

Price-based Max-Min Fair Rate Allocation in Wireless Multi-hop Networks

Liansheng Tan, Xiaomei Zhang, Lachlan L. H. Andrew and Moshe Zukerman

Abstract—When allocating rates in wireless multi-hop networks, one difficulty comes from the so-called MAC (media access control) constraint. To overcome this difficulty, this paper proposes a price-based max-min fair rate allocation scheme. Unlike existing schemes, our scheme is based on the MaxNet principle using the maximum price of all nodes along a transmission path to control the flow's rate. Through theoretical analyses and simulation results, we show that the algorithm is able to meet the MAC constraint and to achieve max-min fairness among multi-hop flows in wireless networks even if the topology varies, as is the case in a mobile environment.

Index Terms—Rate allocation, Max-min fairness, Flow control, Wireless networks.

I. INTRODUCTION

IN recent years, allocating flow rates fairly either in wire-line networks or in mobile/wireless networks has received much attention [4], [7], [8], [10], [12], [13], [14], [17], [18], [19]. As distinct from wire-line networks, in a wireless network an important additional constraint arises from the Media Access Control (MAC) layer [14]. This constraint requires that, the same node should not transmit and receive packets simultaneously [14], [19]. Therefore, when allocating bandwidth for wireless networks, both the link capacity restriction and the time constraint imposed by the MAC layer must be considered.

The work of [4], [14] provides scheduling schemes for max-min fair allocation of bandwidth in wireless ad hoc networks (WANETs) to meet the MAC constraint. These scheduling schemes require a central controller, which hinders their application to distributed wireless networks that may not rely on centralized management or authority. The challenge is to control the flow rates in a fully distributed manner. In this paper, we meet this challenge by providing a fully distributed algorithm for a price based max-min fair bandwidth allocation over a wireless network.

The use of pricing as a means for allocating bandwidth has been initially proposed in [6], [7], [8] for wired networks, where the authors show that the pricing scheme can achieve (in equilibrium) a proportional fair rate allocation. Such a pricing

mechanism has been generalized by [12] and [18] to WANETs. In [12], the authors propose an adaptation algorithm that converges to the unique bandwidth allocation while maximizing the sum of the user's utilities without explicitly addressing the fairness issue. The authors of [18] present a new pricing policy for end-to-end multi-hop flows in WANETs aiming to achieve maximized aggregate utility of flows. A more general set of constraints is investigated in [3], which take into account the need to schedule transmissions globally so that they do not interfere. These constraints are considered in an end-to-end context in [2].

Maximizing aggregate utility is able to approach max-min fairness if the utility function has a particular asymptotic form [9], [18]. Max-min fairness is an important requirement for wireless networks such as multi-hop WANETs, that demand totally equal treatment of users regardless of the number of hops they travel. The inability of approaches such as [12], [18] to achieve max-min fairness may be related to the fact that these schemes are based on the SumNet architecture [6], [7], [8], that is the rate regulation is based on the summation of all the link prices on a given path. In SumNet approaches, there is an inherent shortcoming. Considering those flows crossing many hops use more network resources, a utility optimal solution may penalize those "long" flows more. Recently, Wydrowski *et al.* [15], [16], [17] proposed the so-called MaxNet rate control approach for wireline networks which is able to achieve max-min fairness. This paper achieves max-min fair rate allocation in wireless networks by applying the MaxNet idea and considering the MAC constraint. Mobility of wireless networks generally leads to variations of topology; our proposed algorithm is shown to be adaptive to the routing changeovers arising from the mobility of wireless networks in the sense that the rate allocation is still able to maintain max-min fairness for wireless mobile networks. Furthermore, it facilitates implementation due to its distributed nature.

The rest of the paper is organized as follows. In Section 2, we specify the MAC constraint and describe the pricing policy for wireless networks. In Section 3, we analyze the algorithm and prove that the algorithm reaches max-min fairness. In Section 4, we validate our algorithm by simulation results. Finally, we conclude the paper in Section 5.

II. MAC CONSTRAINTS AND PRICING POLICY

A. MAC constraints

In this paper, we consider a static multi-hop wireless network which the links have a fixed capacity. For this model we assume that the channel's changes are much slower than

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Liansheng Tan and Xiaomei Zhang are with the Computer Science Department of Central China Normal University, Wuhan 430079, PR. China (e-mail: l.tan@mail.ccnu.edu.cn). Moshe Zukerman and Lachlan L.H. Andrew are with the Australian Research Council Special Research Center for Ultra-Broadband Information Networks (CUBIN) an affiliated program of National ICT Australia, EEE Dept., The University of Melbourne, Victoria 3010, Australia (e-mail: mzu@unimelb.edu.au).

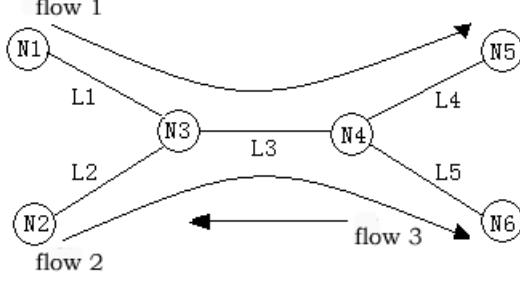


Fig. 1. An example network for MAC constraints

the response of the congestion control scheme. We assume that a reliable link layer is used, and so ignore packet loss. In [14], [19], access constraints at the MAC layers arise due to the fact that each node is not able to transmit or receive packets simultaneously in more than one link. Note that a further restriction whereby a node cannot receive packets when a nearby node is transmitting was considered in [2], [3], [18]. However, these extra constraints do not apply, and hence the MAC constraints are sufficient, if interference between links is negligible. This occurs, for example, in a CDMA system with sufficient spreading in which each node transmits with an independent code. This is reportedly the case for Bluetooth networks [14].

This so-called MAC constraint can be stated as

$$\sum_{s \in S(j)} x_s \left(\frac{1}{C_{l_i(j,s)}} + \frac{1}{C_{l_o(j,s)}} \right) \leq \varepsilon, \quad (1)$$

where we use $S(j)$ to denote the set of flows incident on Node j , x_s to denote the rate of Flow s . The input and output links of Flow s on Node j , in the case Flow s passes through Node j , are denoted by $l_i(j, s)$ and $l_o(j, s)$, respectively. C_l is the capacity of the channel link l . Note that for any source node $C_{l_i(j,s)} = \infty$; while for any destination node $C_{l_o(j,s)} = \infty$. The parameter ε is the efficiency factor in MAC protocol. In this paper, we assume that ε is known and equal for all nodes. For notational simplicity, we take $\varepsilon = 1$, which is equivalent to a suitable scaling of the link capacities. Observe that $x_s/C_{l_i(j,s)}$ can be interpreted as the fraction of time Node j expends to receive data of Flow s over a unit interval of time. $x_s/C_{l_o(j,s)}$ is interpreted as the fraction of time Node j expends to transmit data of Flow s over a unit interval of time. Thus, as total fraction of time expended at Node j cannot exceed $1(\varepsilon)$.

We further expound the concept of MAC constraint by using the model depicted in Fig. 1. In Fig. 1, node 3 can transmit packets of flow 1 or 2 or receive packet from flow 1 or 2 or receive packet from flow 3 or remain idle.

As described in [19], there are link constraints and time constraints in wireless networks. For example in Fig. 1, the network consists of three flows, flow 1, flow 2, and flow 3. Let x_s , $s = 1, 2, 3$ be the data rate of the flows respectively and C_l be the capacity of link l . The link constraints are given as follows:

$$\frac{x_1}{c_1} \leq 1, \quad \frac{x_2}{c_2} \leq 1, \quad \frac{x_1 + x_2 + x_3}{c_3} \leq 1, \quad \frac{x_1}{c_4} \leq 1, \quad \frac{x_2}{c_5} \leq 1. \quad (2)$$

The time constraints are given as follows:

$$\frac{x_1}{c_1} \leq 1, \quad \frac{x_2}{c_2} \leq 1, \quad \frac{x_1}{c_1} + \frac{x_2}{c_2} + \frac{x_1 + x_2 + x_3}{c_3} \leq 1, \\ \frac{x_1 + x_2 + x_3}{c_3} + \frac{x_1}{c_4} + \frac{x_2}{c_5} \leq 1, \quad \frac{x_1}{c_4} \leq 1, \quad \frac{x_2}{c_5} \leq 1. \quad (3)$$

Note that the link constraints can be subsumed into the time constraints. Subsequently, for wireless networks, we can only consider the time constraints; while for the wireline case we can consider only the link constraints.

B. Node pricing: problem formulation

Following [17], we use the same price updating rule as would be used in Kelly's theory of utility maximization [6], [7], subject to additional constraints in the wireless multi-hop network model. Let S be the set of flows, $S(j)$ be the set of flows that uses Node j . The optimization of the aggregated utility function of the set of flows subject to the MAC constraint can be formulated as follows:

$$\mathbf{P} : \max \sum_s U_s(x_s) \quad (4)$$

$$\text{Subject to : } \sum_{s \in S(j)} x_s \left(\frac{1}{C_{l_i(j,s)}} + \frac{1}{C_{l_o(j,s)}} \right) \leq 1. \quad (5)$$

Note that the main difference between the above optimization problem and the wire-line case discussed by Kelly's theory [6] [7] lies in the fact that, for wireless case one needs to consider the MAC constraint. Following a similar procedure, we reach the following price updating rule:

$$p_j(t+1) = \left[p_j(t) + \gamma \left(\sum_{s \in S(j)} x_s(p(t)) \left(\frac{1}{C_{l_i(j,s)}} + \frac{1}{C_{l_o(j,s)}} \right) - 1 \right) \right]^+, \quad (6)$$

where the component γ is a step-size, and the function $[z]^+$ is defined as $[z]^+ = \max\{z, 0\}$. We have put the derivation of the above updating rule in Appendix.

We can interpret p_j as the price per unit time Node j . This is also consistent with the following law: if the total fraction of time at Node j exceeds $1(\varepsilon)$, raise price $p_j(t)$; otherwise reduce price $p_j(t)$. In our approach, the term "price" can be regarded simply as a congestion signal to guide sources' decisions.

III. ALGORITHM DESCRIPTIONS AND MAX-MIN FAIRNESS PROOF

A. Algorithm descriptions

On the basis of the derived node's price updating rule and adopting the idea of MaxNet [17], this section presents the whole algorithm.

Let $A(s)$ be the set of nodes Flow s traverses. In our proposed system, the source of Flow s is charged a price $q_s(t) = \max\{p_j(t), j \in A(s)\}$ per unit bandwidth. Therefore, each node formulates its own price using (6). The network uses price as a signal to reflect the traffic load on the wireless nodes. In our network model, the node prices should be the feedback

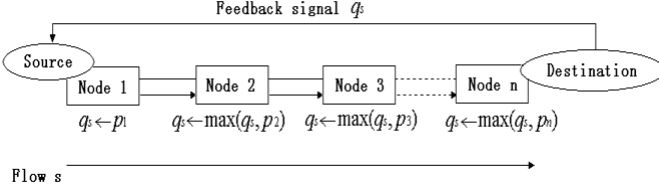


Fig. 2. Calculation of the maximal node price along an end-to-end transmission path

congestion signal to control the flow rates. This congestion signal, q_s , communicated to Flow s , is the maximum of all node prices on the end-to-end transmission path. In this manner our scheme fulfills the aim of controlling and adjusting the flows' rates by using the maximum price of all nodes along a transmission path. This mechanism is illustrated in Fig. 2.

To achieve this, the packet format must include bits to communicate the complete congestion price. Each node replaces the congestion price in the packet with its own congestion price if its own price is larger than the one in the packet. The Flow s is governed by an explicit demand function $D_s(\cdot)$, such that its transmitting rate is $x_s = D_s(q_s)$.

In our network model, the demand functions of all the flows are homogeneous demand functions. We can see that each flow adapts its rate according to the feedback congestion signal, which is the maximum of all node prices on the transmission path.

The algorithm performed by Node j

At times $t = 1, 2, \dots$, Node j performs the following calculations:

1. Estimate rates $x_s(t)$ from all packets (other than acknowledgement) $s \in S(j)$ that go through Node j ;
2. Compute a new price according to (6);
3. For all flows that go through Node j , retrieve the price q_s , from their packet headers, compares q_s and the price $p_j(t+1)$ of Node j . If $p_j(t+1)$ is greater than q_s in the packet, Node j replace the value of q_s with the value of $p_j(t+1)$. This is also performed for packets originating at, or destined for, higher layers at Node j .

The receiver of Flow s takes the value q_s from the packets it receives, and places it in the price field of the acknowledgements it sends.

The algorithm performed by the source of Flow s

At times $t = 1, 2, \dots$, Flow s performs the following calculations:

1. Retrieve the price $q_s(t)$ from the headers of the acknowledgements;
2. Compute a new transmission rate for the next period

$$x_s(t+1) = D(q_s(t)). \quad (7)$$

3. Within one round trip time, the new rate $x_s(t+1)$ will be observed by those nodes $j \in A(s)$ in its path.

B. Max-min fairness proof

In this section we will show that the proposed algorithm leads to max-min fairness, when all flows have equal demand

functions. Assuming all sources are saturated, a rate allocation is max-min fair if it is not possible to increase the allocated rate of a user without lowering the rate of any other user whose rate is already lower than this user's [14].

Max-min fairness results when all flows have the same demand function, $D(q_s)$, which is assumed to be continuous, positive and decreasing. Recall $x_s = D(q_s)$, where q_s is the maximum price of any node traversed by Flow s .

Let us partition the nodes into classes T_i , such that $p_n = p_{n'}$ for all n, n' in T_i , and $p_n > p_{n'}$ for all n in T_i and n' in T_j with $j > i$. Let P_i be the price of nodes in class T_i . The network must have at least one node corresponding to the maximum price, P_0 . All the flows through node(s) in T_0 will be marked with congestion price $q_s = P_0$ because it is maximal in the network. Let $S_i(j)$ be the set of flows traversing Node j which traverse nodes in T_i , but do not traverse nodes in $T_{i'}$ for any $i' < i$. Let S_i be the union of all $S_i(j)$. For $s \in S_i$, the price is $q_s = P_i$. Let $X_i = D(P_i)$ be the rate of any flow $s \in S_i$. In particular, $X_0 = D(P_0)$. Noting that $S_0(j) = S(j)$ for all $j \in T_0$, the updating rule at a node $j \in T_0$ is

$$p_j(t+1) = p_j(t) + \gamma_0 \left(\sum_{s \in S_0(j)} D(P_0) \cdot \left(\frac{1}{C_{l_i(j,s)}} + \frac{1}{C_{l_o(j,s)}} \right) - 1 \right). \quad (8)$$

In steady state $p_j(t+1) = p_j(t) = P_0$ for $j \in T_0$. Thus

$$P_0 = D^{-1} \left(\frac{1}{\sum_{s \in S_0(j)} \left(\frac{1}{C_{l_i(j,s)}} + \frac{1}{C_{l_o(j,s)}} \right)} \right), \quad j \in T_0. \quad (9)$$

The flows of rate X_0 have equal rate at their bottleneck nodes. If we apply the max-min condition only to this set of minimum rate flows, we see that their rates are equal to their max-min fair rates, since they are feasible and on account of the constraint at their bottleneck nodes, no rate can be increased without decreasing another flow within S_0 , and flows not in S_0 have greater rates. We now extend this argument to include flows with price P_1 . The equilibrium price at any node $j \in T_1$ must satisfy

$$P_1(t+1) = P_1(t) + \gamma_1 \left(\sum_{s \in S_1(j)} D(P_1) \cdot \left(\frac{1}{C_{l_i(j,s)}} + \frac{1}{C_{l_o(j,s)}} \right) + \sum_{s \in S_0(j)} D(P_0) \cdot \left(\frac{1}{C_{l_i(j,s)}} + \frac{1}{C_{l_o(j,s)}} \right) - 1 \right). \quad (10)$$

This implies

$$P_1 = D^{-1} \left(\frac{1 - \sum_{s \in S_0(j)} D(P_0) \cdot \left(\frac{1}{C_{l_i(j,s)}} + \frac{1}{C_{l_o(j,s)}} \right)}{\sum_{s \in S_1(j)} \left(\frac{1}{C_{l_i(j,s)}} + \frac{1}{C_{l_o(j,s)}} \right)} \right), \quad j \in T_1. \quad (11)$$

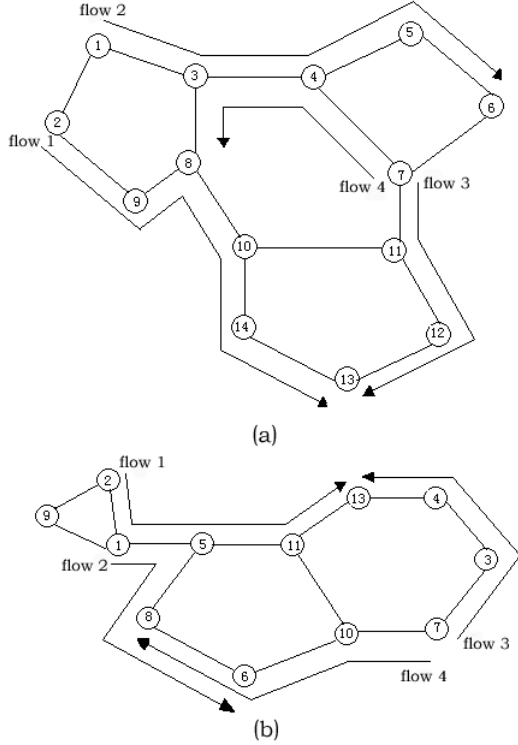


Fig. 3. Network topology (a) before 300 steps, (b) after 300 steps

The flows with price $q_s = P_0$ are not controlled at this node, and we have already shown that their rates are max-min fair. All other flows share the remaining resource of the node and also have equal rate. Their rates are also max-min fair, since they are feasible and on account of the constraint at their bottleneck nodes, no rate can be increased without decreasing another flow having equal or lower rate. Let us assume that the flows with prices $P_{k-1}, P_{k-2}, \dots, P_0$ are max-min fair. Then we prove that the flows with price P_k are also max-min fair. We can conclude, $j \in T_k$:

$$P_k = D^{-1} \left(\frac{1 - \sum_{s \in S_{k-1}(j)} D(P_{k-1}) \cdot \left(\frac{1}{C_{l_i(j,s)}} + \frac{1}{C_{l_o(j,s)}} \right)}{\sum_{s \in S_k(j)} \left(\frac{1}{C_{l_i(j,s)}} + \frac{1}{C_{l_o(j,s)}} \right)} \right. \\ \left. - \dots - \frac{\sum_{s \in S_0(j)} D(P_0) \cdot \left(\frac{1}{C_{l_i(j,s)}} + \frac{1}{C_{l_o(j,s)}} \right)}{\sum_{s \in S_k(j)} \left(\frac{1}{C_{l_i(j,s)}} + \frac{1}{C_{l_o(j,s)}} \right)} \right). \quad (12)$$

Then flows with the price $P_{k-1}, P_{k-2}, \dots, P_0$ have been proven to be max-min fair. The remaining resource of this node $j \in T_k$ is shared by the flows with the price P_k . These flows are also max-min fair. By induction, all flows are max-min fair and global fairness is thus achieved.

IV. SIMULATION RESULTS

In this section we evaluate the performance of our proposed algorithm by simulation. We focus on two objectives here: (1) to compare the rate allocation in the case where a node cannot receive and transmit simultaneously (which is the wireless

TABLE I
COMPARISON OF RATE ALLOCATION IN WIRELINE AND WIRELESS NETWORKS

	x_1	x_2	x_3	x_4
Wireline rate before 300 steps	1	0.5	1	0.5
Wireless rate before 300 steps	0.375	0.25	0.5	0.25
Wireline rate after 300 steps	0.5	0.5	1	0.5
Wireless rate after 300 steps	0.25	0.25	0.5	0.375

case) versus the case where this constraint does not exist (which is the wireline case); (2) to gain insight into how the performance of the algorithm is affected by wireless network topology variations. To meet the first objective, we consider two networks, designated “wireless” and “wireline”. Our two networks are the same in all aspects except that in the wireless network, a node cannot receive and transmit simultaneously. For both cases, we consider a network with flow 1, flow 2, flow 3 and flow 4. We further assume that the channel capacity of wireless networks and the bandwidth of wireline networks are both 1 Mb/s.

In order to investigate the ability of the network to react to topology variations, we change the network topology after 300 updating steps of the power control algorithm. Figure 3 shows the topologies before and after this change. After the changeover of the topology, nodes 8, 12 and 14 become idle. Note that the source and destination of each flow are unchanged, and only the routings of the links are changed. The rate allocations in wireline networks and in wireless networks before and after the topology change at the iteration 300 are shown in Fig. 4 and Fig. 5, respectively.

Consider first the performance before the topology change. Based on link constraints, flow 2 and flow 3 shared the link between node 3 and node 4 equally. So the rate allocation in wireline networks is (1, 0.5, 1, 0.5). This equilibrium is reached after approximately 50 update steps. In wireless networks, because of time constraints, nodes 3, 4, 8, 11 and 12 are the bottleneck nodes. Node 3 offers equal bandwidth to all flows that traverse it. Thus flow 2 and flow 4 are offered 1/4 each at node 3. In the same way, flow 1 cannot receive more than 3/8 on account of constraint at node 8. Flow 3 is offered 3/4 at node 7 and is offered 5/8 at node 13 and offered 1/2 at node 11, 12. Thus, flow 3 cannot receive more than 1/2 on account of constraint at node 11 and node 12. The max-min fair shares are 3/8, 1/4, 1/2, 1/4 for flows 1, 2, 3, 4, respectively. We can see from Fig. 5, the rates of four flows all achieve the max-min fairness, again after approximately 50 updates.

We tabulate all the steady state rates of each flow in Table 1 to present comparisons of the rate allocation in the wireline and wireless networks before and after the network topology changes. In general, as illustrated in Table 1, the rates of flows in wireless networks are smaller than the rates of flows in wireline networks. The reason for this is the MAC constraint i.e., any node can’t simultaneously transmit or simultaneously receive in more than one link in wireless networks. Due to this, the time constraints in wireless networks reduce the utilization of link (channel) capacity.

The simulation results show that the algorithm converges

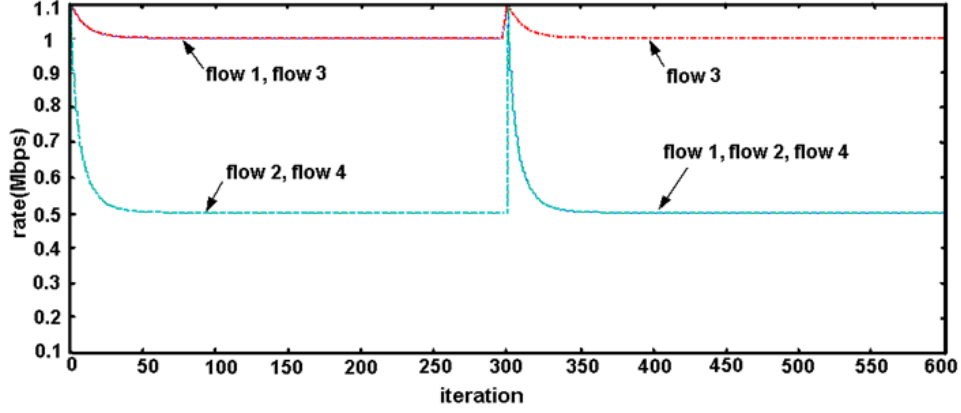


Fig. 4. Transmission rates in wireline networks

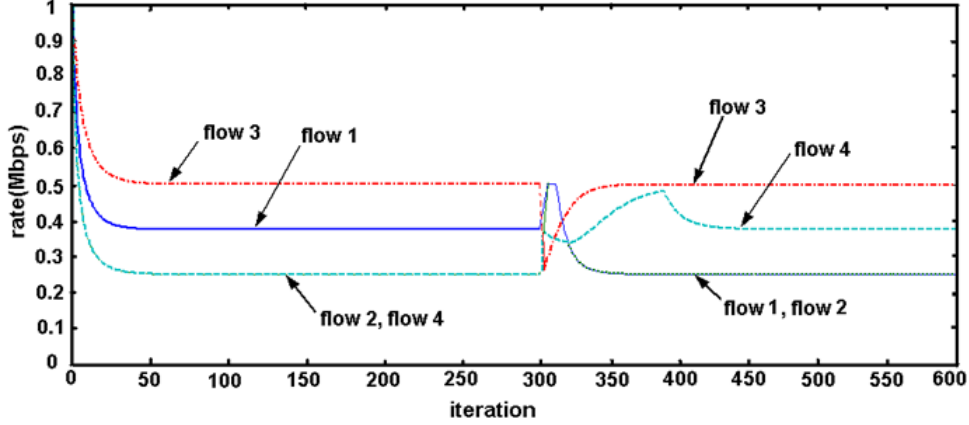


Fig. 5. Transmission rates in wireless networks

in approximately 50 updating steps. The actual speed of the updating depends on the interval between steps. In this simulation, it was assumed that the flow rates at each interval fully reflect the changes in the prices of the previous updating. That requires the update interval be at least one round trip time. If the network has a maximum round trip time of around 100 ms, the algorithm converges in approximately 5 seconds. The algorithm itself does not require that the update interval be less than the round trip time; however, if it is not, convergence may not be monotonic, and there may be oscillation before the equilibrium is reached.

At the 300th iteration, the topology changes. The rates for the wireline network undergo very similar transients to the ones when the flows first started. This is because all of the links used in the new topology are unused in the old topology, and so have price 0. In the wireless network, the transients are less severe, because prices are associated with nodes, and many of the nodes used in the new paths already have non-zero prices.

Once the system reaches equilibrium after the topology change, we can see that four flows again achieve max-min fairness. Under max-min fair allocation in the wireless network, every node offers equal bandwidth to all flows traversing the node. Thus flows 1 and 2 are offered $1/3$ each at node 1 and offered $1/4$ each at node 5. So flows 1 and 2 cannot receive more than $1/4$ on account of constraint at node 5. In the same way, flow 4 cannot receive more than $3/8$ on account

of constraint at node 6 and flow 3 cannot receive more than $1/2$ on account of constraint at node 3 and node 4. Thus the max-min fair shares are $1/4$, $1/4$, $1/2$, $3/8$ for flows 1, 2, 3, 4, respectively. The simulation result is consistent with the theoretical analyses of max-min fairness in wireless networks. Compared with the token-based local scheduling policy at each node to ensure max-min fairness [14], our price-based approach is designed as a fully distributed algorithm to achieve max-min fairness in wireless networks.

V. CONCLUSION

In this paper, we developed a MaxNet-based rate allocation algorithm for multi-hop wireless networks considering a MAC layer constraint whereby a node cannot transmit and receive packets simultaneously. This algorithm is designed to facilitate implementation in wireless networks in a distributed manner. We showed through both theoretical analysis and simulation results that our algorithm achieves max-min fair rate allocation.

APPENDIX

In this appendix, we apply Kelly's more general theory [6], [7] to our particular wireless networks, which involve the MAC constraint. Note that MaxNet [17] uses a price update rule of the same form as derived in [6]-[8] to optimize the aggregate utility, even though MaxNet does not itself seek to maximize the utility. We adopt the same approach here.

The optimization of the aggregated utility function of the set of flows subject to the MAC constraint can be formulated as follows:

$$\mathbf{P} : \max \sum_s U_s(x_s) \quad (13)$$

$$\text{Subject to : } \sum_{s \in S(j)} x_s \left(\frac{1}{C_{l_i(j,s)}} + \frac{1}{C_{l_o(j,s)}} \right) \leq 1. \quad (14)$$

We observe that the objective function of (13) is strictly concave. Similar to [6], [7], we define the Lagrangian:

$$\begin{aligned} L(x, p) &= \sum_s U_s(x_s) - \sum_{j: s \in S(j)} p_j \left(\sum_{s \in S(j)} x_s \left(\frac{1}{C_{l_i(j,s)}} + \frac{1}{C_{l_o(j,s)}} \right) - 1 \right) \\ &= \sum_s \left(U_s(x_s) - x_s \sum_{j: s \in S(j)} \left(\frac{1}{C_{l_i(j,s)}} + \frac{1}{C_{l_o(j,s)}} \right) p_j \right) \\ &\quad + \sum_{j: s \in S(j)} p_j \end{aligned} \quad (15)$$

Following [8], we further have

$$\begin{aligned} D(p) &= \max_{x_s} L(x, p) = \sum_{j: s \in S(j)} p_j \\ &\quad + \sum_s \max \left(U_s(x_s) - x_s \sum_{j: s \in S(j)} \left(\frac{1}{C_{l_i(j,s)}} + \frac{1}{C_{l_o(j,s)}} \right) p_j \right). \end{aligned} \quad (16)$$

Denote $x_s(p)$ is the unique maximizer in (16), so

$$\begin{aligned} D(p) &= \sum_{j: s \in S(j)} p_j \\ &\quad + \sum_s \left(U_s(x_s(p)) - x_s(p) \sum_{j: s \in S(j)} \left(\frac{1}{C_{l_i(j,s)}} + \frac{1}{C_{l_o(j,s)}} \right) p_j \right). \end{aligned} \quad (17)$$

Since U_s are strictly concave, $D(p)$ is continuously differentiable ([1, pp. 669]) with derivatives given by

$$\frac{\partial D}{\partial p_j}(p) = 1 - \sum_{s \in S(j)} x_s(p) \left(\frac{1}{C_{l_i(j,s)}} + \frac{1}{C_{l_o(j,s)}} \right). \quad (18)$$

We will solve the dual problem using gradient projection method [8] where node prices are adjusted in opposite direction to the gradient $\nabla D(p)$. The node prices can be adjusted according to the following equation:

$$p_j(t+1) = \left[p_j(t) - \gamma \frac{\partial D}{\partial p_j}(p(t)) \right]^+, \quad (19)$$

where the component γ is a step-size, and the function $[z]^+$ is defined as $[z]^+ = \max\{z, 0\}$. By substituting (18) into (19) one has

$$\begin{aligned} p_j(t+1) &= \left[p_j(t) + \gamma \left(\sum_{s \in S(j)} x_s(p(t)) \left(\frac{1}{C_{l_i(j,s)}} + \frac{1}{C_{l_o(j,s)}} \right) - 1 \right) \right]^+. \end{aligned} \quad (20)$$

This yields the required price-updating rule. ■

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