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G12: From Solver Independent Models to Efficient Solutions

Peter J. Stuckey

NICTA Victoria Laboratory

University of Melbourne

NICTA is proudly supported by:

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Australian Research Council

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Outline

- **G12 Project Overview**
- Developing Constraint Solutions
- Solver Independent Modelling
  - Zinc example and features
- Mapping models to algorithms
  - Cadmium mapping tentative examples
- Efficient Solutions
  - Mercury discussion
- Concluding Remarks

Underpants Gnomes Business Plan

- Phase 1: Collect underpants
- Phase 2: ??????
- Phase 3: Profit

G12 Project Plan

- Phase 1: Solver Independent Modelling
- Phase 2: ?????
- Phase 3: Efficient Solutions

G12 Overview

- G12: a software platform for solving large scale industrial combinatorial optimisation problems.
  - ZINC:
    - A language to specify solver independent models
  - CADMIUM:
    - A mapping language from solver independent models to solvers
    - A language for specifying search
  - MERCURY: (For our purposes)
    - A language to interface to external solvers
    - A language to write solvers
    - A language to combine solvers
    - Providing debugging support

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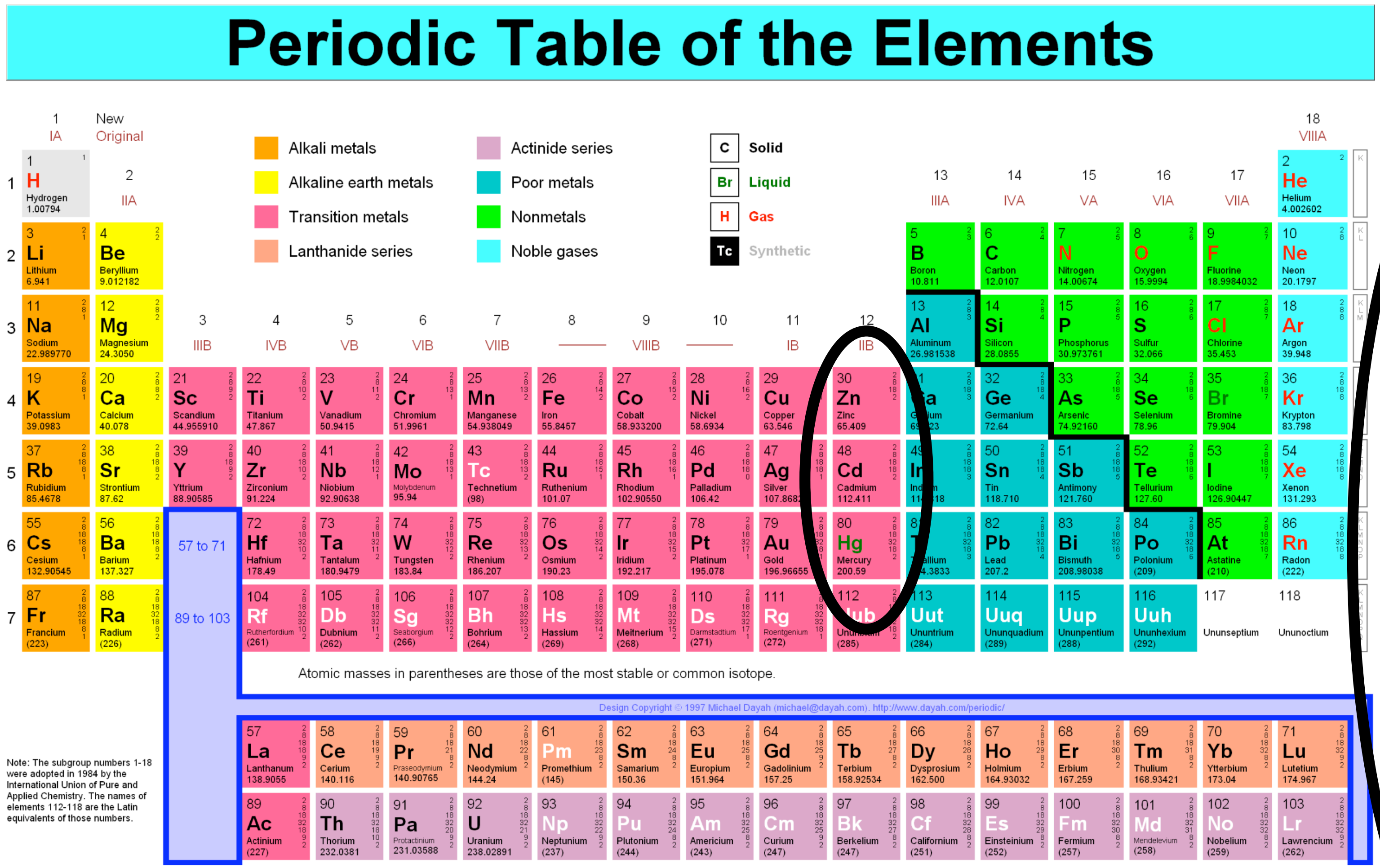
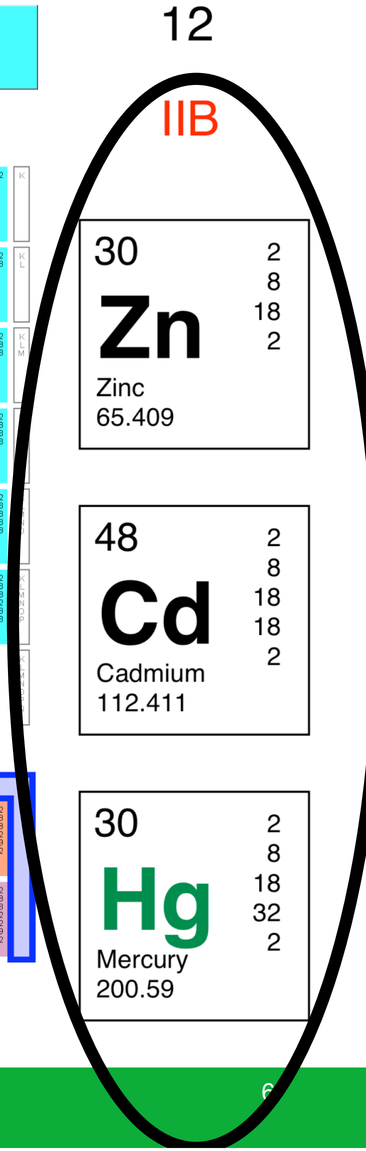
Group 12 of the Periodic Table





G12 Participants

- Peter Stuckey, NICTA Victoria
- Maria Garcia de la Banda, Monash University
- Michael Maher, NICTA Kensington (NSW)
- Kim Marriott, Monash University
- John Slaney, NICTA Canberra
- Zoltan Somogyi, NICTA Victoria
- Mark Wallace, Monash University
- Toby Walsh, NICTA Kensington (NSW)
- and others

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The Problem Solving Process

- "Find four different integers between 1 and 5 which sum to 14"
- Conceptual Model
  - User-oriented "declarative" problem statement
  - $\exists$ S. S $\subseteq$ {1..5} $\wedge$ |S| = 4 $\wedge$ sum(S) = 14.
- Design Model
  - Correct efficient algorithm
  - [W,X,Y,Z] :: 1..5, alldifferent([W,X,Y,Z]), W + X + Y + Z #= 14, labeling([W,X,Y,Z]).
- Solution
  - W = 2 $\wedge$ X = 3 $\wedge$ Y = 4 $\wedge$ Z = 5 → S = {2,3,4,5}

The Problem Solving Process

- Conceptual Model
  – User-oriented "declarative" problem statement

↓

- Design Model
  – Correct efficient algorithm

↓

- Solution

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From Conceptual Model to Design Model

- Conceptual Model: logical specification

S → {W,X,Y,Z}

Logical Transformation

  - Mapping the logical constraints to behaviour

|{W,X,Y,Z}| = 4 → alldifferent([W,X,Y,Z])

  - Adding a specification of search

→ labeling([W,X,Y,Z])

- Design model: algorithmic specification

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Behaviour: Choosing a Solving Technology

- Mixed Integer Programming (MIP)
  - strong optimization, lower bounding
  - limited expressiveness for constraints (linear only)
  - able to handle huge problems 1,000s of vars and constraints
- Finite Domain Propagation (FD)
  - strong satisfaction, poor optimization
  - highly expressive constraints
  - specialized algorithms for important sub-constraints
- DPLL Boolean Satisfaction (SAT)
  - satisfaction principally,
  - limited expressiveness (clauses or Boolean formulae)
  - effective conflict learning, highly efficient propagation
- Local Search: SA, GSAT, DLM, Comet, genetic algorithms
  - good optimization, poorer satisfaction (cant detect unsatisfiability)
  - highly expressive constraints (arbitrary functions?)
  - scale to large problems

Complete Solving Technologies

- Mixed Integer Programming (MIP)
  - strong optimization, lower bounding
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Incomplete Solving Technologies

- Good optimization, poorer satisfaction (cant detect unsatisfiability)
- Highly expressive constraints (arbitrary functions?)
- Scale to large problems
- Local Search:
  - simulated annealing
  - Lagrangian relaxation: DLM, GSAT, ...
  - Comet (language for local search methods)
- Population Methods
  - genetic algorithms
  - ant colony optimization, ...

Behaviour: Hybrid Solving Approaches

- Design model using two or more solving approaches
  - Only need partially model the problem in each part
  - pass constraints from one model to another
    - values of variables $W = 2$
    - bounds of variables $W \geq 3$
    - cuts $2X + 3Y + 4Z \leq 15$
  - pass upper or lower bounds from one technique to another
- Decompose the problem into two or more parts using different solving techniques
  - Dantzig-Wolfe decomposition, Column generation, ...

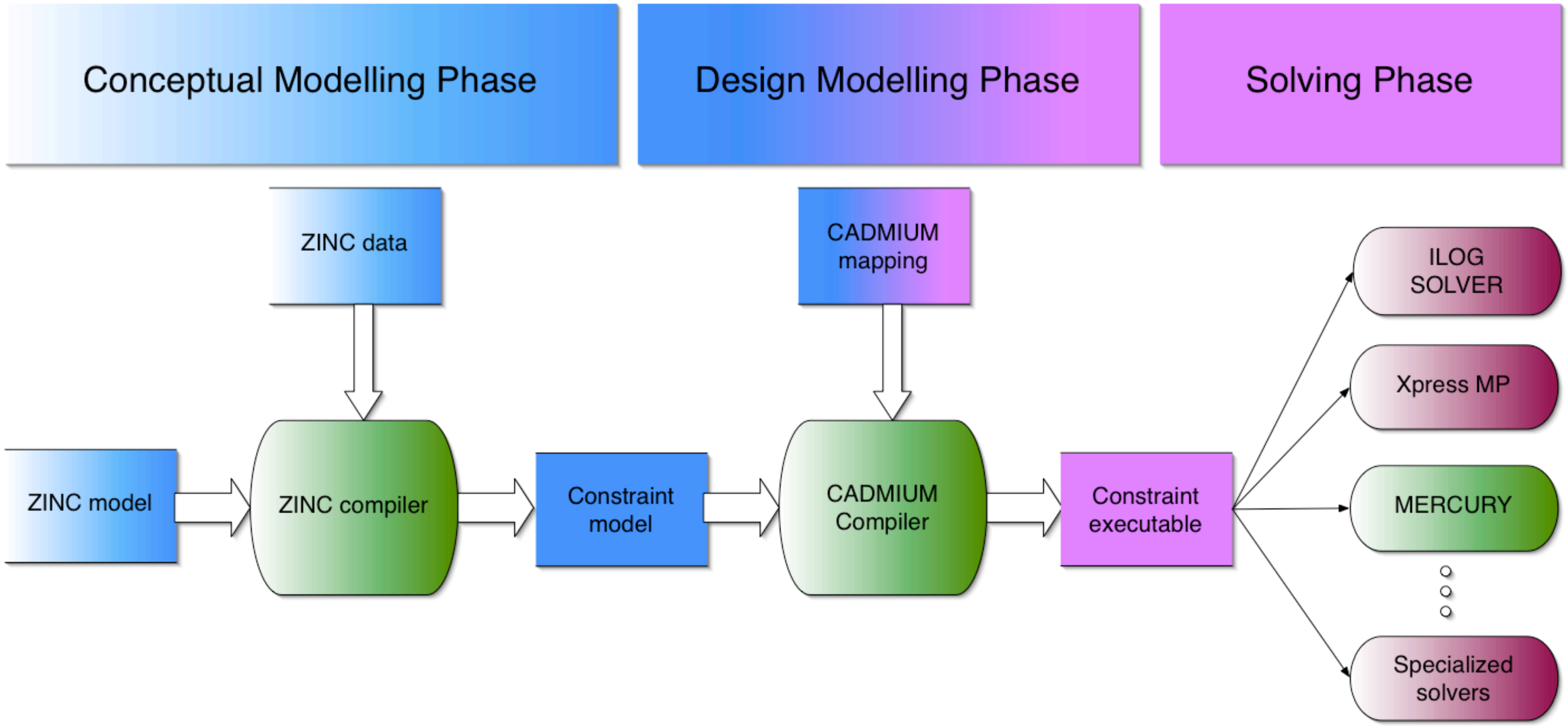
Search:

- Generic search strategy:
  - limited discrepancy search, first fail, maximum regret
  - symmetry breaking,
  - learn parameters
- Specific search strategy (programmed)
- Solving technology may restrict search
- Hybrid search:
  - Support the search of one method with another
  - Define heuristic function with one method
    - support limited discrepancy search of other method
  - Wide area local search, repair based methods

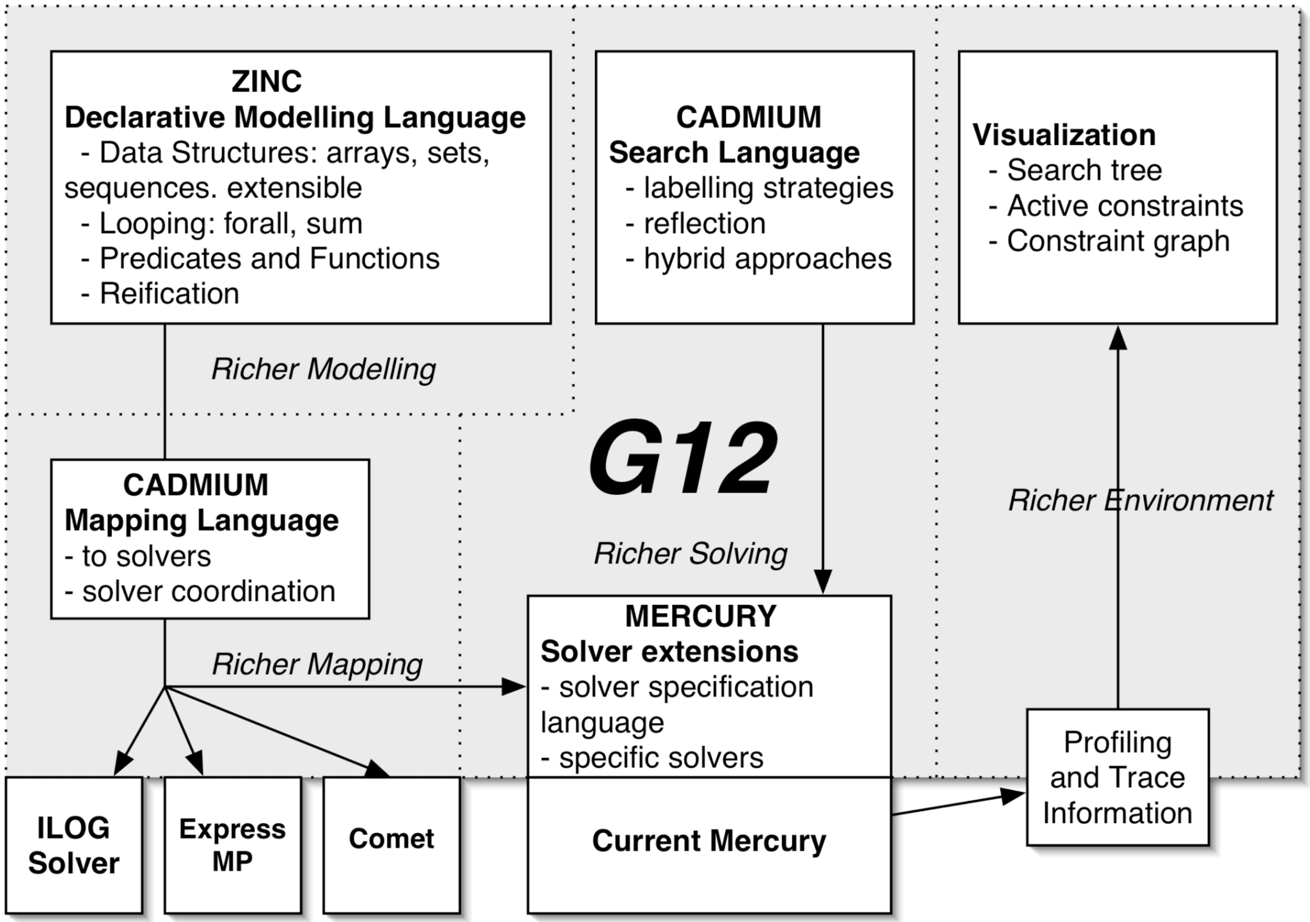
Environment

- The worst answer to a constraint problem?
  - *No*
- An even worse answer to a constraint problem
  - *execution does not terminate in days!*
- (Performance) Debugging the Design Model
  - visualization of the "active" constraints
  - visualization of the solver state (e.g. domains of variables)
  - visualization of the search
  - (preferably) mapped back to Conceptual Model
  - Hybrid approaches complicate this!

G12 development model



G12 Project Diagram



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Developing Constraint Solutions

- What modelling language is best to express the problem naturally?
- How do we map the problem to the most suitable combination of algorithms to solve it
- How do we support the search for the right algorithm, by high-level control and facilities to visualize and interact with the system as is solves?
- G12 aims to support these questions!

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G12 Goals

- Richer Modelling
  - Separate conceptual modelling from design modelling using
    - solver independent conceptual models
    - mapping from conceptual to design models
- Richer Mapping
  - extensible user defined mappings
  - hybridization of solvers
- Richer Solving
  - hybridization of search
- Richer Environment
  - visualization of search and constraint solving

Advantages of G12 model

- Checking the conceptual model
  - trusted default mappings give basic design model
  - test conceptual model on small examples this way
- Checking the design model
  - check optimized mapping versus trusted default mapping
- Remembering good modelling approaches
  - reuse of
    - model independent mappings
    - transformations/optimizations of design models
- Support for algorithmic debugging
  - reverse mapping to visualize in terms of the conceptual model

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- **Solver Independent Modelling**
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What is Solver Independent Modelling

- A model independent of the solver to be used
- Examples
  - .cnf format for SAT
  - AMPL for linear and quadratic programming
  - HAL program using solver classes
  - (?) ECLiPSe program (for eplex, ic, fd,etc solvers)
  - (?) OPL (although it essentially connects to one solver)
- All the above fix the form of the constraints by the model
- All except .cnf fix the "solving paradigm"
- More independent
  - ESRA [Uppsala]
  - Essence and Conjure [York]
    - model and transformation rules

Zinc: a solver independent modelling language

- mathematical notation like syntax (coercion, overloading, iteration, sets, arrays)
- expressive constraints (FD, set, linear arithmetic, integer)
- different kinds of problems (satisfaction, explicit optimisation, preference (soft constraints))
- separation of data from model
- high-level data structures and data encapsulation (lists, sets, arrays, records, constrained types)
- extensibility (user defined functions, constraints)
- reliability (type checking, assertions)
- simple, declarative semantics
- Zinc extends OPL and moves closer to CLP language such as ECLiPSe

Example Zinc model

- Social Golfers
  - Given a set of players, a number of weeks and a size of playing groups.
  - Devise a playing schedule so that
    - each player plays each week
    - no pairs play together twice
  - Many symmetries (ignore for now)
    - order of groups
    - order of weeks
    - order of players
    - ...

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Social Golfers in Zinc 0.1

- Type Declarations (to be read from data file)

```
enum Players = {...};
```

- Parameter Declarations (first 2 from data file)

```
int: Weeks;
int: GroupSize;
int: Groups = |Players| div GroupSize;
```

- Assertions on Parameters

```
assert("Players must be divisible by GroupSize")
   Groups * GroupSize == |Players|;
```

- Variable Declarations

```
array[1..Weeks,1..Groups] of var set of Player: group;
```

Social Golfers in Zinc 0.1

- Predicate (and Function) Declarations

```
predicate maxOverlap(var set of $E: x,y, int: m) =
   |x inter y| =< m;

predicate partition(list of var set of $E:sets,
                    set of $E: univ) =
    forall (i,j in 1..length(sets) where i < j)
        maxOverlap(sets[i],sets[j],0)
    /\  unionlist(sets) == univ;
```

Social Golfers in Zinc 0.1

- Constraints

```
constraint forall (i in 1..Weeks)(
   partition([group[i,j] | j in 1..Groups], Players) /\
   forall (j in 1.. Groups) (
         |group[i,j]| == Groupsize /\
         forall (k in i+1..Weeks; l in 1..Groups)
             maxOverlap(group[i,j],group[k,l],1)
   ));
class("redundant"):: constraint
  forall (a,b in Players where a < b)
    sum (i in 1..Weeks; j in 1..Groups)
        holds({a,b} subset group[i,j])
          =< 1;
```

Social Golfers in Zinc 0.1

```
int: Weeks;
int: GroupSize;
enum Players = {...};
int: Groups = |Players| div GroupSize;
assert("Players must be divisible by GroupSize") Groups * GroupSize = |Players|;
array[1..Weeks,1..Groups] of var set of Player: group;

predicate maxOverlap(var set of $E: x,y, int: m) =
    |x inter y| =< m;
predicate partition(list of var set of $E: sets, set of $E: universe) =
      (forall (i,j in 1..length(sets) where i < j)
          maxOverlap(sets[i],sets[j],0)
  /\  unionlist(sets) == universe;

constraint forall (i in 1..Weeks)(
    partition([group[i,j] | j in 1..Groups], Players) /\
    forall (j in 1.. Groups) (|group[i,j]| == Groupsize /\
        forall (k in i+1..Weeks; l in 1..Groups)
            maxOverlap(group[i,j],group[k,l],1)
));
class("redundant"):: constraint forall (a,b in Players where a < b)
  sum (i in 1..Weeks; j in 1..Groups) holds({a,b} subset group[i,j]) =< 1;
```

Zinc Features

- Types:
  - float, int, bool, string,
  - tuples, records (with named fields), discriminated unions
  - sets, lists, arrays (multidimensional = array of array of ...)
  - var type
    - arrays and lists of var types: `array [1..12] of var int`
    - set var type of nonvar type: `var set of bool`
  - coercion
    - nonvar type to var type: `float -> var float    (x + 3.0)`
    - ground sets to lists: `length({1,2,3,5,8})`
    - lists to one-dimensional arrays:
  - constrained types (assertions)

```
record Task = (int: Duration, var int: Start, Finish)
     where Finish == Start + Duration;
```

Zinc Features

- Comparisons
  - `==, !=, >, <, >= , =<`
  - generated automatically for all types (lexicographic)
- Reification
  - predicates are functions to var bool
  - Boolean operations:
    - `/\ (and), \/ (or), ~ (not), xor, =>, <=, <=>`
  - `ZeroOne = 0..1;`

    `function holds(var bool:b):var ZeroOne:h`
    - h is the integer coercion of the bool b
  - Anything can be "reified"
    - problem for solvers?

Zinc Features

- List and Set comprehensions
  - generators + tests must be independent of vars
  - `list of int: b = [2*i | i in 1..100 where ~(kind[i] in S)]`
  - shorthand
    - `sum (i in 1..Weeks; j in 1..Groups) holds(c) =< 1;`
    - `sum([ holds(c) | i in 1..Weeks; j in 1..Groups ]) =< 1;`
- Functions and predicates
  - local variables
  - (non-recursive) but foldl, foldr, zip
  -

```
function unionlist(list of var set of $E: sets):
                              var set of $E =
      foldl(union,{},sets)
```

  - starting point for mapping language Cadmium

Zinc Features

- Annotations
  - classification constraints: `class(string)`
    - (possible multiple) classifications for constraints
    - used for guiding rewriting, debugging
    - `class("linear") :: constraint x + 3*y + 4*z =< q;`
  - soft constraints: `level(int)` and `strength(float)`
    - lower levels are preferential
    - strength gives relative priority over levels
    - ```
      int: strong = 1;
      level(strong) strength(2.0):: constraint x < 2 /\ y < 9;
      ```
    - map to objective function if not supported by solver
- Objectives
  - `minimize/maximize` <arithmetic expr>
      ```

Zinc Status and Challenges

- Status
  - Initial language design
  - Type checker
  - Compiler in progress
- Challenges
  - Easy to use for mathematical programmers
    - Error messages, syntax
  - Symmetry specification
  - Multi parameter objective and/or robustness objective specification
  - Recursion?
  - Pattern matching

Zinc Challenges

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Cadmium

- Maps solver independent models to solvers
  - extension of Zinc
  - term rewriting/constraint handling rules like features
- Model independent transformations! (as far as possible)
- Trying to extract some of the “internal transformations” performed by solvers, to make them
  - visible
  - reusable
  - replaceable
- Also adds search strategy to model
  - not really discussed here

Cadmium Examples (VAPOR)

- Simple Defaults

```
map = bdd_sets.map;
```

- Overriding Defaults

```
map = bdd_sets.map;
predicate partition(list of var set of $E: sets,
                    set of $E: univ) =
    bdd_partition(sets, univ, [prop = cardinality]);
```

- Using Classes

```
class("redundant") :: c <=> delay(vars(c), c);
```

- Merging Constraints

```
map = bdd_sets.map;
partition(sets, univ), sorted(sets) <=>
          list of var set of $E: sets, set of $E: univ |
    bdd_and_prop(bdd_partition(sets,univ),bdd_sorted(sets));
```

Cadmium Examples (VAPOR)

- Variable Conversion
  - creates mapping `sat` from original variables to new variables

```
var set of $E: s <=> array[$E] of var bool: sat(s);
```

- Mapping of Functions and Predicates

```
function ||(array[$E] of var bool:s): var int =
                sum (e in $E) holds(s[e]);
function inter(array[$E] of var bool:s,t):
    array[$E] of var bool =  [ s[e] /\ t[e] | e in $E ];

function {}: array[$E] of bool = [false | e in  $E]; (?????)
```

- Refinement and Specialization of Constraints

```
s subset t <=> set of $E:s, var set of $E:t |
               forall (e in s) e in t;
maxOverlap(s,t,c1) \ maxOverlap(s,t,c2) <=>
               int: c1, int :c2, c1 =< c2 | true.
```

Cadmium Examples (VAPOR)

- Multiple levels of Mapping
  - Mapping to CNF (conjunctive normal form)

```
x and y == z <=> var bool:x,y,z |
                 (~z \/ x) /\ (~z \/ y) /\ (z \/ ~x \/ ~y)
partition(list of array[$E]of var bool:sets, set of $E:univ)=
      forall (e in univ) sum (s in sets) holds(s[e]) == 1
      /\ forall (s in sets) (s subset univ)
sum( [ holds(b) | b in bs]) <=>
    list of var bool:bs, var bool: b | sumb(bs)

sumb(bs) == c <=> sumb(bs) =< c /\ sumb(bs) >= c
sumb(bs) =< c <=> list of var bool: bs, int:c |
    forall (l in subsequences(bs,c+1)) exists (b in l) ~b;
```

  - `subsequences` in Mercury? or add recursion to Cadmium

Cadmium Examples (VAPOR)

- Multiple Solvers

```
m1 = bdd_sets.map;
m2 = sat_sets.map;
m2::|_| = _ <=> true;
channeling {
    forall (var set of $E:s; $E:e)
    m1::e in bdds(s) ==> m2::sat(s)[e] == true /\
    m1::e notin bdds(s) ==> m2::sat(s)[e] == false /\
    m2::sat(s)[e] == true ==> m1::e in bdd(s) /\
    m2::sat(s)[e] == false ==> m1::e notin bdd(s) /\
}
```

Mapping to Local Search (VAPOR)

```
var set of $E: s, |s| = c <=> int :c | array [1..c] of var $E: local(s);
set of $E: s <=> int:c = |s|,  array [1..c] of $E: local(s);

predicate subset(array[$R1] of $E: t, array[$R2]of var $E s) <=>
    forall (i in $R1) exists (j in $R2) s[j] == t[i];
predicate in($E: e, array[$R] of var $E:s) =
    exists (i in $R) s[i] == e

predicate partition(list of var array[$R] of $E: sets, set of $E: universe) =
    forall (e in universe)
            sum (i in 1..length(sets); j in $R) holds(sets[i][j] == e) == 1;

maxOverlap(_,_,1) <=> true

var int:f = sum [holds(c) | class("redundant") :: c ];
var int:p = sum [holds(c) | c = partition(_,_) ];

.. move definition ..
.. tabu list definition ..
.. search (using f) ..
.. debugging check (using p) ..
```

Mapping to Local Search (VAPOR)

- Variable and Parameter mapping

```
var set of $E:s, |s| == c <=> int:c | array [1..c] of var $E:lcl(s);
set of $E: s <=> int:c = |s| | array [1..c] of $E: lcl(s);
```

- Predicate mapping

```
predicate subset(array[$R1] of var $E: s, t) =
   forall (i in $R1) exists (j in $R2) s[i] == t[j];

predicate partition(list of var array[$R] of $E: sets,
                    set of $E: univ) =
  forall (e in univ)
    sum (i in 1..length(sets); j in $R) holds(sets[i][j]==e) == 1;

maxOverlap(_,_,1) <=> true
```

Mapping to Local Search (VAPOR)

- Defining Penalty Functions

```
violation(a =< b) <=> var int: a,b | max(0,a - b);
var int:f = sum [violation(c) | class("redundant") :: c ];
var int:p = sum [holds(c) | c = partition(_,_) ];
```

- Defining the algorithm

```
.. move definition ..
.. tabu list definition ..
.. search (using f) ..
.. debugging check (using p) ..
```


Cadmium Challenges ∞

- Specification: polymorphism, solver communication
  - model independent mappings (polymorphism)
  - solver communication
  - full hybridization
- Rewriting: control, confluence?, interaction with subtypes
- Search: Salsa, Comet, CLP
- Error messages: unmapped constraints, etc
- Reverse mappings?
- The last step
  - outputing the format required by an external solver

Cadmium Status and Challenges

- Status
  - many discussions
- Challenges ∞
  - Specification:
    - model independent mappings (polymorphism)
    - solver communication
    - full hybridization
  - Rewriting: control, confluence?, interaction with subtypes
  - Search: Salsa, Comet, CLP
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  - Mercury discussion and hybrid example
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Mercury

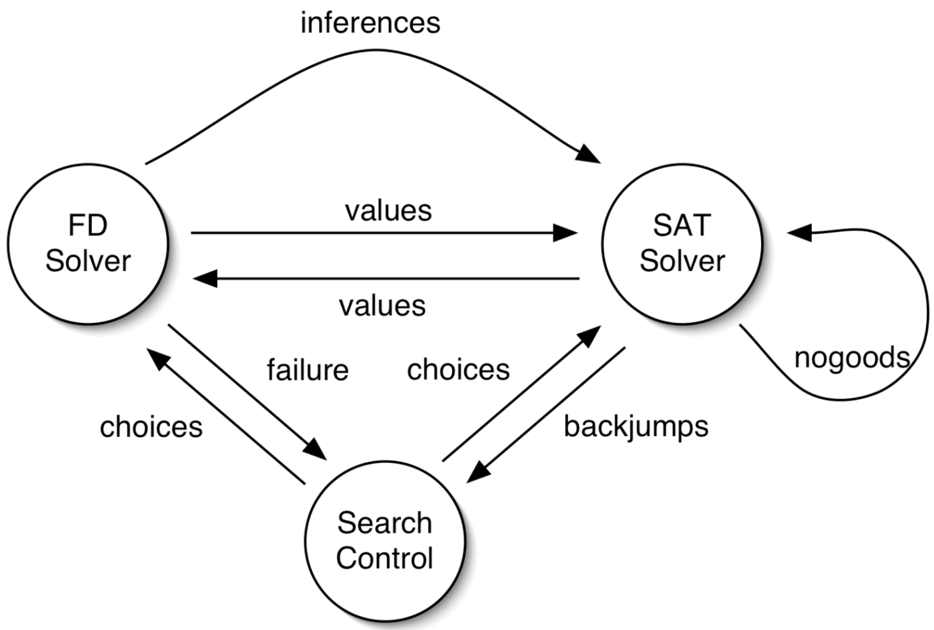
- Purely declarative functional/logic programming language
  - developed since October 1993 at University of Melbourne
  - designed for "programming in the large"
  - strong static typing: Hindley/Milner + type classes with functional dependencies + existential types
  - strong static moding (tracking instantiation of arguments)
  - strong static determinism (number of answers for predicates/functions)
  - strong module system
  - highly efficient, sophisticated compile-time optimizations

Extending Mercury

- No constraint solving (not even Herbrand)
  - added solver types to Mercury
    - Dual view of a type
      - External view: pure declarative solver variable
      - Internal view: data structure representing solver information
  - adding solvers to Mercury
    - herbrand, bdd_sets, sat (MiniSat), lp (cplex, clpr), fd
- Hybridization facilities (currently complete methods only)
  - essentially attach arbitrary code to solver events
    - variable is fixed
    - bounds changes
    - new cut/nogood generated

Mercury hybridization experiment

- bdd FD solver (JAIR 24)
- DPLL based SAT solver (MiniSAT)



BDD based solver

- CP2004, JAIR 24 (2005)
- Essentially a finite domain solver
  - represents variables by "packages of Boolean variables"
    - $\emptyset \subseteq S \subseteq \{1,2,3,4\}$ :: $1 \in S$, $2 \in S$, $3 \in S$, $4 \in S$
    - $0 \leq x \leq 3$ :: $x = 0$, $x = 1$, $x = 2$, $x = 3$ OR x mod 2 = 1, x >= 2
  - represents domains as Boolean formulae (ROBDDs)
    - $D(S) = \{\{1\}..\{1,3,4\}\}$ :: $1 \in S \wedge \neg(2 \in S)$
  - represents constraints as Boolean formulae (ROBDDs)
    - $|S| = x$ :: $(1 \in S \wedge 2 \in S \wedge 3 \in S \wedge \neg (4 \in S) \wedge x = 3) \vee \ldots$
- Propagates constraints using Boolean operations
  - D'(S) = exists x. $D(S) \wedge D(x) \wedge |S| = x$
- Highly competitive for finite set solving
  - not competitive for finite integer solving

SAT DPLL solver (MiniSAT)

- http://www.cs.chalmers.se/Cs/Research/FormalMethods/MiniSat/
- by Niklas Eén, Niklas Sörensson
- DPLL based SAT solver
  - watch literals
  - 1UIP nogood learning, conflict clause minimization
  - (improved) VSIDS dynamic variable order
  - incremental
- Winner of silver medals in 2 Industrial and 1 Handmade classes of SAT 2005
- With preprocessor SatELite winner of gold medals in all 3 Industrial and 1 Handmade classes

Hybridizing BDD and MiniSAT

- Variable to variable propagation
  - fixed variables in BDD <-> fixed variables in MiniSAT
- Scheduling
  - Unit propagation in MiniSAT is one "propagator"
  - higher priority than any BDD propagators
- Modelling
  - all constraints represented in BDD solver
  - NO constraints represented in MiniSAT!

Dynamic clausal representation

- Represent inferences of BDD propagators as clauses
  - $D(S) = \{\{1,2\},\{1,2,4\}\}$ :: $1 \in S \wedge 2 \in S \wedge \neg(3 \in S)$
  - $D(x) = \{0,1,2\}$ :: $\neg(x = 3)$
  - Propagating $|S| = x$
  - Newly inferred propositions
    - $\neg(4 \in S)$, $\neg(x = 0)$, $\neg(x = 1)$, $x = 2$
  - simple inferences
    - $1 \in S \wedge 2 \in S \wedge \neg(3 \in S) \wedge \neg(x = 3) \rightarrow \neg(4 \in S)$
    - $1 \in S \wedge 2 \in S \wedge \neg(3 \in S) \wedge \neg(x = 3) \rightarrow \neg(x = 0)$
    - ...
  - clausal representation
    - $\neg(1 \in S) \vee \neg(2 \in S) \vee 3 \in S \vee x = 3 \vee \neg(4 \in S)$
    - $\neg(1 \in S) \vee \neg(2 \in S) \vee 3 \in S \vee x = 3 \vee \neg(x = 0)$
    - ...

Minimal inferences

- A minimal reason for a new proposition *p*
  is a minimal subset of the reasons that ensure p hold
- Examples
  - $1 \in S \wedge 2 \in S \wedge \neg(3 \in S) \wedge \neg(x = 3) \rightarrow \neg(x = 0)$
  - minimal $1 \in S \rightarrow \neg(x = 0)$
  - $1 \in S \wedge 2 \in S \wedge \neg(3 \in S) \wedge \neg(x = 3) \rightarrow \neg(4 \in S)$
  - minimal $1 \in S \wedge 2 \in S \wedge \neg(x = 3) \rightarrow \neg(4 \in S)$
- Add minimal clauses
  - $\neg (1 \in S) \vee \neg(x = 0)$
  - $\neg (1 \in S) \vee \neg (2 \in S) \vee x = 3 \vee \neg(4 \in S)$
- Efficient BDD operations to determine minimal reasons
  - minimal unsatisfiable subset

Dynamic clause generation

- Propagation in the BDD solver represents inferences
  - Initially D(S) = {{} .. {1,2,3,4}}, D(x) = {0,1,2,3}
  - D(S) = {{1,2} .. {1,2,4}}, D(x) = {0,1,2}, |S| = x
    - gives
    - D(S) = {{1,2}}, D(x) = {2}
  - Simple inference
    - $1 \in S \wedge 2 \in S \wedge \neg(3 \in S) \wedge \neg(x = 3) \rightarrow \neg(x = 0)$
  - Minimal inference
    - $1 \in S \rightarrow \neg(x = 0)$
- Pass the inferences made to the SAT solver
  - $\neg (1 \in S) \vee \neg(x = 0)$

Experiments

- Social Golfers Problems
- Versus bounds propagation bdd set solver using a sequential smallest element is set search strategy (18/20)
  - simple inferences (18/20): fails 1/2 - 1 (0.70), time 4/5 - 2 (1.22)
  - minimal inferences:
    - just inferring (18/20): time 1 - 3 (1.76) (surprisingly low !)
    - using inferences in implication graph only (19/20): fails 1/35 - 1 (0.29), time 1/10 - 2 (0.78)
    - adding clauses (20/20): fails 1/157 - 1 (0.10), time 1/62 - 2 (0.30)
- Versus (improved) VSIDS search strategy from miniSAT (20/20)
  - miniSAT (16/20): fails 0.95 - 186 (10), time 1/14 - 58 (2.7)
  - dual model (20/20): fails 1/12 - 16 (2.3), time 2/3 - 13 (3.0)
  - sequential (20/20): fails 1/55 - 13 (0.52), time 1/5 - 10 (0.95)

Experiments

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    - adding clauses (20/20): fails 1/157 - 1 (0.10), time 1/62 - 2 (0.30)
- VSIDS search strategy (20/20)
  - versus miniSAT (16/20): fails 1/186 - 1.05 (0.10), time 1/58 - 14 (0.37)
  - versus dual model (20/20): fails 1/16 - 12 (0.44), time 1/13 - 3/2 (0.33)
  - versus sequential (20/20): fails 1/13 - 55 (1.9), time 1/10 - 5 (1.05)

What does it mean?

- Conflict directed backjumping in another guise?
- Related work
  - PalM, E-constraints: uses decision cuts not 1-UIP
  - Katsirelos and Bacchus CP2003: only forward checking, (appear to) only use FC inferences in implication graph
- finite domain propagation = clausal cut generation?

Outline

- G12 Project Overview
- Developing Constraint Solutions
- Solver Independent Modelling
  - Zinc example and features
- Mapping models to algorithms
  - Cadmium mapping tentative examples
- Efficient Solutions
  - Mercury discussion
- **Concluding Remarks**

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G12 Progress

- Zinc
  - Language design ✓
  - Type checker ✓
  - Starting compiler
- Cadmium
- Mercury
  - building new solvers: fd, generic propagation structures, value propagation
  - integrate solvers: bdd_sets, minisat, CPLEX ✓
  - solver types ✓

Other Aspects of the G12 Project

- Logical Transformations (Zinc2Zinc): dualization, etc
- Robust solutions: insensitive to change in parameters
- Search
- Master-subproblem decompositions: Benders, Lagrangian relaxation, column generation
- Population search: evolutionary algorithms
- Solver visualization
- Default mappings
- Online optimization
- Scripting

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Conclusion

- G12 is an ambitious project aiming to provide
  - Solver independent modelling
  - Model independent mappings from conceptual to design models
  - Easy experimentation of hybrid approaches
  - A good environment for exploring design models
- We have only just begun!
- The holy grail
  - Default mappings are good enough: only conceptual model

Advertisement

- Constraint Programming positions available
  - see http://nicta.com.au/jobs.html
  - positions in Melbourne (Network Information Processing) and Sydney (Knowledge Representation and Reasoning)
- G12 postgraduates needed
  - apply to University of Melbourne or University of New South Wales
- G12 visitors welcome
  - are you interested in some of the things discussed here?

END