The On-Off Fading Channel

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Why does \( p_1 = p_2 = 1/2 \) achieves capacity for Channel B but not for Channel A? It turns out to be because the transition probability table for Channel B is point-symmetric to its center. Interchange of the values of \( p_i \) and \( 1 - p_i \) for both \( i = 1 \) and \( 2 \), results in \( P(Y)_{p_1=p_2=p_2'} = P(-Y)_{p_1=1-p_2'} = P(Y)_{p_1=1-p_2} = P(-Y)_{p_1=p_2} = H(Y)_{p_1=1-p_2'} = H(Y)_{p_1=p_2} \). Another easily verified fact is \( H(Y|X)_{p_1=p_2} = H(Y|X)_{p_1=1-p_2} = H(Y|X)_{p_1=1-p_2'} = H(Y|X)_{p_1=p_2} \). Thus, for all \( q_1, q_2 \),

\[
I(X;Y)_{p_1=p_2=p_2'} = I(X;Y)_{p_1=1-p_2} = I(X;Y)_{p_1=1-p_2'} \tag{1}
\]

It can also be verified (with some difficulty) that \( I(X;Y) \) is concave on the closed convex set of vectors \((p_1,p_2)\), with \( 0 \leq p_1, p_2 \leq 1 \). This is in addition to the usual concavity on the joint distribution space. Suppose we have found \( \alpha = (p_1^*, p_2^*) \) that maximizes the mutual information. By symmetry (1), there must be a second maximum corresponding to \( \alpha' = (1-p_1^*, 1-p_2^*) \), having the same value. Concavity on this product space implies that all convex combinations, in particular \( (\alpha + \alpha')/2 \) result in mutual information that are no smaller which contradicts our assumption about \( \alpha \) being a maximum, unless \( \alpha = (1/2, 1/2) \). For Channel A however, all of the above is not true, because \( P(Y|X) \) does not have the required symmetry properties (similarly for Channel C).

It turns out that the type of symmetry that we have described here leads to the definition of an interesting class of multiple-access channels for which the uniform distribution is always optimal. This class is a type of generalization of the single-user symmetric channels in [5]. We now consider the \( M \geq 2 \), \( |X| \geq 2 \) extension of Channel D.

Definition 2 (Unique Outputs). The \( M \)-user on-off fading channel with unique outputs has \( X = \{x_1, x_2, \ldots, x_M\} \) such every sum of \( m \leq M \) input symbols is non-zero and unique.

Theorem 1. For a \( M \)-user on-off fading channel with unique outputs, the uniform distribution achieves capacity.

References


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