Capabilities and Limitations of Slow Light Optical Buffers:
Searching for the Killer Application

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Summary

• Slow light and optical data
  - Group velocity and data bit-size compression
• Optical delay lines and buffers
  - Signal bandwidth and information bandwidth
  - FIFO buffers
• Properties of an ideal slow light medium
  - Delay-bandwidth product
• Requirements of practical optical buffers
  - Storage density
  - Dispersion
  - Attenuation
• Busting some slow light myths
• Data storage in high-Q resonators
Group Velocity

Group velocity:

\[ v_g = \frac{\partial \omega}{\partial k} = \frac{c}{n + \omega \frac{dn}{d\omega}} \]

Intrinsic attenuation:

\[ \alpha \approx \frac{1}{\tau_{abs} v_g} = \frac{1}{\tau_{abs} c} \left( n + \omega \frac{dn}{d\omega} \right) \]

<table>
<thead>
<tr>
<th>Waveguide loss (dB/cm)</th>
<th>Absorption time (ns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>30</td>
</tr>
<tr>
<td>0.1</td>
<td>3</td>
</tr>
</tbody>
</table>

Time to attenuate by $e^{-1}$
Electromagnetically-Induced Transparency (EIT)

\[ \frac{dn}{d\omega} \] becomes large

\[ \alpha(\omega) \]

\[ \omega_o \]

\[ \Delta\omega \]

\[ n \]

\[ \omega \]

Background

i.e. Hilbert transform

Transfer Function

Kramers Kronig
Micro-resonator Delay Line

- $n_{avg}$
- FSR
- Passband
- $\Delta \omega$

- $n$ vs $\omega$
- $\omega_{p-1}$, $\omega_1$, $\omega_p$, $\omega_2$
Ideal Slow-Light Material

Effective Index, $n$

Transfer Function

Signal Spectrum

Bandwidth

All-pass function

$\tau_{abs} \rightarrow \infty$

$\omega_{min}$ $\omega$ $\omega_{o}$ $\omega_{max}$

$n_{min}$ $n_{avg}$ $n_{max}$

$0$ $0$ dB

$\Delta \omega$
Signal Bandwidth and Data Bandwidth

<table>
<thead>
<tr>
<th>Signal Bandwidth (Hz)</th>
<th>Data “Bandwidth” or Information Rate (b/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>~ 1/τ</td>
<td>→ 0</td>
</tr>
<tr>
<td>~ 1/τ</td>
<td>→ 0</td>
</tr>
<tr>
<td>~ 1/τ</td>
<td>1/T_{bit}</td>
</tr>
</tbody>
</table>
Group Velocity Change at Boundary

Index = $n_1$

Input

Waveguide 1

Index = $n_2$

Waveguide 2

Output

Slow-down factor:

$$S(\omega) = \frac{v_{g1}}{v_{g2}} = \frac{n_2 + \omega \frac{dn_2}{d\omega}}{n_1 + \omega \frac{dn_1}{d\omega}} = \frac{n g_2}{n g_1}$$

Group indices
Group Velocity and Bit Length

Group Velocity

Information Bandwidth

Bit Period

Bit Length

= Period x Velocity

Field

Regular Waveguide

Slow Light Waveguide

L_{in} \rightarrow \text{ } L_{bit} \rightarrow \text{ } L_{in}
Car Analogy

100 km/h → 100 km/h

Real World

Slow Light World
Reduced Group Velocity
Constant Bitrate

L_{bit} → → →
Tapered Transition Region

Region 1  Transition  Region 2
Region

\[ V_g \]

\[ V_{g1} \]

\[ V_{g2} \]

\[ X_{A1} \]

\[ X_{A2} \]

\[ \Delta V_g \]

Field

\[ L_b(x) \]
Circuit Switching and Packet Switching

Freeway Model

- Car: Packet
- Lane: Wavelength
- Freeway: \{ Waveband, Fiber \}
- Interchange: Router
Statistical Multiplexing in Buffer

Incoming packets

Buffer

load = 0%

Outgoing packets

Nick McKeown  http://tiny-tera.stanford.edu/~nickm/
Storage time, $T_S$

Hold-off time, $T_{HO}$

Bit period, $T_{bit}$

Packet length, $t_{packet}$

Data in

Buffer

Data out

Control

Packet Bit rate

$B_{packet} = \frac{1}{T_{bit}}$

Information rate

$B_{info} = B_{packet} \cdot \frac{t_{packet}}{T_{HO}}$

Minimum time between incoming packets
Optical Packet Switch

First-In-First-Out (FIFO)

Input Buffers

Demux

Wavelength-Interchanging Cross Connect

Mux

Input Fibers

Outgoing Packets

Output Fibers

Incoming Packets

Output Buffers

Single Input

First-In-First-Out (FIFO)

Single Output
Optical Packet Switch

First-In-First-Out (FIFO)

Multiple Inputs

Output Buffers

Wavelength-Interchanging Cross Connect

Multiple Inputs

Single Output

Output buffering:
- optimum contention resolution
- more complicated than input buffering
Single-Input Single-Output FIFO

M cascaded delay lines with controllable delays

For acceptable performance, M > 20
FIFO Buffer Using Controllable Delay Lines

Group Velocity

\[ V_{g1} \]

\[ V_{g2} \]

Packet 1

\[ P_0 \]
FIFO Buffer Using Controllable Delay Lines

Call to Read Packet 1

Group Velocity

\[ V_{g1} \]

\[ V_{g2} \]

Packet 1

\[ P_0 \]

Note: info packet \[ \rightarrow \] HO packet

\[ T_t = \]
FIFO Buffer Using Controllable Delay Lines

Group Velocity

\[ V_{g1} \]

\[ V_{g2} \]

\[ 0 \quad x_N \quad x_M \]

Packet 1

Note: \[ T_{HO} = t_{packet} ; B_{info} = B_{packet} \]
FIFO Buffer Using Controllable Delay Lines

Note: \( T_{HO} = t_{\text{packet}} \), \( B_{\text{info}} = B_{\text{packet}} \)
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FIFO Buffer Using Controllable Delay Lines

Group Velocity

\[ V_{g1} \]

\[ V_{g2} \]

\[ 0 \]

\[ x \]

\[ x_M \]

Note: \[ T_{HO} = t_{packet} \]; \[ B_{info} = B_{packet} \]
Multiple-Input Single-Output FIFO

Control signals

FIFO

FIFO

FIFO

All FIFO’s provide full delay

10 – 100 Inputs

Switch

200 – 10,000 Delay lines

Key issues:
- Delay line utilization (i.e. “void” filling)
- Complexity of control
Optical Pulses in Slow Light Delay Line

Group Velocity Profile

- **Input Region**
  - $v_g$ (Group Velocity)
  - $v_{g1}$
  - $v_{g2}$

- **Slow Light Region**
  - $L = \frac{L}{v_{g2}}$

- **Output Region**

Field

- $L_{in}$
- $L_{bit} = \frac{v_{g2}}{B_{packet}}$

Delay

\[ T = \frac{L}{v_{g2}} \]

Capacity

\[ C = \frac{L}{L_{bit}} \]

Delay-Bandwidth Product

\[ T \cdot B_{info} = T \cdot B_{packet} = L \cdot B_{packet} / v_{g2} = C \]
# Fundamental Limitations of Ideal Slow Light

<table>
<thead>
<tr>
<th>Max. Delay-Bandwidth Product</th>
<th>Minimum Bit Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L(n_{avg} - n_{min}) )</td>
<td>( \alpha L \cdot \frac{\tau_{abs}}{\tau_{bit}} )</td>
</tr>
<tr>
<td>( \lambda_0 )</td>
<td>( \frac{\lambda_0}{(n_{avg} - n_{min})} )</td>
</tr>
</tbody>
</table>

\[ v_{g2} = \frac{c}{n + \omega \frac{dn}{d\omega}} \]

Information bandwidth

\[ 2\pi B_{\text{packet}} \]
Two Classes of Slow Light Delay Line

Class A
- Group velocity profile does not change while data stored
- Data enters and leaves slow-light regions across discontinuities
- All previous examples

Class B
- Bandwidth of medium changed adiabatically with time
- Group velocity changes while data is stored
Characteristics of Class B Slow Light

\[ E(t + dt) = E_0 e^{j(\omega + \delta\omega_s)(t+dt)} \]

\[ \delta\omega_s = (\omega - \omega_0) \frac{d(\omega_{\text{max}} - \omega_{\text{min}})}{\omega_{\text{max}} - \omega_{\text{min}}} \]
Class A and Class B Slow Light

A: “Conventional” - Slow-down in Space

- Group Velocity
- Information Bandwidth
- Bit Period
- Bit Length

= Period x Velocity

B: Adiabatic - Slow-down in Time

- Group Velocity
- Information Bandwidth
- Bit Period
- Bit Length

= Period x Velocity

Tucker et al., JLT, 23, 2005
Car Analogy – Class B Slow Light

“Conventional” Slow Light

100 km/h Speed Limit

20 km/h Speed Limit

Adiabatically-Slowed

slow together

(Bitrate Reduced)

Dangerous Adiabatic Driving

(Bitrate Unchanged)

Solution: Toll Plaza
Operation of Class B FIFO Delay Line

Note: $T_{HO} > t_{packet} ; B_{info} < B_{packet}$
Mixed Delay Lines

Increased Slow-Down Factor

- Increased tuning range (product of tuning ranges)
- Smaller bandwidth constriction in Class B section

Tucker et al., *JLT*, 23, 2005
Class A and Class B Buffers

“Traditional” (Class A): FIFO

Adiabatically Compressed (Class B): FIFO

Tucker et al., JLT, 23, 2005
The Myth-Busters

Myth #1:
Class B Slow Light breaks through the limitation of the Delay-Bandwidth Product.

Myth #2:
Attenuation in slow light waveguides can always be overcome using optical gain.
Myth #1 Busted

Input Region

(Input Region) Slow Light Region

Delay

\[ T = \frac{L}{v_{g2}} \]

Capacity

\[ C = \frac{L}{L_{bit}} \]

Delay-Bandwidth Product

\[ T \cdot B_{info} = T \cdot \frac{B_{g2}}{B_{g1}} B_{packet} \]

\[ = \frac{L \cdot B_{packet}}{v_{g1}} \]

\[ = C \]

Same as in WG1

\[ v_g \]

Interval 1

\[ v_{g1} \]

\[ v_{g2} \]

\[ t_1 \]

\[ t_2 \]

\[ t_3 \]

Bandwidth

\[ B_{g1} \]

\[ B_{g2} \]
Requirements of Practical Optical Buffers

Advanced 100 Tb/s electronic router
1000 ports @100 Gb/s
250 ms buffering per port

Optical packet switch with 1000 ports,
250 ms buffering per port
using optical fibre delay lines

Total buffer capacity of 2.5 TB
~ $10^3$ RAM chips
< US$ 50k in cost
< 1 kW power dissipation

Total fibre length = 40 Gm
150 times distance from Earth to Moon!
Packet Switching with Reduced Buffering

Enachescu et al., ACM/SIGCOMM July 2005: Buffer size can be reduced

2 μs buffering per port (200 kb/port) ~20 packets @ 100 Gb/s

Total fibre length = 400 km (~ 400 m/port)

Buffering with fiber delay lines is a challenge
Is Slow Light a Viable Alternative?

100-Tb/s Optical Router (1000 ports @ 100 Gb/s)
(Input) buffer size: 20 packets (200 kb, or 2 μs) per port → 200 Mb total

Fiber
400 m/port, 400 km total
Storage Density: 1 bit / 2 mm

“Practical” Slow Light Waveguide
Slow-down factor = 100
4 m / port, 4 km total
Storage Density: 1 bit / 20 μm

Ideal Slow Light Waveguide
200 cm/port, 200 m total
Storage Density: ~1 bit / μm

~wavelength
Size Matters

**Ideal Slow Light Waveguide**

- Minimum bit area $\sim 5\lambda^2$ ($\lambda = \sim 1 \mu m$)

**CMOS (2018)**

- eDRAM cell area $80 \text{ nm} \times 80 \text{ nm}$

### Storage Density
- 150 Gbit/m² per wavelength

### Area
- 13 cm²
- 1.3 m²

### Capacity
- 200 Mbit
- 200 Gbit
- 1.3 mm²
- 13 cm²
Size Matters

“Practical” Slow Light Waveguide

- Minimum bit area ~ 50\(\lambda^2\)
  \((\lambda = \sim 1\ \mu m)\)

- Storage Density
  - 15 Gbit/m\(^2\)

- Area
  - 130 cm\(^2\)
  - 13 m\(^2\)

- Capacity
  - 200 Mbit
  - 200 Gbit

CMOS (2018)

- eDRAM cell area
  - 80 nm x 80 nm

- Capacity
  - 150 Tbit/m\(^2\)

- Area
  - 1.3 mm\(^2\)
  - 13 cm\(^2\)
Loss Happens

Fibre: ~0.2 dB/km

```
<table>
<thead>
<tr>
<th>In</th>
<th>Out</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>15 km for 3-dB loss</td>
</tr>
</tbody>
</table>
```

\[ e^{-1} \text{ absorption time} \approx 100 \mu s \]

“Low loss” Planar WG: 0.01 dB/cm

```
<table>
<thead>
<tr>
<th>In</th>
<th>Out</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3 cm for 3-dB loss</td>
</tr>
</tbody>
</table>
```

\[ e^{-1} \text{ absorption time} \approx 20 \text{ ns} \]

- 20 packets (2 μs) → ~0.1 dB
- 20 packets (2 μs) → 400 dB
- 0.0001 dB/cm (10 dB/km) → 4 dB
The Myth-Busters

Myth #1:

Class B Slow Light breaks through the limitation of the Delay-Bandwidth Product.

Myth #2:

Attenuation in slow light waveguides can always be overcome using optical gain.
Overcoming Attenuation with Optical Gain

**Stage 1**

- Attenuation $\alpha$
- Slow light waveguide
- Waveguide dispersion compensation
- Waveguide loss compensation
- Signal
- Noise
- $P_{\text{sat}}$

**Stage m**

- Attenuation $\alpha$
- Slow light waveguide
- Waveguide dispersion compensation
- Waveguide loss compensation
- Signal
- Noise
- $P_{\text{sat}}$

Two key limitations:
- Output SNR
- Amplifier Saturation Power ($P_{\text{sat}}$)

Tucker, *JLT*, 24, 2006
For 20 packets, require loss $< 0.005 \text{ dB/cm}$

$$N_{bit} = \frac{100\sqrt{B(\text{Gb/s})}}{\alpha(\text{dB/cm})}$$

Tucker, *JLT*, 24, 2006
Dispersion-Limited Buffer Capacity

![Graph showing buffer length (L) vs. bit rate, with amplitude and dispersion limits, and buffer capacities labeled as N_{\text{bit}} = 10k and N_{\text{bit}} = 100.]

Length of Stored Bit Versus Capacity

- Maximum (Slow-down factor = 1) and fiber
- EIT dispersion limit
- Coupled Resonator dispersion limit
- EIT amplitude limit
- Minimum (Ideal)

- Delay Line Capacity (b)
  - 1 μm
  - 10 μm
  - 100 μm
  - 1 mm
  - 10 mm

- Length of Stored bit
  - 1 μm
  - 10 μm
  - 100 μm
  - 1 mm
  - 10 mm

- 40 Gb/s
- 0.005 dB/cm
- 0.5 dB/cm
- \( P_{\text{sat}} = 10 \text{ mW} \)

- Tucker, JLT, 24, 2006
Ring Resonator Memory Cell

- Savchenkov et al., LEOS Summer Topical Meeting, 2004.
Coupling Coefficient

![Graph showing coupling coefficient over time with input and output pulses, and coupling coefficients labeled as $K_{\text{write}}$, $K_{\text{read}}$, and $K_{\text{store}}$.)

- Input Pulse
- Output Pulse
- Coupling Coefficient $K$
- Time

$K_{\text{write}}$, $K_{\text{read}}$, and $K_{\text{store}}$ are shown in the graph, indicating the coupling coefficients for write, read, and store processes, respectively.

- ER (Equilibrium Region) is marked in the graph.

The graph illustrates the behavior of the coupling coefficient over time, highlighting the dynamics of the system with input and output pulses, and the coupling coefficients for different operations.
RetentionPolicy

Input 5-ps pulse

Output Pulses

Normalized Amplitude

Retention Time ~ 800 ps

Time (ns)

Cavity Q = 2 \times 10^6

Simulation (VPI)
Resonator RAM

\[ Q_{\text{store}} = \frac{2\pi n_g}{\lambda (\alpha + K_{\text{store}} / L)} \]

Waveguide power loss \quad \text{Switch coupling coefficient} \quad \text{Cavity length}

Retention Time \quad = \quad \frac{Q_{\text{store}}}{2\pi f_0} = \frac{1}{v_g (\alpha + K_{\text{store}} / L)} = \text{Absorption Time}

<table>
<thead>
<tr>
<th>(\alpha) (dB/cm)</th>
<th>Switch Extinction ((L = 100 , \mu\text{m}))</th>
<th>(Q_{\text{store}})</th>
<th>Retention Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>&gt; 40 dB</td>
<td>5 \times 10^6</td>
<td>2 , \text{ns} (0.02 packets)</td>
</tr>
<tr>
<td>0.0001</td>
<td>&gt; 70 dB</td>
<td>5 \times 10^9</td>
<td>2 , \mu\text{s} (20 packets)</td>
</tr>
</tbody>
</table>
The Myth-Busters

Myth #1:
Class B Slow Light breaks through the limitation of the Delay-Bandwidth Product.

Myth #2:
Attenuation in slow light waveguides can always be overcome using optical gain.

Myth #3:
High Q resonators can break through the limitation of the Delay-Bandwidth Product.

Maximum delay = Retention time
Information rate (bandwidth) = 1/Retention time
Delay bandwidth product = 1
# Comparing Technologies for Packet Buffering

## Technology Comparison Table

<table>
<thead>
<tr>
<th>Technology</th>
<th>Fiber</th>
<th>Planar, Slow Light</th>
<th>Resonator</th>
<th>Holographic</th>
<th>CMOS-O/E/O</th>
</tr>
</thead>
<tbody>
<tr>
<td>Access Time</td>
<td>Structure-dependent</td>
<td>Structure-dependent</td>
<td>Small</td>
<td>~ 50 μs</td>
<td>200 ps</td>
</tr>
<tr>
<td>Retention Time</td>
<td>&gt; 500 μs</td>
<td>&lt; 5 μs</td>
<td>1-100 ns</td>
<td>→ ∞</td>
<td>&gt; 50 ms</td>
</tr>
<tr>
<td>Capacity (Packets)</td>
<td>&gt; 2,000</td>
<td>&lt; 20</td>
<td>&lt;&lt; 1</td>
<td>→ ∞</td>
<td>→ ∞</td>
</tr>
<tr>
<td>Energy/bit</td>
<td>~ 1 fJ</td>
<td>~ 1 pJ</td>
<td>~ 1 pJ</td>
<td>~ 1 pJ</td>
<td>~ 1 fJ</td>
</tr>
<tr>
<td>Physical Size</td>
<td>Very Large</td>
<td>Medium</td>
<td>Medium</td>
<td>Small</td>
<td>Very Small</td>
</tr>
<tr>
<td>Chirp Sensitivity</td>
<td>No</td>
<td>Small</td>
<td>Large</td>
<td>Large</td>
<td>No</td>
</tr>
</tbody>
</table>

## Notes
- **Show stopper**
- **Challenging**
Conclusions

- Limitations and capabilities of slow light buffers
  - Dispersion and attenuation
  - Delay bandwidth product (treat with care)
  - Storage density

- Requirements of practical optical buffers
  - Capacity limited to a few thousand bits, at best
  - Very low loss waveguides required

- There are no free lunches