A control-oriented model for cold start
operation of spark ignition engines

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Abstract: The use of mean value models in replicating the key characteristics of automotive
powertrains has been well established. There has been considerable success in the application
of these models to controller design, with improved emissions and performance the primary
benefits. However, these low order models typically must make certain assumptions about the
engine - with constant temperature operation a standard approach. As economy and emissions
at every point in the drive cycle become scrutinised, the cold start operation of the engine
becomes more critical and the constant temperature assumption is limiting. This paper seeks
to develop an engine model framework for capturing the temperature transients and gaseous
concentrations throughout the engine. It is intended for use in controller design and optimisation
studies that incorporate the engine warm up period.

Keywords: Engine modelling, engine systems, reduced-order models, automotive control,
engine emissions

1. INTRODUCTION

The characteristics of automotive engines following a cold
start make this region of engine operation one of the
most difficult. Fuel used to overcome engine friction at
idle is typically two to three times higher relative to a
fully warm engine, while emissions will be high until the
three way catalyst has lit off requiring high exhaust gas
temperatures among other considerations. As emissions
and fuel consumption requirements become increasingly
strict, it is apparent there is a need to model engine
characteristics during a cold start transient.

In general, the approach favoured by many control and
optimisation researchers is to use low order models that
sufficiently capture the basic physics to reduce the need
for significant storage of black box relationships. The
development of low order models for automotive engine
poweredtains began in the 1970s with the work of Hazell
and Flower (1971), but it was not until the 1980s that
the potential to use these models as an effective simula-
tion and control tool became apparent in work such as
that conducted by Dobner (1980), Moskwa and Hedrick
(1992), and Cho and Hedrick (1989). In the early 1990s,
these types of models were termed ‘mean value models’
by Hendricks and Sorenson (1990) and typically based on
simple physical models with approximations that included
treating the pulsating flows through the engine as contin-
uous flows averaged over an entire engine cycle and the
assumption of constant temperature operation.

The success of these models in dealing with various power-
train control strategies is well established with a multitude
of examples in the literature. However, there are several
limitations that preclude the use of existing models in
cold start studies. Most critically, of course, is the typ-
ical absence of temperature as an engine variable. The
removal of a constant temperature assumption requires
the addition of several new states within the model, but
is necessary in order to capture thermal transients. The
pioneering work described in Kaplan and Heywood (1991)
and Sandoval and Heywood (2003) form an excellent basis
in this domain.

Furthermore, as pre-catalyst emissions become an impor-
tant consideration for catalyst light off and modelling, a
framework to incorporate species concentrations within
the low order model is required along with the exhaust
gas temperature. Previous attempts in this regard have
typically been of high order or are phenomenologically
based, e.g. Fiengo et al. (2003).

The aim of this work is to extend the mean value models
through the inclusion of thermal characteristics, as well as
proposing a framework for modelling gas concentrations
through the engine. The model developed is intended
to serve as a validated starting point for further model
reduction to be used in controller design and optimisation
studies. To ensure portability, there is a high emphasis on
physics-based aspects in the model.

2. ENGINE MODEL

The automotive engine can be split into three intercon-
ected physical domains - the mechanical domain where
the dynamics of engine speed are considered, the thermal
domain describing the temperature dynamics of the fluids flowing throughout the engine system and the thermodynamic domain containing both the combustion process and the gaseous pressure and temperature dynamics within the various control volumes in the engine. While the main contribution of this work is incorporating the thermal domain, for completeness and to clearly demonstrate the inter-domain linkages, the complete model is described in this section. The overall system being modelled is represented in the block diagram of Figure 1.

Fig. 1. Overall system model

2.1 Mechanical Domain

The crankshaft velocity, \( \omega_{\text{crank}} \), is dictated by Newton’s second law, with the torques supplied by the individual combustion processes in each cylinder and load torques emanating from mechanical friction and brake torque

\[
J_{\text{total}} \dot{\omega}_{\text{crank}} = \sum_{i=1}^{N_{\text{cyl}}} \tau_{\text{crank},i} - \tau_{\text{fric}} - \tau_{\text{brake}} \tag{1}
\]

Assumption 1. The inertia of the crankshaft, \( J_{\text{total}} \), is sufficiently large to ensure that the speed fluctuations arising from individual cylinder combustion events are small in comparison to the mean engine speed over the cycle.

Remark 1. A standard result of Assumption 1, there is no need for consideration of individual cylinder combustion events, and thus a single approximation of indicated torque may be utilised. Consequently (1) may be reduced to:

\[
J_{\text{total}} \dot{\omega}_{\text{crank}} = \tau_{\text{crank}} - \tau_{\text{fric}} - \tau_{\text{brake}} \tag{2}
\]

The mean torque generated by combustion over an engine cycle, \( \tau_{\text{crank}} \), is an output from the thermodynamic domain, and will be calculated in Section 2.3.

The engine mechanical friction is described in terms of a mean effective pressure, \( P_{\text{fme}} \), which varies with oil temperature, \( T_{\text{oil}} \), and crankshaft velocity. It is used to specify the frictional torque acting on the engine and clearly scales with the number of cylinders, \( N_{\text{cyl}} \), and swept volume, \( V_s \), of the engine:

\[
\tau_{\text{fric}} = \frac{N_{\text{cyl}} V_s}{4\pi} P_{\text{fme}}(T_{\text{oil}}, \omega_{\text{crank}}) \tag{3}
\]

The friction mean effective pressure, \( P_{\text{fme}} \), has four source terms arising from rubbing losses from the crankshaft, reciprocating components, the valve train and other auxiliary devices such as the water and oil pumps. Temperature dependence arises through the viscosity, \( \mu \), being a function of oil temperature. A structure for these equations was developed and validated by Sandoval and Heywood (2003) on multiple engines and is the model adopted here. For convenience, the equations are repeated below:

\[
P_{\text{fme}} = c_1 + c_2 \sqrt{\frac{\mu(T_{\text{oil}})}{\mu_0}} \omega_{\text{crank}}^2 + c_3 \omega_{\text{crank}}^4 \tag{4}
\]

\[
P_{\text{fme}} = c_4 + c_5 \frac{1}{\omega_{\text{crank}}} + c_6 \sqrt{\frac{\mu(T_{\text{oil}})}{\mu_0}} + c_7 \omega_{\text{crank}} \sqrt{\frac{\mu(T_{\text{oil}})}{\mu_0}} \tag{5}
\]

\[
P_{\text{fme}} = c_8 + c_9 \frac{1}{\omega_{\text{crank}}} + c_{10} \sqrt{\frac{\mu(T_{\text{oil}})}{\mu_0}} + c_{11} \omega_{\text{crank}} \sqrt{\frac{\mu(T_{\text{oil}})\omega_{\text{crank}}}{\mu_0}} \tag{6}
\]

\[
P_{\text{fme}} = a_0 + a_1 \omega_{\text{crank}} + a_2 \omega_{\text{crank}}^2 + a_3 \omega_{\text{crank}}^3 \tag{7}
\]

\[
P_{\text{fme}} = P_{\text{fme}} + P_{\text{fric}} + P_{\text{brake}} \tag{8}
\]

The parameters \( c_1 \) to \( c_{11} \) are known functions of the engine geometry (validated for different engines in Sandoval and Heywood (2003)), while \( a_0 \) to \( a_3 \) are obtained as part of the model calibration process for the specific engine of interest.

The brake torque, \( \tau_{\text{brake}} \), is the amount of torque the engine performs to drive the vehicle through a drive cycle. In our experiments, this is simulated by a brake dynamometer, which can be run either in a torque-tracking mode and the throttle angle \( \alpha \) adjusted through closed loop control, i.e., if the dynamometer is run in torque tracking mode with a desired drive cycle torque, \( \tau_{\text{drive}} \):

\[
\alpha = \left( K_p + K_i \int + K_d \frac{d}{dt} \right) (\tau_{\text{brake}} - \tau_{\text{drive}}) \tag{9}
\]

A similar equation to (9) penalising speed deviations may be used for the throttle if the dynamometer is run in a speed-tracking mode.

2.2 Thermal domain

This domain is used to describe the various heat flows in the engine, with source terms being the heat generated by mechanical friction (arising from the Mechanical domain) and the heat generated by combustion (arising from the Thermodynamic domain). Ambient conditions act as a heat sink. The temperature of each of the engine components and fluids must be modelled in the thermal domain.

Assumption 2. The head, block, piston and liner of the engine can be treated as lumped masses.

Remark 2. The actual masses experience temperature gradients as a result of proximity to the heat source (friction or combustion). The lumped mass temperature will therefore not necessarily be indicative of the mean temperature across the entire body, but the mean temperature of the body at the point of contact with the coolant or oil. This subtle difference is taken into account in the procedure for calibrating the heat transfer coefficients in Section 3.
Assumption 3. The coolant and oil are treated as lumped fluids existing in the block and head (coolant) and the head, block and sump (oil).

Remark 3. As with Assumption 2, there will still be temperature gradients within the fluid in practice. The possibility of local boiling is not considered under this assumption, as it is assumed the flow velocity of the fluids, which is set by the pump characteristic and typically a function of engine speed, is sufficient to avoid this effect occurring. Despite the lumped fluid assumption there is still enthalpy flow as will be described in the system’s dynamic equations. The temperature of the fluid is the same as the outlet temperature for each of the volumes.

Assumption 4. The thermostat remains closed during engine warmup.

Remark 4. Since the thermostat is closed there is no need to model the entire coolant circuit during thermal transients, and thus only the head, block, piston, liner and the fluid flows within them are considered relevant. Once the engine operating temperature reaches steady condition, the whole thermal behaviour of the engine can be neglected with the exception of dynamics associated with the opening and closing of the thermostat valve - i.e. a constant temperature model such as the one described in Cho and Hedrick (1989), becomes a valid approximation.

From Assumptions 2 to 4 there are nine states in the engine corresponding to the temperatures of the four lumped masses (head, block, piston, liner, and oil flowing through the block, Tc,b; and the head, To,b; and oil flowing through the block, To,b the head, Tm,h, and remaining in the sump, To,s).

Each of the nine temperature states in the thermal model has dynamics dictated by the standard heat capacitance relationship:

\[ C_i \Delta T_i = Q_{in,i} - Q_{out,i} \]  

where the total heat capacitance of each lumped element is the product of its mass and the specific heat capacity, i.e. \( C_i = m_i c_p_i \).

Naturally, the heat flowing out of one state flows into another state (or exits the system to the environment). The thermal flow diagram presented in Figure 2 depicts the interconnections between the lumped parameters in the thermal domain.

The values of heat transfer, \( \dot{Q}_{i,j} \), between lumped masses \( i \) and \( j \) is calculated from the relation

\[ \dot{Q}_{i,j} = G_{i,j}(T_i - T_j) \]

When forced convection takes place between a fluid and solid, the thermal conductance, \( G_{i,j} \) of (11) is given by the product of the contact surface area \( A_{i,j} \) and the convective heat transfer coefficient, \( h_{i,j} \):

\[ G_{i,j} = h_{i,j} A_{i,j} \]

Furthermore, the heat transfer coefficient is well described by the Nusselt correlation (see e.g. Bohac et al. (1996)) in terms of kinematic fluid viscosity, \( \nu \), mean velocity, \( \bar{v} \), of the fluid, and the characteristic diameter, \( D \) and fluid thermal conductivity, \( \kappa \), as:

\[ h_{i,j} = a_c \frac{k}{D} \left( \frac{\bar{v} D}{\nu} \right)^{b_c} \]

For the fluid in the block and head (coolant), and the head, block and sump (oil), the values of heat transfer, \( \dot{Q}_{i,j} \), are calculated from the relation

\[ \dot{Q}_{i,j} = h_{i,j} A_{i,j}(T_i - T_j) \]

where the total heat transfer rate is the product of its contact surface area \( A_{i,j} \) and the effective heat transfer coefficient, \( h_{i,j} \).

As an example using Figure 2, applying (15) to the oil in the block results in:

\[ \dot{Q}_{o,b} = \dot{m}_{oil}(\omega_{crank})c_p_{oil}[T_{inlet} - T_{outlet}] \]

The losses in mechanical energy due to the friction torque were described in (3), and represent the first source of heat. This effective power is given by:

\[ \dot{Q}_{fric} = \tau_{fric}(\omega_{crank}) \left( \frac{N_{cy} V_s}{4\pi} \right) P_{fme}(T_{oil}, \omega_{crank}) \omega_{crank} \]

Assumption 5. The heat flow from the cylinder may be treated as a continuous process (i.e. averaged over the four engine strokes) and furthermore the heat transfer coefficients between the combustion by-products in the cylinder and each of the cylinder walls is the same. Also the gas temperatures in the exhaust port and the cylinder may be considered equal.
Remark 5. Given the time scales of the combustion process relative to the mean value time scales considered in the remainder of the model this approximation is reasonable.

The heat source due to the combustion results in heat transfer from within the cylinder to the surrounding lumped masses, $Q_{cyl}$, as well as from the exhaust port to the head, $Q_{port}$. Using Assumption 5 and the convection equation (11), the following relationships hold:

$$Q_{port}(T_{cyl}, T_h) = h_{exh,h} A_{port}(T_{cyl} - T_h)$$
$$Q_{cyl}(T_{cyl}, T_h, T_p, T_{1i}) = h_{cyl,walls}(A_p(T_{cyl} - T_p) + A_h(T_{cyl} - T_h) + A_i(T_{cyl} - T_{1i}))$$

The heat transfer coefficient between the gas in the exhaust port and the head, $h_{exh,h}$, is given in Bohac et al. (1996). Averaged over all four engine strokes, the mean heat transfer coefficient to the cylinder walls, $h_{cyl,walls}$, is given by equation (13), where the characteristic diameter is taken to be the cylinder bore, and the characteristic velocity is calculated from the mean value mass flow rate through the cylinders, $\dot{m}_{cyl}$.

The cross sectional areas used in (18) - (19) are taken for all $N$ cylinders, while the area of the liner, $A_l$, is the average area over the combustion stroke. The derivation of the gas temperature in the cylinder, $T_{cyl}$, will be detailed more fully in the next section dealing with the thermodynamic domain.

2.3 Thermodynamic domain

The thermodynamic domain describes the combustion process occurring in the cylinders resulting in the production of heat and mechanical work required for the thermal and mechanical domains. To model the process completely, it is necessary to include the temperature, pressure and species concentrations in the intake and exhaust manifolds as states of the domain, and the combustion process itself. In modelling this domain it is useful to make some simplifying assumptions and define some terminology.

Assumption 6. The temperatures and pressures throughout each manifold are constant throughout the volumes.

Remark 6. This is a standard assumption in mean value models, and is equivalent to Assumptions 2 and 3 in the thermal domain.

Assumption 7. The dynamics in the intake and exhaust manifolds induced by the in-cylinder events including valve opening and heat release are of a different time scale to the mean flows. Consequently their effects on engine temperature, manifold pressures are assumed negligible.

Remark 7. Again, this is a standard assumption in mean value models and can be rigorously justified using singular perturbation techniques, e.g. Sharma et al. (In press). The significance of the mean value approach is that each control volume can be modelled using three internal state equations for pressure, temperature and composition.

Assumption 8. The closed loop control of air fuel ratio is sufficiently accurate that the engine controller is able to deliver the requested air fuel ratio.

Remark 8. This simplifying assumption negates the need for fuel pooling dynamics to be considered in the model, and requires the closed loop lambda controller to either have fuel puddle information or a reasonable online estimate, see e.g. Choi and Hedrick (1998). The main purpose of this assumption is to allow air fuel ratio to be used as a control input. Relaxation of this assumption would require an additional fuel pooling state equation(s), which are typically a variant of the form originally presented in Aquino (1981), as well as the fuel injection controller dynamics to be included in the model.

Definition 1. Define $x_i$ as an $n$-vector whose elements represent the relative composition in the volume $i$ of each of the $n$-gaseous species considered in the model. The mass flow into the volume, $\dot{m}_i$, is also an $n$-vector representing the mass flow of each considered gaseous species from control volume $i$.

Definition 2. Define $c_p(T)$, $\kappa(T)$ and $R$ as $n$-vectors whose elements represent the specific heat and gas constants for each element of $x_i$. Further, let the ratio of dot products, $\gamma(x_i, T) = \frac{c_p(T)}{\kappa(T)\gamma}$, represent the ‘mean’ ratio of specific heats in the volume $i$ with temperature, $T$.

Definition 3. For notational convenience, let $||.||$ represent the 1-norm of the $n$-vector.

To begin, the intake manifold state equations can be found by considering the flows into the manifold (consisting of flow through the injectors, $\dot{m}_{inj}$, fresh charge, $\dot{m}_{fc}$, and exhaust gas recirculation $\dot{m}_{egr}$) as well as the mass flow leaving the manifold (and entering the cylinder $\dot{m}_{cyl}$). Only the element of the $\dot{m}_{inj}$ vector corresponding to fuel will be non-zero, and the flow rate is specified by the engine controller in order to deliver the requested air fuel ratio by Assumption 8.

The mass flow into the manifold from ambient, $\dot{m}_{fc}$, due to internal exhaust gas recirculation, $\dot{m}_{egr}$, and out of the tailpipe, $\dot{m}_{tp}$, are static functions of the throttle angle, $\alpha$, and pressure differences:

$$\dot{m}_{fc} = \frac{P_{amb}}{\sqrt{RT_{amb}}} A_{cyl}(\alpha) \Psi_1(P_{amb}, P_{em})$$
$$\dot{m}_{egr} = \frac{\Psi_1(P_{em}, P_{amb}, \theta_{vo})}{\lambda AFR_s}$$
$$\dot{m}_{tp} = \frac{\dot{m}_{fc}}{\lambda AFR_s}$$

The nonlinear functions $\Psi_1()$, $\Psi_2()$, $\Psi_3()$ have been previously identified in Heywood (1988), Fox et al. (1993) and Shayler et al. (1993) respectively.

The total mass flow from the intake manifold into the cylinders is represented by the well-known speed-density relationship, which uses a static mapping of volumetric efficiency $\eta_v(\cdot)$ to give:

$$\dot{m}_{cyl} = \left( \frac{N_{cyl} V_s}{4\pi} \right) \frac{1}{RT_{im}} \eta_v(\cdot) P_{im}\omega_{crank}$$

There are enthalpy and mass flows into the intake manifold from the fresh charge ($\dot{E}_{h,fc} = \dot{m}_{fc} c_p(x, T_{im}) T_{amb}$ and $\dot{m}_{fc}$), fuel ($\dot{E}_{h,fuel}$ and $\dot{m}_{fuel}$) and exhaust gas recirculation ($\dot{E}_{h,egr}$ and $\dot{m}_{egr}$). The flow out of the intake manifold is into the cylinders, $\dot{E}_{h,cyl}$ and $\dot{m}_{cyl}$, resulting in net enthalpy and net mass flows into the intake manifold given by:
\[ \dot{E}_{\text{em}} = \dot{E}_{\text{h,fu}} + \dot{E}_{\text{h,egr}} - \dot{E}_{\text{h,cyl}} \quad (25) \]
\[ \dot{M}_{\text{em}} = \dot{m}_{\text{fu}} + \dot{m}_{\text{egr}} - \dot{m}_{\text{cyl}} \quad (26) \]

In Appendix A of Keynejad (2009) it is shown from the first law of thermodynamics, mass conservation and the ideal gas law that the following dynamic equations hold in the intake manifold:

\[ \dot{P}_{\text{im}} = \left( \frac{\gamma (x, T_{\text{im}}) - 1}{V_{\text{im}}} \right) \dot{E}_{\text{im}} \]
\[ \quad + \left( \frac{R}{\gamma - 1} \frac{\dot{x}_{\text{im}}}{c_v(T_{\text{im}})} \right) \dot{M}_{\text{im}} \]
\[ \dot{T}_{\text{im}} = \left( \frac{\gamma (x, T_{\text{im}}) - 1}{T_{\text{im}}} \right) \frac{T_{\text{im}}}{P_{\text{im}} V_{\text{im}}} \]
\[ \times \left( \dot{E}_{\text{im}} - \left( \frac{c_v(T_{\text{im}})}{c_v(T_{\text{im}})} \right) \dot{M}_{\text{im}} \right) \quad (27) \]
\[ \dot{x}_{\text{im}} = \frac{RT_{\text{im}}}{P_{\text{im}} V_{\text{im}}} \left( \dot{M}_{\text{im}} - \dot{x}_{\text{im}} \| \dot{M}_{\text{im}} \| \right) \quad (28) \]

The state equations for the exhaust manifold can be defined similarly, but must include terms reflecting the losses from the exhaust manifold to the environment, \( Q_{\text{em,amb}} \), and the net flows into the exhaust manifold includes gain from the combustion process occurring in the cylinders and losses due to exhaust gas recirculation and the tailpipe:

\[ \dot{E}_{\text{em}} = \dot{E}_{\text{h,cyl}} - \dot{E}_{\text{h,egr}} - \dot{E}_{\text{h,tp}} \quad (30) \]
\[ \dot{M}_{\text{em}} = \dot{m}_{\text{cyl}} - \dot{m}_{\text{egr}} - \dot{m}_{\text{tp}} \quad (31) \]

Consequently, the state equations for the exhaust manifold are:

\[ \dot{P}_{\text{em}} = \left( \frac{\gamma (x, T_{\text{em}}) - 1}{V_{\text{em}}} \right) \dot{E}_{\text{em}} - \dot{Q}_{\text{em,amb}} \]
\[ \quad + \left( \frac{R}{\gamma - 1} \frac{\dot{x}_{\text{em}}}{c_v(T_{\text{em}})} \right) \dot{M}_{\text{em}} \]
\[ \dot{T}_{\text{em}} = \left( \frac{\gamma (x, T_{\text{em}}) - 1}{T_{\text{em}}} \right) \frac{P_{\text{em}} V_{\text{em}}}{c_p(T_{\text{em}})} \]
\[ \times \left( \dot{E}_{\text{em}} - \left( \frac{c_v(T_{\text{em}})}{c_v(T_{\text{em}})} \right) \dot{M}_{\text{em}} \right) \dot{Q}_{\text{em,amb}} \quad (32) \]
\[ \dot{x}_{\text{em}} = \frac{RT_{\text{em}}}{P_{\text{em}} V_{\text{em}}} \left( \dot{M}_{\text{em}} - \dot{x}_{\text{em}} \| \dot{M}_{\text{em}} \| \right) \quad (33) \]

Assumption 9. The rate of change of gas composition, \( x \), in each manifold is assumed negligible. 

Remark 9. With this assumption, equations (29) and (34) become redundant and the second terms on the right hand side of (27) and (32) disappear. Consequently, the manifolds can be modelled in the thermodynamic domain using only equations (28), (33), and the following two simplified versions of (27), (32):

\[ \dot{P}_{\text{em}} = \left( \frac{\gamma (x, T_{\text{em}}) - 1}{V_{\text{em}}} \right) \dot{E}_{\text{em}} \quad (35) \]
\[ \dot{P}_{\text{im}} = \left( \frac{\gamma (x, T_{\text{im}}) - 1}{V_{\text{im}}} \right) \left( \dot{E}_{\text{im}} - \dot{Q}_{\text{em,amb}} \right) \quad (36) \]

With the intake and exhaust manifold dynamics modelled, the in-cylinder behaviour can now be considered.

Assumption 10. The in-cylinder dynamics can be replaced by static maps.

Remark 10. Torque and heat production as well as composition change all take place within the cylinder, primarily dictated by chemical reaction rates in the order of milliseconds. Meanwhile, the inertia of the engine is sufficient that the mechanical domain is characterised by time constants of the order of seconds, while the thermal domain has even slower time constants. The other dynamic equations in the thermodynamic domain, (27) - (34), are mean value approximations and by Assumption 7 will be of the order of several engine cycles. With this degree of time scale separation, a static map approximation to the dynamic equations in the cylinder is well justified.

The control signals for the static maps are valve overlap, \( \theta_{ov} \), and spark advance, \( \theta_{sa} \), while the influence of the conditions in the bounding volumes must also be reflected, so that the static maps resulting from Assumption 10 have the form:

\[ \tau_{\text{crank}} = f_1(P_{\text{im}}, \omega_{\text{crank}}, x_{\text{im}}, \theta_{ov}, \theta_{sa}) \quad (37) \]
\[ T_{\text{cyl}} = f_2(P_{\text{im}}, T_{\text{im}}, x_{\text{im}}, \theta_{ov}, \theta_{sa}) \quad (38) \]
\[ x_{\text{cyl}} = f_3(x_{\text{em}}, T_{\text{im}}) \quad (39) \]

While (37)-(39) are static maps whose outputs feed into the other domains, it is useful to impose a physics-based structure to these mappings to simplify the calibration process. They can also be easily related to other required quantities (such as the temperature and enthalpy of the exhaust gas) using these relations.

A chemical balance can be used to relate the power entering the cylinder (given as the mass of fuel multiplied by its lower heating value, \( Q_{LHV} \)) to the power delivered as mechanical power (including the power delivered to the crankshaft and that required to overcome pumping losses, \( W_{\text{pump}} \)) and that released as heat. The indicated efficiency, \( \eta_i(P_{\text{im}}, x, \theta_{sa}) \), is the fraction of mechanical power to the chemical energy entering the cylinder, and is a calibrated quantity.

\[ \eta_i \dot{m}_{\text{fuel}} Q_{LHV} = \tau_{\text{crank}} \omega_{\text{crank}} + W_{\text{pump}} \quad (40) \]

The pumping losses can be written in terms of the pumping mean effective pressure, \( P_{\text{pme}}(P_{\text{amb}}, P_{\text{im}}) \) defined in Sandoval and Heywood (2003) to give:

\[ W_{\text{pump}} = \frac{N_{\text{cyl}} V_s}{4 \pi} P_{\text{pme}}(P_{\text{amb}}, P_{\text{im}}) \omega_{\text{crank}} \quad (41) \]

Equations (40) and (41) can be combined to model the torque delivered to the crankshaft as:

\[ \tau_{\text{crank}} = \frac{\eta_i \dot{m}_{\text{fuel}} Q_{LHV}}{\omega_{\text{crank}}} - \frac{N_{\text{cyl}} V_s}{4 \pi} P_{\text{pme}}(P_{\text{amb}}, P_{\text{im}}) \quad (42) \]

To calculate the cylinder temperature, the other side of the chemical balance equation is used, where the heat flows to be considered are the heat transfer from the cylinder to the piston, liner, head and exhaust port.

\[ (1-\eta_i)\dot{m}_{\text{fuel}} Q_{LHV} = \dot{Q}_{\text{cyl}}(T_{\text{cyl}}) + \dot{Q}_{\text{port}}(T_{\text{cyl}}) + \dot{E}_{\text{h,cyl}} \quad (43) \]

Thus, defining the enthalpy flow \( \dot{E}_{\text{h,cyl}} = \dot{m}_{\text{cyl}} c_p T_{\text{cyl}} \), it follows from Assumptions 6-10, (19), (24) and (43) that the cylinder temperature is given by:

\[ T_{\text{cyl}} = \left( \frac{\gamma (x, T_{\text{cyl}}) - 1}{T_{\text{cyl}}} \right) \frac{P_{\text{cyl}} V_{\text{cyl}}}{c_p(T_{\text{cyl}})} \times \left( \dot{E}_{\text{cyl}} - \dot{Q}_{\text{cyl}}(T_{\text{cyl}}) \right) \quad (44) \]
\[ T_{cyl} = \frac{1}{h_{cyl, walls} \sum_i A_i + h_{exh,h} A_{port} + \dot{m}_{cyl} c_P} \times \left( (1 - \eta_i) \dot{m}_{fuel} Q_{LHV} + h_{exh,h} A_{port} T_h \right) + h_{cyl, walls} \sum_i A_i T_i \] (44)

It is worth noting that the mass of fuel entering the cylinder, \( \dot{m}_{fuel} \), may be calculated using the total mass flow rate into the cylinder, \( \dot{m}_{cyl} \), and the composition, \( x_{\text{im}} \).

The final static map representing the effect of combustion on composition, \( x_{\text{im}} \), can only be specified when the elements in the vector \( x \) are explicitly specified. For demonstration purposes, the following assumption allows a static map for \( x_{\text{im}} \) to be postulated.

**Assumption 11.** Only three gaseous species are assumed present throughout the engine and these are regarded as fuel (C\(_5\)H\(_{12}\)), air (STP concentrations of nitrogen and oxygen) and exhaust (consisting of mainly CO\(_2\) and H\(_2\)O at stoichiometry). Further, the gaseous species in the cylinder following combustion are assumed functions of air-to-fuel ratio only.

**Remark 11.** Assumption 11, along with the usual definition of \( \lambda \) as the ratio of actual and stoichiometric air-to-fuel ratios, allows the following structure for (39) to be proposed:

\[ x_{cyl} = \frac{x_{fuel}}{x_{air} x_{exh}} = \begin{bmatrix} 1 - \min(\lambda, 1) & 0 & 0 \\ 0 & 1 - \min(\lambda^{-1}, 1) & 0 \\ \min(\lambda, 1) & \min(\lambda^{-1}, 1) & 1 \end{bmatrix} \times x_{\text{im}} \] (45)

### 2.4 Sensor domain

While not strictly necessary to model the dynamics inside the engine during warm up, a model for the exhaust gas thermocouple mounted at the exhaust gas port is present here as the thermocouple response is typically quite slow and hides some of the faster dynamics presented by the model.

As the sensor will be subject to both convection from the exhaust gas and radiation to the head, the structure of equation is:

\[ C_{\text{c}} T_{\text{c}} = h_{\text{c}, \text{exh}} A_{\text{c}} (T_{\text{cyl}} - T_{\text{c}}) - A_{\text{c}} \varepsilon (T_{\text{c}}^4 - T_{\text{amb}}^4) \] (46)

The parameter \( h_{\text{c}, \text{exh}} \) in (46) can be determined using a Nusselt number correlation for a cylindrical object in a fluid stream as described in Churchill and Bernstein (1977), while \( \varepsilon \) was taken to be 0.04 (corresponding to polished aluminium).

### 3. ENGINE TESTBED AND MODEL CALIBRATION

The test engine used was a 4.0 litre, Ford Falcon BF inline 6 cylinder engine with 10.3:1 compression ratio, and was mounted on a 460kW transient AC dynamometer. Aside from the standard engine sensors and dynamometer load feedback, a thermocouple was mounted at the exhaust port to measure \( T_{\text{cyl}} \), and in-cylinder pressure was recorded to measure indicated mean effective pressure.

From manufacturer data and CAD drawings, many of the engine parameters could be either directly measured or inferred. Consequently there are only eleven heat transfer coefficients parameters and three surfaces (\( P_{\text{fmc}} \), \( h_b \) and \( \eta_i \)) in the model described in Section 2 that require calibration across the three domains using data measurements.

In the mechanical domain, the coefficients of (7), describing the auxiliary friction mean effective pressure polynomial, require calibration. Since this quantity is not directly measurable, the total friction mean effective pressure was estimated using measurements of indicated (from in-cylinder pressure measurements) and braking mean effective pressure (from measurements of brake torque and engine speed), i.e.

\[ P_{\text{fmc}} = P_{\text{imp}} - P_{\text{bme}} = \dot{P}_{\text{fmc}} + \dot{P}_{\text{fme}} + \dot{P}_{\text{fmc}} + \dot{P}_{\text{fme}} + e \] (47)

The estimates of \( \dot{P}_{\text{fmc}}, \dot{P}_{\text{fme}} \) and \( \dot{P}_{\text{fmc}} \) were calculated based on Sandoval and Heywood (2003). Although these estimates will have some error, the approach suggested here is to associate all estimation error, \( e \), with the estimate of \( P_{\text{fmc}} \). Consequently, the best estimate of the coefficients of the \( P_{\text{fmc}} \) polynomial is obtained by solving the following optimisation problem

\[ [a_0, \ldots, a_3] = \arg \min_{a_0, \ldots, a_3} \sum e^2 \] (48)

The data set used in solving (48) was obtained using a constant set temperature of 373K, with engine speed varied in 50 rpm increments from 1000rpm to 3000rpm. The resulting parameters following the implementation of an SQP algorithm were obtained as:

\[ a_0 = 6.01 \times 10^{-3}; \quad a_1 = 2.91 \times 10^{-3} \]
\[ a_2 = -4.74 \times 10^{-6}; \quad a_3 = 7.5 \times 10^{-9} \]

The thermal domain has eleven parameters to be calibrated, consisting of three heat transfer coefficients from the head, block and oil sump to ambient (\( h_{h,amb}, h_{b,amb} \) and \( h_{o,amb} \)); the heat transfer coefficients from the coolant to the block and head and from inside the cylinder to the cylinder walls (specified by two sets of coefficients \( a_c \) and \( b_c \)); the heat transfer from the piston to oil sump, and the head and block to the oil (\( h_{p,o}, h_{h,o} \) and \( h_{b,o} \)); and the heat transfer coefficient from the piston to the liner (\( G_{p,l} \)).

Existing thermocouples in the coolant path in the head and the oil path in the block were used to pose the following optimisation problem:

\[ J = \int_{t_0}^{t_f} \left( T_{\text{h,b}} - T_{\text{h,b}} \right) (T_{\text{e,b}} - \dot{T}_{\text{c,b}}) dt \] (49)

\[ \left[ h_{h,amb}^*, \ldots, a_{c,(c-b)}, \ldots, G_{p,l}^* \right] = \arg \min_{h_{h,amb}, \ldots, G_{p,l}} J \] (50)

A steady speed-load warm up test was conducted at 1000 rpm and 50 Nm with initial conditions of \( T_{\text{c},(0)} = 296K, T_{\text{h,b}}(0) = 289K \). Ambient temperature was recorded as \( T_{\text{amb}}(t) = 288K \). The constant heat transfer coefficients resulting from the optimisation were:

\[ h_{h,amb} = 25W/(m^2K); \quad h_{b,amb} = 15W/(m^2K) \]
\[ h_{s,amb} = 200W/(m^2K); \quad h_{h,o} = 80W/(m^2K) \]
\[ h_{b,o} = 130W/(m^2K); \quad h_{p,o} = 400W/(m^2K) \]
\[ G_{p,l} = 30W/K \]
The heat transfer for the coolant flow to the head and block, and for the cylinder gas to the cylinder walls are described by (13) and the parameters of this equation were found to be:

\[
\begin{align*}
\alpha_c(c\rightarrow b) &= \alpha_c(c\rightarrow h) = 0.46; \\
\beta_c(c\rightarrow b) &= \beta_c(c\rightarrow h) = 0.11
\end{align*}
\]

Simulating the model over the NEDC cycle, this leads to the engine speed dependant heat transfer coefficients to fall within the range \(h_{c,b} \in [300,550] W/(m^2 K)\), \(h_{c,h} \in [280,480] W/(m^2 K)\) and \(h_{cyl,w} \in [8,30] W/(m^2 K)\). Note that the different heat transfer coefficients in the head and block arise from the different diameter of the flow passages, and hence the different fluid velocities, while heat transfer coefficient from the cylinder to walls is effectively averaged over the four engine cycles.

Finally, the thermodynamic domain requires calibration of the maps for volumetric, \(\eta_v(P_{in},\omega_{crank},\lambda,\theta_{vo})\) and indicated efficiency, \(\eta_i(P_{in},\omega_{crank},\theta_{vo},\lambda)\). Static sweeps of spark timing, valve timing, engine speed, air fuel ratio and engine load were conducted over a wide range of engine steady state operation and curve fits are produced.

Assumption 11 and (45) were used in the modelling with fuel, air and exhaust assumed the only species present.

4. VALIDATION

The model from Section 2 with parameters identified in Section 3 was then produced in the Modelica simulation environment. It’s accuracy was validated by running the engine over constant speed-load and NEDC drive cycles. Modelled and experimental results are compared in the following subsections.

4.1 Validation of mechanical domain model

The default calibration strategy of the OEM was used during all testing to set the engine input variables such as air-fuel ratio and spark timing. With the engine attached to the dynamometer, the brake torque was specified explicitly and the engine speed was controlled by the throttle in closed loop to match the specified drive cycle. Consequently, the friction mean effective pressure is considered as a useful quantity to validate the proposed model. The friction mean effective pressure cannot be measured directly, but is implied from the difference between the indicated mean effective pressure and the losses due to pumping and brake mean effective pressures.

The comparison of the modelled and experimental friction mean effective pressure is presented in Figure 3. Good agreement between the modelled and experimental data is observed across both engine speed and oil temperature ranges. An exception to the high accuracy is the spike in \(P_{fric}\) seen at 15\(^\circ\)C for both constant engine speeds when the engine starts (possibly due to the initial oil distribution before the oil pump is operational not completely coating the surfaces).

4.2 Validation of thermal domain model

The thermal domain was validated by placing thermocouples in the oil and coolant flow pathways, and the engine was run over the NEDC cycle until the thermostat opened. The left plot in Figure 4 illustrates very good correlation between the predicted and experimental temperatures in both the coolant and oil.

As further validation, the cumulative fuel use was both logged from the engine computer and measured using an external fuel flow meter, and compared to the actual fuel used. The requested air-fuel ratio was maintained between the modelled and simulation results. This comparison, seen in the right hand side of Figure 4, is a good indicator that the friction model and static maps in the engine model reflect accurately the real situation. The cumulative errors in oil, coolant and fuel use can be quantitatively measured as 0.24%, 0.54% and 1.6% respectively.

4.3 Validation of thermodynamic domain model

The key aspect of the thermodynamic model presented here is the inclusion of the gas composition coupling, and the mean value cylinder model represented as a set of static maps. The composition of the gases in the various reservoirs could not be measured due to a lack of equipment, however its effect can be implicitly ascertained through the functional dependence of \(\gamma\) on \(x\).

The manifold pressure dynamics have been well established in the literature previously, so their validation is not repeated here. Instead, the focus is given to the temperature of the gas in the cylinder and how well the model predicts the actual value as measured at the exhaust port. Figure 5 shows the relationship between the temperature predicted by the model and the experimental data. There are significant high frequency dynamics predicted by the
model that are not reflected in the experimental data, however when the thermocouple model is included there is very good agreement between the two over both drive cycle and steady state operation.

![Fig. 5. Exhaust gas temperature predicted from model, corrected using equation (46) and measured for NEDC (left) and constant speed (right) warm up](image)

4.4 Sensitivity analysis of the model

As a basic sensitivity analysis, the parameters of the model identified in Section 3 were randomly perturbed over a large number of trials by up to 20%. The effects on the states and cumulative fuel consumption is summarised in Figure 6, where it is observed that a wide operating envelope exists. On the surface, this indicates that the calibration process is effective, and the sensitivity of the model to changes in the parameters is significant in some instances, although further study is required to make quantifiable statements regarding the critical parameters and states of the model.

![Fig. 6. Envelopes resulting from Monte-Carlo perturbations on model parameters over NEDC (left) Oil and coolant temperatures (right) Cumulative fuel use](image)

5. CONCLUSIONS

The model presented in this paper is able to accurately reproduce the thermal, mechanical and thermodynamic characteristics of a spark ignition engine during the warm up region of operation. This model makes use of several simplifying assumptions, most notably the elimination of the in-cylinder dynamics (the fastest dynamics present in the engine) and their replacement with a static map, as well as the development of previous models from the literature.

The model also proposes a structure for considering the composition of the gas species in different volumes in the engine, and discusses how these are coupled to the dynamics of pressure and temperature in the intake and exhaust manifolds.

While the accuracy of the multi-domain model is sufficient to faithfully reproduce the engine dynamics during warm up, the complexity of the model is low enough to lend itself to offline optimisation studies (simulating 500 seconds of the warm up duration using the model implemented in Modelica on a Pentium 4 processor takes approximately 17 seconds). For closed loop model-based control purposes this is still too complex, but the model provides an excellent basis for further simplification using engine model reduction techniques such as proposed in Sharma et al. (In press).

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