NICTA Victoria Laboratories
Department of Computer Science and Software Engineering
The University of Melbourne
433-637 Constraint Programming
Second Semester, 2010

Mixed Integer Programming + Boolean Satisfiability

1 Branch and Bound

Solve the following problem using branch and bound.

maximize $x + 2y$
subject to

$$2x + 3y \leq 21$$
$$3x + 7y \leq 42$$

assuming x and y are non-negative integers. You should use MiniZinc LP backend to solve the linear programs by writing MiniZinc models where x and y are floating point variables and using `mzn -b mip` to solve the systems.

You should check your answer by writing an integer MiniZinc model.

2 Cutting Planes

The simplex form for the linear relaxation of the problem

maximize $x + 2y$
subject to

$$2x + 3y \leq 21$$
$$3x + 7y \leq 42$$

at optimality is

$$\begin{array}{rcrrr} z & = & 63/5 & -1/5s1 & -1/5s2 \\ x & = & 21/5 & +7/5s1 & -3/5s2 \\ y & = & 21/5 & -3/5s1 & +2/5s2 \end{array}$$

Generate the cutting plane that results from the equation for x and for y using the method in the slides.

3 Dual Simplex

After solving the simplex to find the optimal solution of the linear relaxation of the problem

maximize $x + 2y$
subject to

$$2x + 3y \leq 21$$
$$3x + 7y \leq 42$$

and then adding the constraint $x \geq 5$ we obtain an infeasible optimal tableau of the form

$$\begin{array}{rcrrr} z & = & 63/5 & -1/5s1 & -1/5s2 \\ x & = & 21/5 & +7/5s1 & -3/5s2 \\ y & = & 21/5 & -3/5s1 & +2/5s2 \\ s3 & = & -4/5 & -7/5s1 & +3/5s2 \end{array}$$

Use the dual simplex algorithm to solve the resulting problem.

4 Preprocessing

Give your best preprocessing result on the 0-1 problem with constraints:

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \geq 2$
$x_1 - x_2 + 2x_3 + x_7 \leq 4$
$-x_1 + x_2 + x_3 - x_5 \geq 1$
$-x_1 + 3x_5 + x_7 \geq 2$
$x_1 + 2x_9 - 2x_{10} + x_{11} \leq 1$
$2x_2 + x_7 + 3x_8 \leq 4$
$2x_4 + 3x_6 + 3x_{10} \leq 4$
$x_8 + x_9 + x_{10} + x_{11} \leq 2$

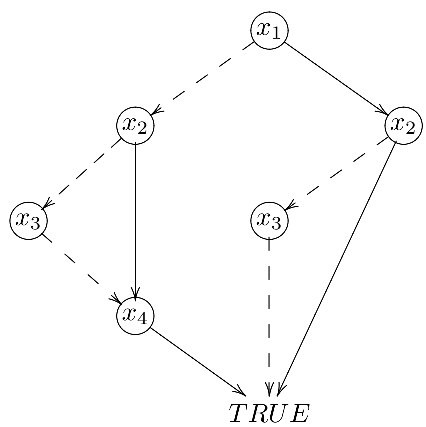
5 1UIP

Given the clauses: $\{-b1, -b3\}$, $\{-b1, b4\}$, $\{-b2, b3, -b6\}$, $\{-b2, -b4, b8\}$, $\{b3, -b5, -b8, -b10\}$, $\{b3, -b12, -b13\}$, $\{-b4, b12, -b15\}$, $\{-b5, b6, -b7\}$, $\{b6, -b16\}$, $\{b7, b9, b10\}$, $\{-b9, b13\}$, $\{-b9, b15\}$, $\{b10, b14\}$, $\{b11, b12, b13\}$, $\{b12, -b15\}$, Trace the building of the implication graph where the search strategy always chooses the lowest unset Boolean variable and sets it true, until failure is detected. Generate the 1UIP nogood, and backjump to the appropriate place and show the implication graph resulting after propagating the new nogood.

6 Cardinality Encodings

1. Draw the Binary Decision Diagram that encodes $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 \leq 3$

   An Binary Decision Diagram encoding $(x_1 \vee x_4) \wedge (x_2 \vee \neg x_3)$ is shown below

   Full arrows indicate the path if the variable is true, dashed arrows indicate the path if false. Missing arrows implicitly lead to a FALSE node. Note the assigment $x_1 = true$, $x_2 = false$, $x_3 = false$, $x_4 = false$ leads to $true$.

2. Draw the odd-even merge sorting network of size 8.

   Give a Boolean sorting network that encodes $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 \leq 3$

   Can you simplify the resulting network?

Work out how big (number of clauses, and total number of literals) the two encodings are. Which do you think is preferable?