NICTA Victoria Laboratories
Department of Computer Science and Software Engineering
The University of Melbourne
433-637 Constraint Programming
Second Semester, 2010

Linear Programming

1 Gauss-Jordan Eliminaton

Solve the following system of linear equations by Gauss-Jordan elimination

$$\begin{array}{rrrrcl} x_1 & +x_2 & +x_3 & & = & 2 \\ & 3x_2 & +x_3 & & = & 6 \\ x_1 & & & -x_4 & = & 0 \\ 2x_1 & -2x_2 & -2x_3 & +2x_4 & = & 8 \end{array}$$

Write down an example soluton.

2 Standard Form

Put the following system of linear inequalities in standard form:

$$\begin{array}{rrrcr} x_1 & +x_2 & +x_3 & \leq & 4 \\ x_1 & & & \leq & 2 \\ & & x_3 & \leq & 3 \\ & -3x_2 & -x_3 & \geq & -6 \\ x_1 & & & \geq & 0 \\ & x_2 & & \geq & 0 \\ & & x_3 & \geq & 0 \end{array}$$

3 Pivoting

Consider the tableau represented below (where all variables are non-negative):

$$\begin{array}{rcrrrr} z & = & -34 & +x_1 & +14x_2 & +6x_3 \\ \hline x_4 & = & 4 & -x_1 & -x_2 & -x_3 \\ x_5 & = & 2 & -x_1 & & \\ x_6 & = & 3 & & & -x_3 \\ x_7 & = & 6 & & -3x_2 & -x_3 \end{array}$$

perform pivots to find the maximal value of objective function z. If you choose the earliest entry variable you can avoid fractions until the last couple of steps!

4 Two Phase Simplex

Solve the problem (where all variables are non-negative):

$$\begin{array}{lrrrrl} \text{maximize} & -3x_1 & -2x_2 & +x_3 & & \\ \text{subject to} & & & & & \\ & x_1 & +x_2 & & & = 3 \\ & -x_1 & -3x_2 & +2x_3 & +x_4 & = 1 \end{array}$$

You should first introduce artificial variables and find a basic feasible solution. Once you have this remove the artificial variables and solve by pivoting. Give an optimal solution.

5 Corner Cases

Consider the tableau represented below (where all variables are non-negative):

$$
\begin{array}{rcrrrr}
z & = & 12 & +x_1 & -x_2 & -x_3 \\
\hline
x_4 & = & 0 & +2x_1 & -x_2 & -x_3 \\
x_5 & = & 1 & +x_1 & +2x_2 & -2x_3 \\
x_6 & = & 3 & & -x_2 & +x_3 \\
x_7 & = & 1 & & 3x_2 & -x_3
\end{array}
$$

What is the result of the the simplex algorithm. What is the maximal value of objective function z.

6 Standard Form Again

Put the following problem in standard form

$$
\begin{array}{rrrrcr}
\text{minimize} & 2x_1 & -2x_2 & & & \\
\text{subject to} & & & & & \\
& x_1 & +x_2 & & = & 3 \\
& x_1 & & & \geq & 3 \\
& & x_2 & & \geq & -1 \\
& & & x_3 & \geq & 0 \\
& x_1 & -x_2 & +x_3 & \leq & 6
\end{array}
$$

where the variables are not necessarily non-negative.