Not covered in Bertoline
- Scalars, Line Vectors, Free Vectors
- Dot and Cross Products
- Vector specifications of lines and planes
- Solving problems
Vector fundamentals

- **Scalar**
  - quantity specified entirely by magnitude (e.g., speed 50 km/h)

- **Vector**
  - quantity specified entirely by magnitude and direction (e.g., velocity 50 km/h North)
  - a directed line segment
Components of a vector

Coordinate system $Oxyz$

Directed line segment $PQ$
Coordinates of $P = (x_1, y_1, z_1)$
Coordinates of $Q = (x_2, y_2, z_2)$

Vector $\overrightarrow{PQ} = \vec{A} = \vec{A}$

Components of vector $\vec{A}$
$a_1 = x_2 - x_1$, $a_2 = y_2 - y_1$, $a_3 = z_2 - z_1$

Unit vectors $\mathbf{i}, \mathbf{j}, \mathbf{k}$

$\vec{A} = a_1 \mathbf{i} + a_2 \mathbf{j} + a_3 \mathbf{k}$
Vector equation of a straight line

Given:
- a point $P_0$ on the line, defined by $\mathbf{r}_0$
- and a unit vector $\mathbf{u}$, in the direction of the line

Consider a point $P$ on the line, a distance $\lambda$ from $P_0$

Position vector of $P$ is:

$$\mathbf{r} = \mathbf{r}_0 + \lambda \mathbf{u}$$
Vector equation of a straight line

Note that any vector in the direction of the line, \( \mathbf{a} \) say, can be used to construct \( \mathbf{r} \):

\[
\mathbf{r} = \mathbf{r}_0 + \lambda \mathbf{u} = \mathbf{r}_0 + \lambda/\mathbf{a} \mathbf{a}
\]

i.e., \( \mathbf{r} = \mathbf{r}_0 + \mu \mathbf{a} \)

where \( \mu \) is proportional to the distance from \( P_0 \) to \( P \).
Vector equation of a straight line

The vector \( \mathbf{a} \) might be defined by two points on the line, \( P_0 \) and \( P_1 \):

\[
\mathbf{a} = \mathbf{r}_1 - \mathbf{r}_0
\]

Position vector of \( P \) is:

\[
\mathbf{r} = (1 - \mu)\mathbf{r}_0 + \mu\mathbf{r}_1
\]
The straight line through 
(1, 1, 1) in the direction 
\( \mathbf{u} = (1/\sqrt{3}, -1/\sqrt{3}, 1/\sqrt{3}) \) is

\[
\mathbf{r} = (1, 1, 1) + \lambda (1/\sqrt{3}, -1/\sqrt{3}, 1/\sqrt{3})
\]

i.e.,

\[
\begin{align*}
    x &= 1 + \lambda/\sqrt{3} = 1 + \mu \\
    y &= 1 - \lambda/\sqrt{3} = 1 - \mu \\
    z &= 1 + \lambda/\sqrt{3} = 1 + \mu
\end{align*}
\]

where \( \mu = \lambda/\sqrt{3} \)
Examples

The straight line through \((1, 0, 2)\) and \((2, 1, 1)\) is:
\[
r = (1 - \nu) (1, 0, 2) + \nu (2, 1, 1)
\]
i.e,
\[
x = 1 + \nu, \quad y = \nu, \quad z = 2 - \nu
\]
This line intersects the previous one
\[
r = (1, 1, 1) + \mu (1, -1, 1)
\]
when
\[
x = 1 + \nu = 1 + \mu \quad \Rightarrow \nu = \mu
\]
\[
y = \nu = 1 - \mu \quad \Rightarrow \nu = 0.5 = \mu
\]
i.e at: \((1.5, 0.5, 1.5)\)
Common perpendicular to two skew lines

Each line defined by:
• a point on the line, \( P_i \)
• a unit vector in the direction of the line, \( \mathbf{u}_i \) (\( i = 1, 2 \))

A vector perpendicular to both lines is \( \mathbf{u}_1 \times \mathbf{u}_2 \).
Hence, unit vector in direction of common perp is
\[
\mathbf{u} = \frac{\mathbf{u}_1 \times \mathbf{u}_2}{|\mathbf{u}_1 \times \mathbf{u}_2|}
\]

Vector from \( P_1 \) to \( P_2 \) is:
\[
\mathbf{r}_2 - \mathbf{r}_1 = P_1S \mathbf{u}_1 + SD \mathbf{u} - P_2D \mathbf{u}_2
\]

Component in \( \mathbf{u} \) direction is:
\[
(r_2 - r_1) \cdot \mathbf{u} = SD, \text{ since } \mathbf{u}_i \cdot \mathbf{u} = 0
\]

Hence
\[
SD = \frac{(\mathbf{r}_2 - \mathbf{r}_1) \cdot (\mathbf{u}_1 \times \mathbf{u}_2)}{|\mathbf{u}_1 \times \mathbf{u}_2|}
\]
Vector equation of a plane

- Plane defined by a point $P_0$ on the plane, and a unit normal vector $u$
- $P_0$ defined by position vector $r_0$ from origin $O$
- Consider arbitrary point $P$ in plane, with position vector $r$

$r - r_0$ is a vector in the plane, and hence normal to $u$

i.e., $(r - r_0) \cdot u = 0$

Then, $r \cdot u = r_0 \cdot u = p$

where $p$ is the perpendicular distance from origin $O$ to the plane
Example: equation to plane

- Consider plane
  - Passing through point $P_0$ with $r_0 = (1, 1, 1)$
  - Normal to $u = (1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$

- Point on plane P has $r = (x, y, z)$
- We have $r \cdot u = r_0 \cdot u = (1, 1, 1) \cdot (1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3}) = \sqrt{3}$

  - Hence $r \cdot u = x/\sqrt{3} + y/\sqrt{3} + z/\sqrt{3} = \sqrt{3}$

- i.e., equation to plane is $x + y + z = 3$