Modeling the Effects of Combustion Variability for Application to Idle Speed Control in SI Engines

Chris Manzie, Harry C. Watson and Paul Baker
The University of Melbourne
ABSTRACT

Combustion in the cylinder of a spark ignition engine, particularly under low load conditions, is subject to cycle-by-cycle variations due to factors such as mixture quality and quantity and internal exhaust gas recirculation. The major result of this phenomenon is an increase in the variability of indicated engine torque at a given engine operating point. Automotive control problems dealing with torque production at low engine loads, particularly the control of idle speed, rely on accurate information about the transfer functions of different engine subsystems, however combustion variability and the effect it has on torque production is often overlooked. In this paper we illustrate the effects that combustion variability at idle has on different transfer functions related to indicated torque, and propose new models for torque production at constant operating points. We also present a model of an in-line, six-cylinder, 4.0 litre Ford engine for use in idle speed simulations and control applications. The integration of combustion variability into an idle speed control framework is also discussed.

INTRODUCTION

Even at identical fixed operating conditions, torque production through combustion in the cylinder of a spark ignition engine is not an identical process at each combustion occurrence. The cyclic variations in the cylinder flow field, turbulence level and the distribution of fuel and air as well as the quantity, composition and thermodynamic state of residual gas in the cylinder will dictate how the flame front progresses through the cylinder volume. The mixture quality is dependent on a range of diverse factors including exhaust gas scavenging, non-uniform evaporation of the fuel film on the intake manifold walls and internal exhaust gas recirculation (IEGR). While the effect of these factors can be averaged over a number of cycles to produce an expected value at a given operating point, and indeed this is the case with mean value engine modeling as in [1], the cycle to cycle variations are not accounted for or eliminated by a mean valued approach. This is a major limitation of using a mean value model for simulation studies and controller design when cyclic variation is important. Variations in combustion, or torque production, directly influence the engine speed. It is therefore of most interest to examine the effects of cyclic variations when engine speed is trying to be controlled. One obvious example of this situation is during idle operation when engine speed should be maintained at a set level in order to optimize both the fuel economy and the vehicle noise vibration and harshness quality (NVH) despite possible torque disturbances from accessory loads.

It is also known that at low or no load, when the air-fuel mixture in the cylinder is at its lowest density, the cyclic variation in indicated torque is at its highest. An equivalent quantity to indicated torque, often used as a measure of combustion is the Indicated Mean Effective Pressure (IMEP). Figure 1 illustrates the coefficient of variation of IMEP (referred to as COVIMEP and calculated by dividing the standard deviation by the mean) for the engine of interest throughout the normal operating range. In this figure a distinct spike is apparent at low manifold absolute pressure (MAP) and engine speed.

Variations in combustion, or torque production, directly influence the engine speed. It is therefore of most interest to examine the effects of cyclic variations when engine speed is trying to be controlled. One obvious example of this situation is during idle operation when engine speed should be maintained at a set level in order to optimize both the fuel economy and the vehicle noise vibration and harshness quality (NVH) despite possible torque disturbances from accessory loads. It is also known that at low or no load, when the air-fuel mixture in the cylinder is at its lowest density, the cyclic variation in indicated torque is at its highest. An equivalent quantity to indicated torque, often used as a measure of combustion is the Indicated Mean Effective Pressure (IMEP). Figure 1 illustrates the coefficient of variation of IMEP (referred to as COVIMEP and calculated by dividing the standard deviation by the mean) for the engine of interest throughout the normal operating range. In this figure a distinct spike is apparent at low manifold absolute pressure (MAP) and engine speed.

Figure 1: Coefficient of variation of IMEP as a function of engine operating point, measured over 300 cycles.

The engine speed fluctuations induced by the torque variability may lead to unnecessary control action being taken by an optimal idle speed control strategy. For example, consider the situation when one low energy...
release combustion event, or even engine misfire, occurs. The engine speed will drop slightly, and the control algorithm may attempt to boost torque to overcome an anticipated increase in load, whereas in reality a larger than normal torque production will follow (because the reduced enthalpy of the residual gas causes the induction of more fresh charge). To avoid this sort of problem, the closed loop gain is often decreased, which has the undesirable effect of increasing controller response times. The variability of engine torque at constant operating points will be modeled in the following section.

The increased level of variability at the low manifold pressures in turn effects how the transfer functions of different engine characteristic functions develop. The accuracy of these transfer functions is very important in applications like optimal idle speed control, where the spark advance is required to be set in order to reject disturbances quickly and correctly. The development of the transfer functions in the low manifold pressure, high variability regions will also be examined in a later section of the paper.

Whilst in depth models for combustion variability have been proposed (e.g. [2] and the references contained within) their complexity precludes their use in a real time control environment. What control engineers seek is the ability to predict combustion variations and the subsequent indicated torque variations, without requiring additional engine sensors. The purpose of this paper is to develop an auto regressive model for these indicated torque fluctuations so that they can be:

i. identified as natural fluctuations in an engine model at idle
ii. included in the model based idle speed control algorithm
iii. applied in the algorithm to reduce their effect.

(Note: Readers unfamiliar with ARMA models will find an introduction in any linear filtering or digital systems textbook, e.g. [3]).

This is a baseline study on gasoline before moving to determine fuel effects with LPG and CNG.

INDICATED ENGINE TORQUE VARIABILITY AT FIXED OPERATING POINT

As detailed above, the variation in combustion efficiency leads to varying levels of torque being produced by the combustion of fuel and air charge in the cylinder. However the variation is not a completely random event, as a combustion with lower than expected torque production is often followed by a greater than average torque production the next time combustion occurs in the cylinder. The process has been previously hypothesized to be due to the temperature of the residual gas in the cylinder, termed the residual gas dynamic effect in [4], although it is far from completely understood. Briefly, the residual gas dynamic effect is explained as when one low-efficiency (i.e. slow or late) combustion occurs, the energy that was not converted into mechanical energy used to drive the piston instead raises the temperature of the exhaust gas. Through the process of internal exhaust gas re-circulation, the residual gases in the cylinder will be of higher temperature and therefore lead to better combustion during the next firing stroke for that cylinder.

As an example of the variability in torque production at fixed operating point, Figure 2 demonstrates the indicated torque variation in the number 6 cylinder for the engine running at approximately 600 r/min with a manifold absolute pressure of 35 kPa. The spark advance was retarded 10 degrees from MBT. The indicated torque is equal to zero if the work produced by the combustion equals the work required to drive the piston through its range of movement, hence a misfire will result in a negative indicated torque. Note that throughout this paper MBT refers to the minimum advance for best torque, and was determined from measurements of the indicated torque, rather than the brake torque. This approach was used since at low absolute manifold pressures the engine dynamometer was disconnected and brake torque could not be measured directly.

Little attention has been paid to modeling the phenomenon observed above in an ARMA form. The only previous attempt is by Ford and Collings [4], who had some success using an infinite impulse response (IIR) model of the form shown in the following equation.

\[ T_c \left( P_m, \alpha^{k+1} \right) = \frac{b_0 + b_1 z^{-1}}{1 + a_1 z^{-1}} T_c \left( P_m, \alpha^{k+1} \right) \]

\[ (1) \]

![Figure 2: Indicated torque for consecutive events at steady state conditions of 600 r/min and 35 kPa.](image)
Here \( \bar{T}_c \left( p_m^k, \alpha^{k+1} \right) \) represents the expected indicated torque at a given manifold pressure \( (p_m) \), spark advance \( (\alpha) \) operating point while \( T_c \left( p_m^k, \alpha^{k+1} \right) \) is the observed indicated torque at the same operating point. The parameter \( z^{-1} \) represents a time shift operator, i.e. \( z^{-1}T_c(k) = T_c(k-1) \).

In this paper we will consider modeling combustion variations by looking at the difference between the expected and indicated torque explicitly. In developing the ARMA model, it was considered that the previous two combustion events would be sufficient for the effects of the residual gas temperature to be noticed, and so the following two parameter model in equation (2) was proposed to describe the effect.

\[
\begin{align*}
    &\left[ T_c \left( p_m^k, \alpha^{k+1} \right) - \bar{T}_c \left( p_m^k, \alpha^{k+1} \right) \right] \\
    &= \begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} T_c \left( p_m^{k-1}, \alpha^{k} \right) - \bar{T}_c \left( p_m^{k-1}, \alpha^{k} \right) \\
    0 & 1 \end{bmatrix} \left[ T_c \left( p_m^{k-2}, \alpha^{k-1} \right) - \bar{T}_c \left( p_m^{k-2}, \alpha^{k-1} \right) \right] + \left[ \sigma_w \right] w_k
\end{align*}
\] (2)

In the above equation \( w_k \) represents zero mean, Gaussian-distributed noise with variance equal to 1, i.e. \( w_k \sim N(0,1) \). Conceptually, the model states that the current deviation from the expected torque will depend on the previous two deviations, with a white Gaussian noise term used to invoke a stochastic nature into the problem (without it the system would enter into a limit cycle whose stability depends on the parameters \( a \) and \( b \)).

In order to obtain estimates for the parameters \( a \) and \( b \) of this model, the indicated torque from a series of 150 to 200 consecutive combustion events at different fixed engine operating points was recorded. The expected combustion torque was calculated by taking the average of a sliding window of the surrounding indicated torques. Using a least squares analysis to minimize the effects of the noise term, the parameters at the operating points were determined, with several listed in the Table 1.

It is clear that the parameters appear fairly constant despite the changing operating points. Given that they will be multiplying the difference between expected and actual combustion torques, rather than the explicit torque as in the IIR model of equation (1), it seems reasonable to approximate these values as constant throughout the idle operating range. Figure 3 is an example of how the proposed model fits the data at one of the specified operating points.

### Table 1: Torque model parameter estimates at sample operating points

<table>
<thead>
<tr>
<th>Manifold Pressure (Abs)</th>
<th>Engine Speed (r/min)</th>
<th>( a )</th>
<th>( b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>36</td>
<td>600</td>
<td>-1.17</td>
<td>-0.55</td>
</tr>
<tr>
<td>40</td>
<td>600</td>
<td>-0.97</td>
<td>-0.46</td>
</tr>
<tr>
<td>40</td>
<td>700</td>
<td>-1.19</td>
<td>-0.59</td>
</tr>
<tr>
<td>45</td>
<td>700</td>
<td>-1.08</td>
<td>-0.55</td>
</tr>
<tr>
<td>55</td>
<td>700</td>
<td>-1.01</td>
<td>-0.56</td>
</tr>
</tbody>
</table>

A quantitative comparison was now made between the estimation schemes. The parameters for the IIR model were also determined via least squares estimation, and as a further comparison the average value only was also used as an estimator. Table 2 gives the root mean square (RMS) error and standard deviation of the error between the predicted and observed indicated torques for each of the three methods.

### Table 2: Quantitative comparison of different models

<table>
<thead>
<tr>
<th>Using mean only</th>
<th>IIR model. Equation (1)</th>
<th>Proposed model. Equation (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMS error (Nm)</td>
<td>3.956</td>
<td>2.983</td>
</tr>
<tr>
<td>Std Dev of Error (Nm)</td>
<td>3.960</td>
<td>2.995</td>
</tr>
</tbody>
</table>
These statistics indicate that the proposed two-parameter ARMA model is around 8% more accurate in both performance metrics (RMS error and standard deviation of the error) than the previous IIR model, which gives some validation for the new model proposed in (2).

A further point worth noting is that the model has been derived using only one cylinder. If an event is considered as a combustion in any of the cylinders in an engine with $N_{\text{cyl}}$ cylinders, then the model proposed in (2) will have to be updated to reflect this. Since we assume there is no correlation between torque deviations in different cylinders, the new model would look like equation (3).

$$\begin{align*}
  T_c \left( p_m^k \alpha^{k+N_{\text{cyl}}} \right) - T_c \left( p_m^k \alpha^{k-N_{\text{cyl}}} \right) - T_c \left( p_m^k \alpha^{k-N_{\text{cyl}}} \right) &= \begin{bmatrix} a & b \\ 1 & 0 \end{bmatrix} \times \\
  \begin{bmatrix} \sigma_w \\ 0 \end{bmatrix}
\end{align*}$$

The remaining unknown parameter in (2) and (3), $\sigma_w$, will now be investigated.

THE STANDARD DEVIATION OF THE INDICATED TORQUE

In this section we will investigate the standard deviation of the indicated torque, and attempt to formulate a relationship with operating point. The units of standard deviation will be Nm, rather than normalized to the mean as in the case with COV IMEP, for use in equations (2) and (3).

The variability of indicated torque was recorded over 300 engine cycles at a range of fixed operating points. From this data, the Minimum advance for Best Torque (MBT) were logged and surfaces generated for the MBT spark advance and the torque production at MBT. The spark advances in the obtained standard deviation data were then normalized to give retard from MBT. The families of curves in Figure 4 and Figure 5 illustrate the progression of standard deviation at a number of selected operating points.

From the three curve families in Figure 4 it appears that the standard deviation has almost no dependence on engine speed, since at a given manifold pressure the curves all appear to be roughly overlapping. The earlier graph in Figure 1 indicates that above a certain engine speed the COV IMEP starts to decrease, however we will restrict our investigation to the idle speed region of operation.

Upon examination of Figure 5 it appears that there is some dependence on manifold pressure in this region, although a strictly separated set of curves is only obvious at 600 r/min. In fact manifold pressure seems to exhibit quite a critical effect, since at 30 kPa (Figure 4a) the magnitude of the standard deviation is significantly larger than all other cases (and the indicated torque is smaller, compounding the effect of having a larger standard deviation). It also should be noted that in Figure 5 the 35 kPa curves are consistently at the top of the family shown (note for clarity the 30 kPa curves have not been included in these plots).

Furthermore, in both Figure 4 and 5 it can be stated quite categorically that there is a clear dependence on the spark retard from MBT, which appears almost linear until the spark advances past MBT.

![Figure 4: Standard deviation of indicated torque as a function of spark retard from MBT and engine speed at (a) 30 kPa, (b) 40 kPa, and (c) 50 kPa.](image-url)
the engine would not feasibly be run in a situation where the spark is advanced past MBT (the idle speed controller would apply a saturation function with MBT or some small retard from MBT as a limit) we will focus only on the spark retard from MBT. A linear dependence on spark retard is assumed, with the y-axis intercept dependant on the manifold pressure. An exponential dependence was assumed for manifold pressure to encompass the sharp increase once manifold pressure dropped below the critical level.

\[ \hat{\sigma}_w (\alpha_{\text{MBT}} - \alpha, P_m) = 2.4 + 0.22 (\alpha_{\text{MBT}} - \alpha) + 160 \exp(-0.12 P_m) \ (\text{Nm}) \]  

(4)

The presence of an exponential in the manifold pressure dependence deserves some explanation. Investigation of some of the relationships allows the following points to be made:

1. The residual mass fraction decreases linearly with increasing manifold pressure.
2. The heat capacitance effect gives a linear relationship between flame temperature and mass of the residuals in the cylinder.
3. The square of the laminar flame speed is exponentially related to the flame temperature, i.e. \( u_L^2 \propto \exp\left(-\frac{E}{RT}\right) \), where \( E \) is the activation energy, \( R \) is the universal gas constant and \( T \) is the absolute gas temperature.
4. The turbulent flame speed is proportional to the laminar flame speed for a given turbulence intensity.
5. The crank angle for total burn is inversely proportional to the turbulent flame speed (by flame travel over time) although in the region of operation the low degree of curvature means this is almost a linear relationship.
6. The torque produced by the combustion is linearly related to crank angle for total burn, i.e. \( T_c \propto -\theta_{\text{burn}} \).

Hence we can conclude that since the mass fraction of the residuals affects the engine torque exponentially, so in regions where there are high residual mass fractions (i.e. at low manifold absolute pressures) we expect to see an exponential increase in the combustion variability.

By the same reasoning we might expect the engine speed to have a similar impact on torque variability for a given MAP (since there is more time for accumulation of residuals through IEGR at lower engine speeds). Such a relationship is noted from the COVIMEP in Figure 1 along the 30kPa contour. However as the COVIMEP is normalized with respect to the mean torque produced, while the standard deviation, \( \sigma_w \), is a finite rather than non-dimensional quantity. Thus the increase in \( \text{COV}_{\text{IMEP}} \) as the engine speed is reduced is countered by a reduction in indicated torque along the 30 kPa contour. Therefore the finite quantity, \( \sigma_w \), may not reveal the increase in \( \text{COV}_{\text{IMEP}} \).
Furthermore, as the obtained data from Figures 4 and 5 lies within the range 600 to 1000 r/min, the range of engine speed (and the subsequent time range for IEGR) may be insufficient to notice the effect on torque variability.

The “level of fitness” for the model was measured using the following non-dimensional parameter, \( \eta \). This parameter is defined for a given set of \( N \) measured data points, \( y_i \), and modeled data points, \( \hat{y}_i \), as

\[
\eta = 1 - \frac{1}{\sqrt{\sum_{i=1}^{N} \left( 1 - \frac{\hat{y}_i}{y_i} \right)^2}}
\]

Values of \( \eta \) close to 1 indicate a very good fit while a value of 0 would indicate no correlation between the model and the observed data. For the model described by equation (4) it was found that \( \eta = 0.714 \). This is lower than expected but understandable given the non-smooth nature of the observed data in Figures 4 and 5.

LIMITATIONS OF THE MODEL

Clearly there are several limitations of the existing model shown in equations (2) and (3). Primarily that the first deviation from the expected value cannot be predicted, only the subsequent response to it. This is unavoidable given the nature of the process and the information available to the estimator.

Also there was some evidence in [4] that on the change of spark advance from a point close to MBT to a set point retarded far from MBT that the indicated torque dropped further than expected for the first event following the change, which the authors attributed to the residual gas dynamic effect. Attempts to reproduce this on the test engine are so far inconclusive, and appear to be hidden by the fluctuations in torque changing engine speed. More work is required to examine the dynamic effects of changing the spark advance.

THE EFFECTS OF COMBUSTION VARIABILITY ON TORQUE PRODUCTION

The influence of spark timing on torque production at a given operating point is well known. As mentioned above the maximum torque produced at a given engine operating point is achieved by setting the spark to MBT. When the spark is retarded from MBT, less torque is produced. As the spark retard from MBT is increased, the combustion variability generally also increases. In high variability regions, this causes a faster drop in average torque produced than would be seen in low variability regions. Similarly if the spark is advanced past MBT the combustion variability also increases, and hence the average indicated torque drops.

It is the variation in torque as a function of spark advance (or more conveniently spark retard from MBT, as this provides a more transparent view of the situation) that we are interested in here. The influence of spark advance on torque production is represented by a spark influence function, \( SI(\alpha - \alpha_{MBT}) \). Most of the previous approaches describing the spark influence function in the literature have centred around a look up table that is quadratic in spark retard from MBT, while the only functional representation that we are aware of is given in [5], where the spark influence function was proposed as the following equation.

\[
SI(\alpha - \alpha_{MBT}) = \frac{T_c}{T_{MBT}} = \cos^{2.875}(\alpha - \alpha_{MBT})
\]

However, at a given engine speed, the evolution of the spark influence function is not constant as demonstrated in Figure 6, where the spark influence function has been plotted by normalizing the torque production and spark advance to the relative MBT quantities.

It appears that as the load, or manifold pressure, is decreased the torque drops much faster from MBT. From the previous graphs it was observed that the variability in torque increased rapidly as the manifold pressure dropped below some critical point around 35 kPa. Here we note that at the higher manifold pressures (45 kPa, 50 kPa and 55 kPa) the curves are very similar while they start to diverge as the manifold pressure drops. An increase in torque variability would lead to a drop in average torque production, and this is noted in the curves in Figure 6. Invariably this means that equation (6) will not hold at low loads, which for idle speed control is the most critical time.

![Figure 6: Indicated torque normalized to MBT torque as a function of spark retard from MBT and manifold pressure at an engine speed of 1000 r/min.](image)
combustion variability, the indicated torque data from different engine speeds and manifold pressures was normalized with respect to the MBT torque for that operating point as the spark advance was varied to produce families of curves like that seen in Figure 6.

Since the cosine structure of equation (6) fitted the data well at high loads, it was decided to try and fit all the data by adjusting the power to which the cosine was raised. It was quickly apparent that the power term was nonlinear in manifold pressure, and the following form was derived from a least squares estimate and found to fit the observed data reasonably well for the spark influence function,

\[ \alpha - \alpha_{MBT} = \left( \frac{T_c}{T_{MBT}} \right)^{f(P_m)} \]  

(7)

where \( f(P_m) = 2.5 + 950 \exp(-0.115P_m) \)  

(8)

As in the case when the standard deviation of indicated torque was considered, the exponential form of equation (8) is not entirely unexpected given the exponential increase in combustion variability as the manifold pressure decreases. Whilst the form of equation (8) is not completely physically explained, it is useful for a quantitative analysis and modeling the test engine in a simulation environment, as well as in an idle speed control strategy.

The level of fitness was measured using equation (5) across the range of engine speeds for the proposed model and was found to be \( \eta = 0.922 \). This indicates a very good correlation between the model proposed in equations (8) and (9) and the actual data.

A COMPLETE IDLE SPEED MODEL

The additional information so far can be easily incorporated into an event-based idle speed model for the test engine as shown in Figure 7. An event domain model, with one combustion corresponding to one event, is typically used for idle speed problems since torque production is a discrete occurrence rather than a continuous one, and this technique is employed here.

The blocks of the model represent the following systems:

- \( m_{ai} \) represents the air flow into the manifold. At idle with closed throttle, the bypass air valve (BPAV) controls this quantity. At other times the throttle angle will be responsible for controlling the air flow into the manifold. Its transfer function is dependent on the driver circuit for the valve, and the BPAV duty cycle, \( u_{BP} \). Using a modified LM1949 injector driver circuit to control the BPAV led to the following transfer function

\[ m_{ai} = 0.2102u_{BP} - 2.2928 \text{ (g/s)} \]  

(9)

- \( m_{ao} \) represents the air out of the manifold (and into the cylinders). This is dependent on the operating point of the engine, for the engine of interest the following equation was derived:

\[ m_{ao} = 3.87 \times 10^{-1}(0.935P_m - 13.4)N \text{ (g/s)} \]  

(10)

Both the mass air flow into and out of the manifold are converted to the event domain units (g/event) after multiplying by \( \frac{20}{N} \) for the 6 cylinder engine.

- \( \Delta P_m \) is the state equation for intake manifold pressure, and updates the manifold pressure estimate. It is calculated from the ideal gas law and the requirement of conservation of mass in the intake manifold. For the given manifold volume of the engine, and assuming constant in the manifold the state equation is

\[ P_m(k+1) = P_m(k) + 1.69 \times (m_{ai} - m_{ao}) \text{ (kPa)} \]  

(11)

- \( f(P_m) \) represents the effect of manifold pressure on spark influence function, given by equation (8).

- \( MBT \) calculates the torque at MBT spark for the given operating point. For the engine of interest the following surfaces for MBT spark advance and torque at MBT were found to apply in the tested range of operation:

\[ \alpha_{MBT} = 27.03 - 0.322P_m + 0.00955N \text{ (°BTDC)} \]  

(12)

\[ T_{MBT} = 4.01P_m + 0.0413N - 126.4 \text{ (Nm)} \]  

(13)
These relationships were obtained after plotting the indicated torque as a function of spark advance over a wide range of engine speeds and manifold absolute pressure. As the torque curve is fairly flat about its peak, the minimum spark advance for best torque, $\alpha_{MBT}$, is taken as the leading edge of the flat section. The torque at this operating point and spark advance is taken as $T_{MBT}$. As both surfaces appeared to be linear in the parameters $P_m$ and $N$, a least squares estimation was used to determine the coefficients in the surface equations (12) and (13). The level of fitness was measured for both of these surfaces and was found to be $\eta = 0.952$ and $\eta = 0.921$ for the spark advance for MBT and the subsequent torque at MBT respectively. Since the level of fitness is above 0.9 in both cases this represents a very good fit. It should be noted that the torque actually rolls off slightly faster than linearly at low manifold pressure, once again due to an increase in the combustion variability as noted previously, and this accounts for the slightly lower fitness in the modeling of this parameter.

- $\bar{T}_c$ calculates the expected torque at the given operating point, given a certain spark advance and MBT torque. The spark influence function is given in equation (7). If variations in lambda are assumed then air fuel ratio will also affect the fraction of MBT torque that is produced, represented by an air fuel influence function. A standard result for the air fuel influence function, $AFI$, was found in [5] to have a sinusoidal dependence on air fuel ratio, $AFR$, up to a limit of 0.9. The function is represented in the following equation:

$$AFI(-) = \max \left\{ 0.9, \cos \left( 7.3834(AFR - 13.5) \right) \right\}$$ (14)

- $\sigma_u$ is the standard deviation of combustion torque at the given operating point and is calculated by equation (4).

- $T_c - T_c$ gives the actual produced torque and is calculated from equation (3).

- $\Delta N$ is the engine speed state equation. It updates the engine speed estimate based on the current indicated torque and the load torque, $T_l$, (which includes friction). The following equation updates the engine state:

$$N(k + 1) = N(k) + \frac{900}{\pi \cdot I \cdot N(k)} (T_c(k) - T_l(k)) \quad \text{(r/min)}$$ (15)

The inertia of the engine was estimated using deceleration testing in [6] to be $I = 0.145 \text{ kg.m}^2$.

The model is now compared to the actual engine by changing the spark advance and observing the effect of the changing engine speed and manifold pressure on indicated torque. The engine is initially run at 1000 r/min with a spark advance of 5 degrees BTDC. The manifold pressure is approximately 34 kPa. After 20 cycles (combustion events), the spark advance is changed to 15 degrees BTDC. The sudden shift closer to MBT causes a rise in indicated torque and a subsequent increase in engine speed. The increase in engine speed increases the air flow rate out of the manifold and manifold pressure is reduced. As a result the indicated torque decreases from its peak value and settles at a slightly higher value than at 5 degrees BTDC because of the small engine speed increase.

Figure 8: Comparison of measured and modeled indicated torque on change of spark advance from 5 degrees BTDC to 15 degrees BTDC at event 20 at a nominal engine speed of 1000 r/min. (a) The underlying model only i.e. no combustion variability is modeled. (b) Combustion variability included in model.

Figure 8a shows the measured data and the expected torque calculated from the underlying model, that is the complete model described above neglecting the effects of combustion variability. It is observed that the underlying model appears to have quite a close correlation to the observed data. In Figure 8b combustion variability is now included in the model and the result is compared to the measured data. An exact match between the measured and modeled data is not
expected given the stochastic nature of the combustion model described in (3), however there is suitable correlation in the spread of data to have confidence in using the presented model as a tool for idle speed simulation and control.

As a final point about the model derived here it should be noted that the camshaft in the test engine has 17 degrees of overlap at 0.25mm valve lift and (in a two valve per cylinder arrangement) may be considered as low valve overlap. For engines with significantly larger valve overlap, the effects of exhaust back flow into the cylinder may have some effects on combustion stability, and thus result in different values of the coefficients \(a\) and \(b\) in the model (3). Alteration of the location of the spark plug away from the centre of the combustion chamber may also influence burn duration and the variability coefficients.

Further research on an engine with inlet and exhaust valve cam phasing is required to properly establish a relationship between valve overlap and the parameters of the combustion variability model. The present engine has a single cam (but with variable timing) only.

**THE EFFECT OF COMBUSTION VARIABILITY ON IDLE SPEED CONTROL ALGORITHMS**

Incorporating the combustion variability effects into an idle speed control algorithm would involve different levels of difficulty. The effect on the spark influence function could easily be handled by simply including manifold pressure as one of the inputs to the calculation of spark advance.

On the other hand, to account for the torque deviations at a particular operating point would require knowledge of the indicated torque after each combustion event. The method presented in this paper for calculating indicated torque involves 180 samples per revolution at two crank angle degree increments, before a piecewise integration is performed to obtain the IMEP, which in turn may be used to calculate the indicated torque. This is not a feasible approach for real time operation in an existing production engine ECU, as the computations involved are too great and would require an in-cylinder pressure sensor in addition to the existing hardware.

Some researchers (eg [7]) have proposed methods for estimating the in-cylinder pressure from measurements of the instantaneous engine speed, which may prove beneficial. However increasing the number of cylinders considered from 4 to 6 with this method is likely to result in a drop in accuracy as there will be more overlap between the input forces and thus their speed influences. Further work is still required in this area, but availability of indicated torque after each combustion event will have further potential benefits such as online estimation of MBT surfaces given in equations (12) and (13), as well as the cyclic variability parameters through methods similar to those used in [8], which would serve to further improve engine operation.

**CONCLUSIONS**

Combustion variability has been investigated at low manifold pressures and found to have significant effects on torque production. This is of particular interest for idle speed operation, when the engine is necessarily at low manifold pressure and engine speed in order to minimize fuel consumption. Torque disruptions at idle are one factor that limits the minimum speed at which an engine can be safely run without compromising the vehicle NVH, so prior knowledge of forthcoming combustion variability could usefully be employed in an idle speed reduction and control strategy.

A new model for the variability of indicated torque has been presented. It uses an ARMA model of the previous deviations from the average, or expected, torque at a given operating point to predict the next deviation. The effect of torque variability on spark influence function has also been quantitatively examined. In both cases it was found that there appears to be a critical manifold pressure below which the indicated torque variability increases exponentially, and new models for the standard deviation of indicated torque at constant operating point and the spark influence function have been proposed to simulate this effect.

An idle speed model has been presented (for constant air fuel ratio operation) that incorporates the effects of combustion variability. The issues associated with using the extra information in the context of an improved idle speed control algorithm have also been discussed.

**ACKNOWLEDGMENTS**

This research was supported with the assistance of an Australian Research Council (ARC) grant. The authors would also like to thank Gavin Dober for his useful comments.

**REFERENCES**


CONTACT

For further information contact Chris Manzie at c.manzie@ee.mu.oz.au.

DEFINITIONS

\( \alpha \) Spark advance, degrees BTDC.

\( \alpha_{MBT} \) Spark advance for MBT, degrees BTDC.

\( AFI() \) Air fuel influence function.

\( AFR \) Air fuel ratio.

\( \sigma_w \) Standard deviation of indicated torque, Nm.

\( \text{COV}_{\text{IMEP}} \) Coefficient of variation of IMEP.

\( I \) Effective inertia of the engine, kg.m\(^2\).

\( \dot{m}_{ai} \) Mass flow of air into manifold, g/s.

\( \dot{m}_{ao} \) Mass flow of air out of manifold, g/s.

\( \eta \) Level of fitness indicator.

\( N \) Engine speed, r/min.

\( N_{cyl} \) Number of cylinders.

\( P_m \) Manifold absolute pressure, kPa.

\( SI(\cdot) \) Spark influence function.

\( T_c \) Indicated torque, Nm.

\( \bar{T}_c \) Expected indicated torque, Nm.

\( T_{MBT} \) Indicated torque at MBT, Nm.

\( u_{BP} \) BPAV duty cycle, %.

\( w_k \) I.I.D. Gaussian variable at time \( k \).

ACRONYMS

ARMA Auto-regressive moving average

ABDC After bottom dead center

ATDC After top dead center

BBDC Before bottom dead centre

BTDC Before top dead centre

BPAV Bypass air valve

IEGR Internal exhaust gas re-circulation

IID Independent and identically distributed

IIR Infinite impulse response

IMEP Indicated mean effective pressure

MAP Manifold absolute pressure

MBT Minimum advance for best torque

NVH Noise, vibration and harshness

RMS Root mean square

APPENDIX - SPECIFICATIONS FOR THE TEST ENGINE (FORD AU FALCON)

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Displacement</td>
<td>3.9835 litres</td>
</tr>
<tr>
<td>Bore and stroke</td>
<td>92.26 x 99.31 mm</td>
</tr>
<tr>
<td>No. cylinders</td>
<td>6, inline</td>
</tr>
<tr>
<td>Valves</td>
<td>2 per cylinder</td>
</tr>
<tr>
<td>Valve timing: Int.</td>
<td>Opens at 12 degrees BTDC</td>
</tr>
<tr>
<td></td>
<td>Closes at 72 degrees ABDC</td>
</tr>
<tr>
<td></td>
<td>Exh. Opens at 58 degrees BBDC</td>
</tr>
<tr>
<td></td>
<td>Closes at 24 degrees ATDC</td>
</tr>
<tr>
<td>Maximum Torque</td>
<td>370 Nm @ 4200 r/min</td>
</tr>
<tr>
<td>Maximum Power</td>
<td>172 kW @ 4800 r/min</td>
</tr>
</tbody>
</table>