NICTA Victoria Laboratories
Department of Computer Science and Software Engineering
The University of Melbourne
433-637 Constraint Programming
Second Semester, 2010

Propagation

1 Propagation Rules

Gives bounds propagation rules for the constraint $x = abs(y)$ that are as strong as possible.

2 Consistency

1. Explain how a solver that maintains node, arc and hyper-arc (or domain) consistency will treat the constraint
$$Y \leq 5 \wedge Z \leq 5 \wedge Z \neq 1 \wedge X = Y \times Z \wedge Y \geq Z + 1 \wedge X \neq 9$$
given the initial domains are [0..10]. Give domains for X, Y, and Z after any changes, and explain why the change was made. [4 marks].

2. Explain how a solver that maintains bounds consistency will treat the constraint
$$Y \leq 5 \wedge Z \leq 5 \wedge Z \neq 1 \wedge X = Y \times Z \wedge Y \geq Z + 1 \wedge X \neq 9$$
given the initial domains are [0..10]. Give domains for X, Y, and Z after any changes, and explain why the change was made.

3 Idempotence

Recall the bounds propagation rules for $A = B \times C$ (when they are all non-negative) are

$$\begin{array}{lcl} A & \geq & min_D B \times min_D C \\ A & \leq & max_D B \times max_D C \\ B & \geq & \frac{min_D A}{max_D C} \\ B & \leq & \frac{max_D A}{min_D C} \\ C & \geq & \frac{min_D A}{max_D B} \\ C & \leq & \frac{max_D A}{min_D B} \end{array}$$

Is this propagator idempotent? Justify your answer.

4 Reification

Below are two constraint expressions defining $x = min(y, z)$. Write out how they will appear after decomposition by reification. Which do you think propagates more strongly. Justify your answer.

$$x \leq y \wedge x \leq z \wedge (x = y \vee x = z)$$

$$(y \leq z \wedge x = y) \vee (y > z \wedge x = z)$$

5 Alldifferent

Determine the resulting domain for applying the (a) naive and (b) domain consistent `alldifferent` propagator for the `alldifferent`($[x, y, z, t, u, v]$) to domain $D(x) = \{1, 2\}$, $D(y) = \{1, 2, 3, 4, 5\}$, $D(z) = \{1, 3, 4, 5\}$, $D(t) = \{3, 4, 5, 6, 7\}$, $D(u) = \{2, 3, 5\}$ $D(v) = \{2\}$.

For part (b) you should show diagrams illustrating a maximal matching, and the strongly connected components and reachable nodes.

6 Cumulative

Consider the cumulative constraint `cumulative`($[s_1, s_2, s_3, s_4, s_5]$, $[2, 5, 6, 3, 4]$, $[3, 2, 2, 4, 1]$, 4) And assume the all tasks must start after time 0 and end before time 12. Assume also the precedence that task 1 must end before task 2 begins i.e $s_1 + 2 \leq s_2$, and task 3 must end before task 4 begins, i.e. $s_3 + 6 \leq s_4$. Give the start time domains after propagation of the precedence constraints. Determine the compulsory parts of each task. Determine what propagation the timetable propagator makes on each task using this timetable. Give the start time domains after propagation.

More challenging Note that the propagation will create a new profile of compusory parts, which will in turn cause more changes in start time domains. Determine the final start times after all propagator from the cumulative propagator is finished.