with transportation needs in an ad-hoc manner [16]. Today's centralized systems cannot cope with ad-hoc shared-ride trip planning on a large scale, since this planning problem concerns a complex, non-deterministic transportation network. Complexity arises from the unpredictability of transportation capacities: vehicle drivers are autonomous and do not follow schedules, and vehicle seats are quickly occupied.

But ad-hoc shared-ride trip planning can be efficiently realized in a decentralized manner, establishing a mobile geosensor network [14] with nodes representing transportation client and host agents. This approach was demonstrated in principle by using a simple, sub-optimal planning strategy [16].

An optimal trip, for example the quickest trip, can take any route, and can involve multi-step rides. Consequently, a planning agent seems to require knowledge of the full current transportation network before coming up with an optimal trip. Since the planning node must collect this knowledge ad-hoc from other network nodes, it becomes clear that the critical question is how the communication effort to collect this knowledge can be reduced. The motivation for any reduction is manifold [20, 17]:

- Communication costs are a major concern in mobile sensor networks. Nodes are battery-powered, and short-range radio communication is the most energy-consuming activity of a node.
- Bandwidth of the communication channel is another concern. Mobile sensor networks communicate in relatively short communication windows (to save power), and this limits the number of messages to be exchanged.
- The memory of the planning nodes (e.g., handheld mobile devices) is another limited resource in this type of application. It can be assumed that all agents locally store the street network and have routing algorithms for their own purposes. But in shared-ride trip planning the network is dynamic, with the number of edges equaling the number of hosts times the lengths of their travel plans. For an inner-urban traffic situation, this number can easily exhaust the storing and analyzing capacity of the planning agent.

In this paper we are interested in the (at the time of planning) optimal trip for a client. The hypothesis is that the required knowledge, and hence, the necessary communication in the geosensor network, can be restricted significantly. This means our focus is on enriching the reasoning capabilities of the agents in the geosensor network, and we are not so much concerned with communication routing strategies or protocols.

Specifically, we develop a heuristic based on spatio-temporal criteria that enables the planning clients to identify potentially relevant transportation hosts before any communication, and also enables contacted hosts to decide whether their routes may contribute to an optimal trip before responding. The theoretical framework that underlies this fil-
tering approach is time geography. It was introduced by [2] and focuses on the question of how peoples locations in space at given times affect their abilities to be at other locations at other times.

The next section gives an overview of previous work and the resources needed for the rest of the paper. We then approach the problem first by identifying what information is potentially relevant for the trip planning process (Section 3). Since this leads to time geographic elements, we introduce the required concepts and formalize the information needs (Section 4). The formal model will be demonstrated by an example that confirms the theoretical results (Section 5). The paper closes with a discussion of the results and conclusions (Section 6).

2. Previous work

2.1. Shared-ride trip planning

Ad-hoc shared-ride trip planning has recently been proposed as one application for mobile geosensor networks [16, 18, 17]. Conventional central shared-ride systems cannot deal with ad-hoc trip planning and are in principle not scalable for large numbers of concurrent transportation clients and hosts. The peer-to-peer approach of mobile geosensor networks [14] turns out to provide an effective, efficient and elegant alternative. It is effective because it can provide trips close to optimal trips (note that the globally optimal trip can be determined in a non-deterministic system only in hindsight). It is efficient because it does so for low communication costs [17]. And it is elegant because it requires low computational effort. Details depend on the chosen combination of communication and wayfinding strategies.

Ad-hoc shared-ride trip planning is a problem defined on a complex, non-deterministic transportation network. Therefore the approach taken to show the properties of trip planning in a mobile geosensor network was by simulation. This simulation is specified in [18] and results are presented in [17]. The reality of urban traffic was thereby simplified to a grid world, in which hosts travel at constant velocities and clients look for rides along a predefined route. A suitable protocol allowed directed messaging in the peer-to-peer communication network. The simulation realized a negotiation process of three steps:

1. A client broadcasts a request message with the specified route.

2. Hosts with travel plans overlapping this route broadcast an offer message for this particular overlap.

3. Clients, after collecting all offers and selecting the best, broadcast booking messages.

In this simulation the authors investigated how different communication strategies—different depths of communication into the transportation network—affect the average trip lengths. Note that all three negotiation steps must happen within one communication window, which in practice radically limits the communication depth, and thus also the knowledge of the planning client agent.

While this simulation was suited to investigate the effect of different communication strategies, it did not provide a way to compute optimal solutions for the shared-ride trip problem. The limitation of the clients knowledge was not based on relevance criteria, and the chosen wayfinding strategy—following a predefined route—does not necessarily deliver the optimal (e.g., quickest) trip.

The optimal trip can take any route. In the beginning of the planning process, the client does know only start and destination. But if a client’s request contains only start and destination, it seems difficult for the hosts to determine whether their travel plans are potentially relevant for the client. This problem will be addressed in the following by utilizing elements of time geography during the planning process.

2.2. Time geography

People and resources are available only at a limited number of locations and for a limited amount of time. The ability to be present at a particular location in time is therefore an essential human requirement. Time geography defines the space-time mechanics by considering different constraints for such presence, i.e., the capability, coupling, and authority constraint [2]. The possibility of being present at a specific location and time is determined by peoples ability to trade time for space, supported by transportation and communication services.

Every individual (person or object) can be characterized by its space-time path. Such paths are available at various spatial (e.g., house, city, country) and temporal granularities (e.g., decade, year, day) and can be represented through different dimensions. Figure 1 shows a persons space-time path during a day, representing her movements and activity participation at three different locations. The tubes depict space-time stations, i.e., locations that provide resources for engaging in particular activities, such as sleeping, eating, and working. The slope of the path indicates how fast a person can move through her environment. If the path is vertical then the person is engaged in a stationary activity.

Three classes of constraints limit a persons activities in space and time. Capability constraints limit an individuals activities based on her abilities and the available resources. For example, a fundamental requirement for many people is to sleep between six and eight hours at home. Coupling constraints require a person to be at a specific location at a
particular time. For example, if two persons want to meet at a Café, then they have to be there at the same time. In time-geographic terms, their paths cluster into a space-time bundle. Certain domains in life are controlled through authority constraints: A person can only shop at a mall, when the mall is open, such as between 9am and 8pm.

All space-time paths must lie within space-time prisms. These are geometrical constructs of two intersecting cones [6]. Their boundaries limit the possible locations a path can take based on peoples abilities to trade time for space. In order for a person or activity to be accessible, its space-time station must intersect the space-time prism for a minimal temporal duration. The projection of the space-time prism to geographic space results in the potential path area—all locations that can be reached by the individual [7].

Time geography has been applied in the areas of Geographic Information Systems regarding transportation networks to model and measure space-time accessibility [8, 11, 19]. It has also been advocated to integrate time geography with both Geographic Information Systems and Location-Based Services to achieve more user-centered systems [10, 13]. Further applications in the geo-domain concern the structuring of dynamic wayfinding environments [4] and the modeling of geospatial lifelines [3]. Analytical formulations of basic entities and relationships from time geography can be found in [9].

3. An optimal shared-ride trip planning strategy

In the best case, an ad-hoc shared-ride trip planning agent disposes of all currently available transportation. With this information it can apply a time-dependent shortest path algorithm [1, 5, 12] to identify the, let us say, quickest route, and the hosts providing this trip. The dynamics and unpredictability of the transportation capacities make the quickest route a temporally limited one: at other times the solution can be different. Even from hindsight the quickest route at the requested time may have been another one. The temporal limitation cannot be overcome by any strategy; hence, the “quickest” route is the optimal solution at time t with all the transportation capacities available (known) at that time.

Learning about all currently existing travel plans requires communication, i.e., battery power and bandwidth, and computation, in particular memory. Since all of them, battery power, bandwidth, and memory, are scarce resources in mobile geosensor networks, any effort must be made to minimize communication and collected information. Key is filtering the potentially relevant travel plans from all travel plans, and to communicate and collect only this subset.

In [16] it was reasonable to initiate communication by a request from the client, containing a route plan. Hosts responded with an offer only if they could contribute to the request, i.e., if their own travel plans overlapped with the route planned by the client. In our scenario the client does not know a route; she knows only her current position s and the desired destination d. Hence, it is not obvious for a host to decide whether her travel plans could contribute to the demand of the client expressed through s and d.

In this situation the most conservative solution is to request from all hosts broadcasting their travel plans. By this way each client gets a full picture of the network and can find an optimal (quickest) trip, but clients also collect many host travels that are irrelevant for their trip. However, any limitation of this set has to ensure that no travel plan that could be part of a quickest trip is excluded from communication.

Any heuristics will work with some sort of spatial buffer: near transportation hosts are more probably relevant for a client than far ones, in adaptation to the First Law of Geography [15]. This argument can be sharpened by looking into realistic travel demand. Nobody is willing to travel infinite time in an urban environment. There is an upper time limit beyond which a travel offer is no longer accepted. Either people get to their destination within a reasonable time, or they are not willing to pay for the offered transportation. They will either give up their travel demand and stay, or travel on their own.

For the following it is important to find an upper limit that is realistic and conservative: a latest arrival time. If it is chosen too short, the trip planning attempt of the client can be unsuccessful: there is no trip within this time frame. If it is chosen too long its filtering effect suffers: there are many more trips than the quickest one. For the time being,
and without limiting generality, we propose to take walking time as the upper limit in urban environments, i.e., on trips of walking distances, assuming that people only pay for transportation if they have a benefit, i.e., if they are faster than by foot.

An upper travel time limit yields a first sharp criterion to identify potentially relevant transportation hosts. Only hosts that can reach the client’s destination within this travel time potentially contribute to the client’s trip. Otherwise they are too far off to allow the client to reach the destination within time. Hence, one can delimit the hosts responding to a client’s request by the circle drawn from the client’s destination with a radius derived from the upper limit of the client’s travel time. In a realistic, heterogeneous urban street network the circle deforms to a potential path area [7].

Although this argument was given here by intuition, it already uses an element from time geography. In the next section we develop more systematically the required instruments from time geography, and re-consider and improve upon this first result.

4. Time geography for optimal shared-ride trip planning

In this section those elements from time geography are utilized that help to narrow down the hosts relevant for a declared demand of a client to an absolute minimum. The elements are combined to simple computations, which hosts can apply to decide whether their travel plan is relevant and should therefore be communicated to the client.

4.1. The cone of latest arrival time

Consider Figure 2. It shows a client’s current position and destination in a space-time diagram. The client wants to reach the destination in any case before the marked latest arrival time (upper limit of travel time). Since this arrival time was determined by the walking time for the client, and transportation is assumed to be faster than walking, the client’s start point is somewhere inside a conic volume containing all locations from which the destination can be reached before the marked arrival time. We call this cone dl-cone, being placed in the destination at the latest arrival time.

All hosts under the cone can contribute to transportation to the destination before the latest arrival time. All hosts outside cannot contribute, since the cone is constructed based on the maximum velocity of vehicles. Hosts on the surface of the cone can only contribute if the client waits until his space-time station intersects with the host’s space-time path on the surface of the cone, and if the host then heads directly to the destination with maximum velocity. Hosts within the base circle, but far from the start, can contribute only very late in the process, i.e., close to the top of the cone.

The mathematics for the cone base circle is simple: if the latest arrival time is $t_h = t_0 + h$ then the circle is defined by its radius $r$:

$$r = h \cdot v_{\text{max}}$$

If the client collects at time $t_0$ the travel plans of all hosts within the base circle at $t_0$, the client gets complete transportation network knowledge for trip planning as available at $t_0$. Note however that the method specifies all potential locations of hosts, not hosts. If, by chance, there are no hosts at these locations there is no trip available within this cone, i.e., before the latest arrival time.

As discussed by [16] shared-ride trip planning is an iterative process, due to the dynamics of the transportation supply: new hosts enter the traffic, other ones get booked, or get their bookings canceled and are available again. In this situation a client revises travel plans in regular time intervals. The more time proceeds the smaller the base circle of the cone becomes. In Equation 1 $h$ is decremented with every iteration. A linearly reducing radius means that the number of locations—or, assuming equal distribution of hosts over all locations, the number of hosts—reduces quadratically from iteration to iteration. The potential path area $a$ is calculated by:

$$a = \pi \cdot r^2$$

Two other measures are needed for the following: the volume of a cone, $c_v$, and the volume of a frustum, $c_f$, the portion of a cone which lies between two parallel planes.
cutting the cone.

\[ c_v = \frac{1}{3}\pi hr^2 \]  
\[ (3) \]

\[ c_f = \frac{1}{3}\pi h \left(r_t^2 + r_t \cdot r_b + r_b^2\right) \]  
\[ (4) \]

In these formulas \( h \) stands for the height of the cone, and \( r \) for the radius (\( r_t \) for the radius of the top circle, and \( r_b \) for the radius of the bottom circle).

4.2. The cones of earliest arrival time

The *earliest* arrival time is determined by constructing the cone originating in the start point and looking for the intersection with the destination station (Figure 3). The cone cut at this level by a parallel plane contains all locations to be reachable within the travel time from now to the earliest arrival time. We call this cone the *se*-cone (start, earliest).

The cone originating in the earliest arrival time in the destination contains all possibilities to reach the destination up to the earliest arrival time (Figure 3). We call this cone the *de*-cone (destination, earliest).

![Figure 3. The cones of earliest arrival time.](image)

4.3. The cone of host locations from start

Now imagine that the client is happy to look only parts of the route ahead. Instead of planning the whole trip she is content with planning for the next segment(s).

Hosts that can take the client from her current location have to be on a space-time-path that crosses the client’s space-time station inside of the *dl*-cone. That means they have to be within the cone centered on the client’s location that is tangential proper part of the *dl*-cone (see Figure 4). The client needs only to query all hosts in the base circle of this smaller cone to find a host to start traveling; other hosts within the base circle of the *dl*-cone cannot contribute to start a trip reaching the destination before the latest arrival time.

![Figure 4. Identifying hosts that can offer timely transport from start.](image)

4.4. The space-time prism of the client’s travel possibilities

So far we have identified locations of hosts only, not considering their travel plans. In this set there are still many hosts that have trajectories irrelevant for the problem, for example if their travel plans lead to the outside of the *dl*-cone, or if they end before reaching the client. Hence, the next question is: can a host decide whether his travel plan is a relevant one, i.e., whether his travel plan *bundles* with the potential trips of the client? If the hosts can do this, they can filter out irrelevant offers before they are made.

Clients can move only within a subspace of the *dl*-cone, a space-time prism defined by the intersection of the *se*-cone and the *dl*-cone (Figure 5). They cannot move outside of this space-time prism without giving up on reaching the destination before the latest arrival time. With other words, if there is a trip for the client, it must be in the space-time prism. If there are several trips, they must all be in the space-time prism, including the optimal. This means only hosts with trajectories intersecting the space-time prism are relevant. It is the smallest volume in the space-time diagram that theory—in this case time geography—can specify.

Consequently, the next question is whether hosts can determine the space-time prism of a client for cheap computational costs before broadcasting their offer for transport. Efficiency is important since communication windows in sensor networks are short, and hence, responses on requests have to be broadcasted quickly. Furthermore, one does not want to loose battery power in extensive computations that was gained in communication. For the computation, the hosts need to know only the street network, but nothing about other agents in the network. In order to solve this problem no communication among hosts is needed.

The remaining question is: what is the gain in efficiency
of communication? In the previous section (Section 3) we have seen that without time geographic concepts a client would at best request all travel plans from hosts within the base circle of the \(dl\)-cone (if not requesting offers from all hosts). Previously all hosts would send an offer. Figure 6 demonstrates that hosts in the base circle of the \(dl\)-cone can reach any point in the frustum, defined by the maximum velocity of the hosts, within the travel time to latest arrival. The frustum and the \(dl\)-cone have the following relationship (note that the frustum is turned upside down, such that the top circle of the frustum is equal to the bottom circle of the \(dl\)-cone; both are of the same height):

\[
\begin{align*}
  r_b(\text{frustum}) &= 2r_t(\text{frustum}) \\
  r_t(\text{frustum}) &= \frac{1}{2}r_t(\text{frustum})
\end{align*}
\]

A simple calculation with Equations 3 and 4 shows that the volume of the \(dl\)-cone is one-seventh of the volume of the frustum. This means many hosts—on average six out of seven—will make an offer although their routes are outside of the \(dl\)-cone and thus in any case irrelevant for the client.

After introducing elements from time geography we see that a client still has to send a request to all these hosts. The number of request messages will not change. However, now only hosts send an offer that have a travel plan intersecting with the space-time prism of the client. Figure 6 illustrates also this difference by comparing the volumes of the frustum with the client’s space-time prism. The client’s space-time prism is again significantly smaller than the \(dl\)-cone (a subspace of the \(dl\)-cone). Note that the volume of the space-time prism varies according to the chosen latest arrival time, \(h\), and its asymmetry. It is smallest for a start point on the surface of the cone (which implies that clients can travel on their own with the maximum velocity of hosts, which is unrealistic in our case): in this case the space-time prism is a straight line only. The volume is largest for a symmetric prism of two cones of half the height \(h\) of the \(dl\)-cone (which implies that a client starts at the destination, which is also unrealistic): this space-time prism has a quarter of the volume of the \(dl\)-cone, with Equation 3:

\[
\begin{align*}
  c_w(\text{frustum}) &= \frac{1}{7} \pi h v^2 \\
  c_p(\text{client at destination}) &= \frac{2}{7} \pi h^2 \left(\frac{r_t}{2}\right)^2 = \frac{1}{4} c_w(\text{frustum})
\end{align*}
\]

With other words, the client’s space-time prism is in our case always smaller than \(\frac{1}{7}\) of the \(dl\)-cone, and thus, at most \(\frac{1}{7} \cdot \frac{1}{7} \approx 3.5\%\) of all hosts in the base circle of the \(dl\)-cone will make an offer to the client.

5. Example by cellular automata

Now let us consider an example to demonstrate the theoretically shown effects. The example assumes a grid environment and autonomous agents on the grid nodes. Movements are possible from each grid node in the four cardinal directions (4-neighborhood). For the example we postulate random direction choices by the hosts at each step and homogeneous velocity of all host agents. The velocity \(v = v_{\text{max}}\) is fixed to 1 grid segment per time interval.

Consider Figure 7. In the environment the client has to travel four grid segments to reach the destination on the shortest route. Then the earliest possible arrival time of the client is four time intervals (if she gets immediate rides on the shortest route).

Let us further assume that the walking velocity of the client is three times slower than \(v\) of the hosts. Then the latest arrival time of the client is twelve time intervals (if she has to walk all the segments on the shortest route because she did not get any ride).

![Figure 5. The space-time prism of a client.](image)

![Figure 6. The space-time prism compared to all possible routes driven by hosts in the base circle.](image)
Applying Equation 1 yields a $dl$-cone base circle of $r = 12$ grid segments, and Equation 2 yields an area of the circle of $a = 452$ grid cells. However, Equation 2 works with the Euclidean distance, while we deal with 4-neighborhood. In our discrete grid environment Equation 2 changes to:

$$a = (r + 1)^2 + r^2$$

—the area of a diamond. This formula yields $a = 313$ grid cells for the base circle of the $dl$-cone (or $a = 41$ for the base circle of the $de$-cone; compare with Figure 7).

With Equation 5 we can compute volumes of discrete cones by integration (again, we do not apply the equations for continuous cones, Equations 3-4):

$$c_v = \sum_{i=0}^{h} a_i$$

For example, the volume of the $dl$-cone is computed by the sum of the base circles over all time instances $t_0, t_1, \ldots, t_{12}$:

$$c_v(dl) = \sum_{i=0}^{12} a_i = 1469 \text{ voxels}$$

Accordingly, the volume of the frustum in Figure 6 is:

$$c_f(dl) = \sum_{i=12}^{24} a_i = 9269 \text{ voxels}$$

For the space-time prism of the client, $cl$, the $se$-cone is proper part of the $dl$-cone up to $t_4$ and contains the $dl$-cone from $t_8$ onwards (see Figure 8). With that its volume is:

$$c_p(cl) = \sum_{i=0}^{12} a_i = 1 + 5 + 13 + 25 + (5 \cdot 41) + 25 + 13 + 5 + 1 = 293 \text{ voxels}$$

Now take into account that hosts move randomly. Some hosts reach their own destination during the considered time period, and other ones enter traffic. That means we may assume an equal distribution of host density in all grid nodes of the frustum. With other words, of 9269 possible host locations only 293 are relevant for the client, which is 3% (which is close to the expected value).

It can be expected that the real number of relevant hosts is even smaller since the relevant voxels are not randomly distributed, but autocorrelated. A host that will travel through one of the locations of the space-time prism will most probably also travel through other ones.

6. Discussion and conclusions

The results show convincingly that the space-time prism of clients imposes a clear criterion for hosts to determine whether their travel plans are potentially relevant for a client. They also demonstrate that this criterion is efficient by filtering out about 97% of all hosts that are irrelevant for the planning process. We have argued that computational costs for determining the space-time prism by hosts are comparably low. Finally the results gave a theoretical motivation for specifying the area for spreading a request. The resulting model is theoretically founded, but becomes a heuristic by choosing arbitrarily a suitable latest arrival time. The choice of the latest arrival time should be carefully investigated. Its arbitrariness can cause situations where the $dl$-cone has no connected path from $s$ to $d$. 
Hence, one of the open questions is a suitable wayfinding strategy of the clients. The clients collect offers, and these offers form a time-dependent transportation network. If this network does not need to have at every time instance a connected path to the destination the client needs a strategy for a risky decision: what is the most promising direction, or the most promising intermediate node for waiting in this network?

Related to this problem is the question when a client who deliberately limits its route planning to the next segments comes up with a global fastest route. Competition for transportation capacity (seats) among clients interferes with incremental trip planning.

Furthermore, we are working on a large-scale simulation of the model in a real street network to confirm the theoretically derived results. In realistic street networks cones still exist, but they have irregular shape due to the travel time geometry. They can be determined, though, by average travel time costs along street edges. Another aspect to study is the density of hosts in this simulation.

Relaxing the constraints of a simulation to real networks and hosts of a variety of velocities is another extension.

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References