Reception Range Estimation in CDMA Multihop Cellular Networks

A. A. N. Ananda Kusuma Lachlan L. H. Andrew Stephen V. Hanly
ARC Special Research Centre for Ultra-Broadband Information Networks (CUBIN)
Department of Electrical and Electronic Engineering
The University of Melbourne, Victoria 3010, Australia
{a.kusuma,l.andrew,s.hanly}@ee.mu.oz.au

***Abstract*— In multihop networks, there is a trade-off between the transmission errors and hop size. In a system with base stations overlay, traffic is aggregated towards base stations and there is a need to investigate the impact of relaying to the user's own traffic. The optimal hop size and transmission strategy to maximise minimum user's throughput is studied in the context of a spread-spectrum CDMA multihop cellular networks. A heuristic and receiver-based routing algorithm is proposed to achieve the target hop size. To estimate user's throughput, an approximate model is introduced, in addition to simulation. We observed that the improvement in minimum user's throughput can be made by allowing target radius for base station and the other nodes to be different.**

I. Introduction

There has been a lot interest in deploying ad-hoc networks (packet radio networks) where a set of nodes can be self-organised and help other nodes' data to their destination. Recently, ad-hoc networks have also been used in the context of cellular or multihop cellular networks (MCN), to enhance performance in terms of power requirements,self-organisation, coverage and capacity [1–7].

In this work, the uplink of MCN is analysed where all traffic goes to the base stations . This causes traffic aggregation towards the base stations, where nodes near the base station carry more traffic than those further away. This raises the issue of fairness which we address by seeking to maximise the minimum throughput over all nodes in the networks (excluding traffic relayed on behalf of other nodes). In particular, our work is to investigate whether the transmission/reception range should be a function of distance from the base station. While there have been many studies into optimal range in ad-hoc networks with uniform traffic distribution [8–11], limited results are available for those with aggregated traffic [4,5].

II. Related Work

Early work on optimal transmission ranges in ad-hoc networks focused on TDMA system [8,9], and the concept of range is coupled with the notion of average number of terminals in a particular area of a random networks.

In the context of spread-spectrum, a model which includes the total contribution of received interference powers (affected by propagation model) was introduced in [10]. Assuming that the terminals can be positioned exactly at the chosen distance from a receiver, an optimal range can be determined. The model ignored thermal noise since it was assumed to be insignificant compared to the multi-user interference, and the optimal range is shown to be a function of processing gain. The concept for considering the total received powers [10] is adopted in [11] for the model from transmitting side which includes forward progress routing towards destination and transmit power adjustment. In [12], the work of [10] is extended by including fading. Nevertheless, in all of above, the transmit probabilities are assumed to be spatially uniform.

In our work, transmit probabilities are spatially non-uniform, which is influenced by the routing and transmission policy. A routing scheme is developed where the target range can be adjusted by varying the routing's radius.

III. Problem Formulation

The general problem in multihop networks is complex as it involves topology management, routing, MAC, power control, etc. We use the following simplified model.

A single cell with unit radius ($r_{max} = 1$) is assumed as shown in Fig. 1, with no influence from the neighbouring cells. The nodes are spatially distributed as a two-dimensional Poisson process. This follows [8–11], but it is contrast to [4,5] where the nodes are assumed to form a continuum and relays always found in the straight line to destination. Node density is given by λ per cell or λ/π per unit area. All the nodes transmit with constant (equal) transmit powers and no fading is assumed; thus, the power received from a unit-power transmitter at distance d is simply d^{-n} where n is the path loss exponent. This offers simplicity and potential implementation of networks with distributed and bursty nodes. Without fading and power control, the effect of interference is more localised, which justifies the single-cell assumption. Relevant studies with ideal power allocation are in [6,7].

Transmission is time-slotted, and a node can not transmit and receive simultaneously. Traffic sources are greedy and nodes always have data to forward. At each time slot, with probability p_o a node i will transmit its own data (new or retransmitted), and with a time-invariant probability $p_t(i) - p_o$ it will forward another node's data. Note that as p_o is fixed, the throughput is unchanged by the node retransmitting its own data. In contrast, $p_t(i)$ is set such that packets which are not receiving correctly are retransmitted.

Spread-spectrum multiple access with fixed carrier to interference ratio (CIR) requirement α is assumed. The effect of capture is ignored, and any packet received will be successfully decoded if its CIRs is greater than α. Capture in spread-spectrum ad-hoc networks is dependent on the employed spreading-code protocol. The difficulty typically comes due to the lack of central controller. In this work, there is central controller (base station), and all links are assumed to have independent spreading codes. Furthermore, multiple packet reception is a potential feature for future ad-hoc networks [13]. The Probability that $CIR > \alpha$ is denoted by p_s, and by [14,10,15],

$$CIR = \frac{Y_0}{\frac{2}{3L}\sum_i Y_i + N_o}, \tag{1}$$

where Y_0 is the target received power, Y_i the received power from the ith interferer, L is the processing gain, and N_o is the thermal noise. Our power unit is in the unit of N_o.

Nodes are stationary and the paths from all nodes to the base station form a tree T. T is parameterised by a target range (average hop size) $R(r)$ at distance r from the base station. A user's throughput u is defined as the probability of successfully transmitting its own data in a given slot. Recall that relays will retransmit the data to guarantee its arrival. Successful own's transmission requires three independent events: the node transmits (probability p_o), the receiver is not transmitting with probability $(1-p_t)$, the interference at the receiver is sufficiently low that, if node j transmits to i, it has $CIR > \alpha$ (probability $p_s(i)$). For a given p_o and T, user j's throughput is thus $u(j) = p_o(1-p_t(i))p_s(i)$. Averaging over realisation of the Poisson process, the throughput of a user is determined by its receiver's distance to the base station, r, by circular symmetry. On average a user is at distance $R(r)$ (the reception range) of the receiver. Using similar notation for average value, we write

$$u(r) = p_o \cdot (1-p_t(r)) \cdot p_s(r) \tag{2}$$

The aim is to find p_o and a means to generate T which will maximise the minimum expected transmission rate. Given $R(r)$, T depends on the routing algorithm. When transmit powers are constant, but traffic is spatially non-uniform, the success of transmission depends on the interference at the receiver, but no property of the transmitter. Thus, the optimal hop size is a property of the receiver, and we use the term "reception range" instead of the more familiar term "transmission range". The fundamental question is whether or not the reception range should be a function of distance from base station r, due to the aggregation of traffic. The performance objective is formulated as

$$\max_{p_o,R}\{\min_r u(r)\}. \tag{3}$$

To make this concrete, the following section introduces a specific routing algorithm, for which we will determine the optimal reception range $R(r)$. However, we are not advocating this as the optimal algorithm; it is hoped that the function $R(r)$ that we find will provide a tool for finding better algorithm.

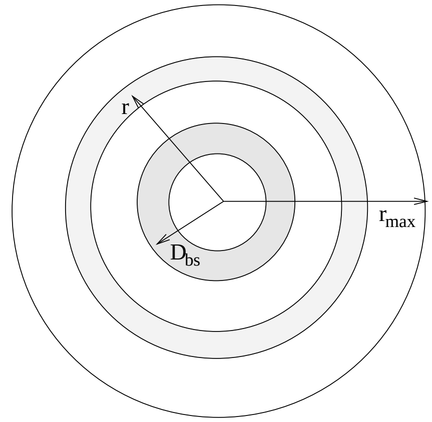

Fig. 1. Single-Cell Multihop Model

IV. Heuristic Routing Algorithm

Traditional routing assigns a fixed cost to each link. However, for wireless networks, the appropriate costs depend on the route eventually chosen, and so other methods must be used. Many algorithms [16] seek a path from transmitter to receiver to achieve its target transmission range. Because our target hop size is (potentially) dependent on the receiver's location, we propose a receiver-oriented algorithm.

The proposed routing algorithm grows a tree starting from the base station. Nodes are added in increasing order of their distance to the base station. A node is added by selecting the node (relay or BS) to which it will transmit. The choice of relay seeks to optimise multiple objectives, in decreasing order of priority: a receiver, j, should not receive from a node further than D_j away; loads should be balanced; the maximum hop length should be minimised; the hop count should be minimised.

The algorithm uses the following quantities: $d(i,j)$ is the distance between nodes i and j; the path $P(j) = (P_{j,N_j},\ldots,P_{j,0}) = (j,\ldots,bs)$ is the ordered list, of length N_j, of nodes which carry a packet from node j to the base station; $m_j = \max_n(d(P_{j,n},P_{j,n-1}))$ is the maximum hop length on path $P(j)$; $m_{i,j} = \max(d(i,j),m_j)$ is the maximum hop length which *would* result from relaying packets from i via j; $f(j)$ is the number of flows relayed by the most heavily loaded node on path $P(j)$, that is, the number of nodes n such that $P_{j,1} = P_{n,1}$; A is the set of allocated nodes. Note that $f(j)$ increases as more nodes are added to the tree, while the other quantities are constant.

Two greedy passes are used; nodes which cannot be connected in the first pass without violating the receive range constraint are pushed onto a queue to be processed in the second pass. In each pass, a subroutine AdOneOf() is called. The algorithm can now be stated as follows:

$P(bs) \leftarrow (bs)$, $A \leftarrow bs$, $f(j) \leftarrow 0$ for all j
foreach node i, in increasing $d(i,bs)$
  $J_1 \leftarrow \{j \in A : d(i,j) < D_j\}$
  if $J_1 = \emptyset$
    push i into the queue
  else
    AddOneOf(J_1)
  endif

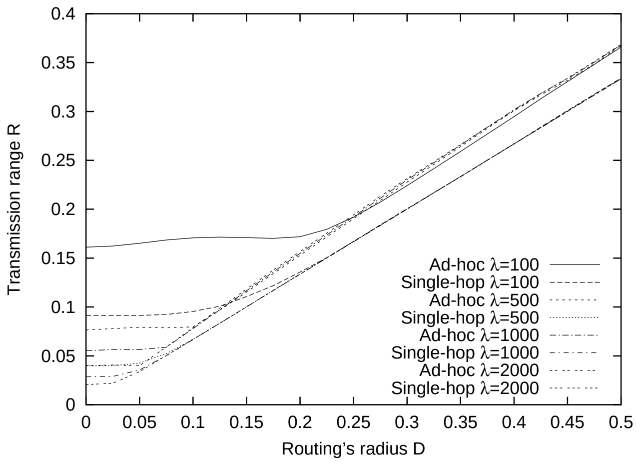

Fig. 2. Relationship between R and D.

endfor
while the queue is not empty
  pop i
  AddOneOf(A)
endwhile
Subroutine AddOneOf(S)
$J_2 \leftarrow \{j \in S : f(j) = \min_{k \in S} f(k)\}$
$J_3 \leftarrow \{j \in J_2 : m_{i,j} = \min_{k \in J_2} m_{i,k}\}$
$J_4 \leftarrow \{j \in J_3 : N_j = \min_{k \in J_3} N_k\}$
$j = \arg\min_{k \in J_4} d(i, k)$
$P(i) \leftarrow (i, P(j))$
$N_i \leftarrow N_j + 1$
$m_i \leftarrow m_{i,j}$
$A \leftarrow A \cup i$
foreach k such that $P_{k,1} = P_{j,1}$
  $f(k) \leftarrow f(k) + 1$
endfor

The algorithm is centralised and it is still appropriate for table driven and base-centric routing protocol, e.g. [17], where the complete topology is available. Designing the distributed version of the algorithm is our future work.

The algorithm maps the specified radius D to the target range (average hop size) R. Fig. 2 shows the linear relationship between R and D. Due to the nature of Poisson process, the position of the terminals are uniform, and for single-hop case it can be shown that $R = 2/3 \cdot D$. In ad-hoc region, R can be larger than $2/3 \cdot D$ since the routing policy allows a link to hop over a particular terminal. The algorithm has an edge-effect when no terminal is found within its target radius, it then connects to the next nearest terminal. Thus, R is clipped at $D_{min} \approx 1/\sqrt{\lambda}$ in single-hop region and $D_{min} \approx 2/\sqrt{\lambda}$ in ad-hoc region, where λ is the terminal density per cell of π unit area. We consider the deviation from $R = 2/3 \cdot D$ is not too much, and for simplicity we assume $R = 2/3 \cdot D$ in both region as long as $D > D_{min}$.

V. Benefits of spatially varying reception ranges

For computationally tractability, we limit ourselves to having one reception range, R_{bs} for the base station and another R_{ms} for all other nodes. Both R_{bs} and R_{ms}, together with p_o are optimised by exhaustive simulation (as described in VII).

λ	p_o	D_{ms}	D_{bs}	u
100	8.504e-2	4.333e-1	4.333e-1	2.82E-02
100	8.541e-2	3.500e-1	4.600e-1	3.136E-02

TABLE I
Comparison of uniform and spatially varying R for $\lambda = 100$.

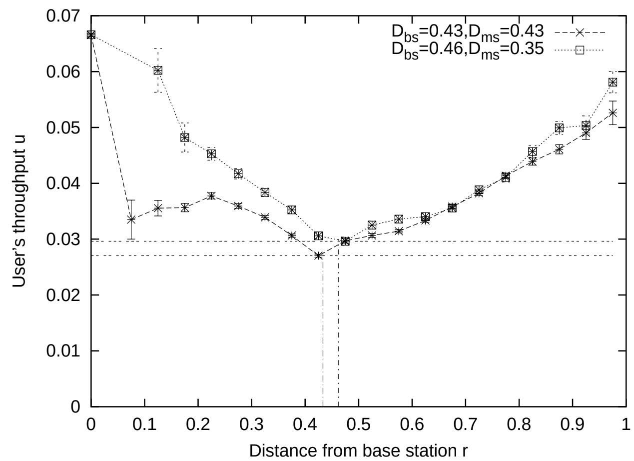

Fig. 3. User's throughput, with $\lambda = 100$ per cell

Table I and Fig. 3 demonstrate that under optimal condition there is a benefit from allowing R to vary spatially. However, exhaustive search by means of simulation is prohibitively slow. The rest of this paper will investigate a model which is simple enough to allow R_{bs} and R_{ms} to be determined more efficiently.

VI. Analytical Modelling

A. Two-hop model

From Fig. 3, it can be seen that the nodes whose data being relayed by the receiver at $r \approx D_b s$ are the worst. The intuitive reason is because all nodes at $r < D_{bs}$ will share all the load from traffic generated in ad-hoc region, and the relay nodes at $\approx D_{bs}$ have the worst wireless channel to the base station. Thus, the task reduces to modelling the throughput of two-hop path as shown in Fig. 4. A proportion of traffic generated in ad-hoc region is pumped from A, forwarded by O to bs. The throughput of A, $u_A = p_o \cdot (1 - p_t(D_{bs})) \cdot p_s^A$, and $p_t(D_{bs}) = (p_s^A \cdot p_f^O)/p_s^O + p_o$ where p_f^O reflects the amount of traffic from A to be forwarded by O to bs. The parameters used in this computation are affected by the choice of p_o and the reception ranges. The following section explains how those parameters are estimated.

B. Transmit Probability

To directly model $p_t(r)$ by including transmission policy is complicated. An easier way is to estimate based only on routing configuration. Assuming p_o is very small, then $p_s \approx 1$ and $p_t(r)$ is basically found by counting the number of flows at r.

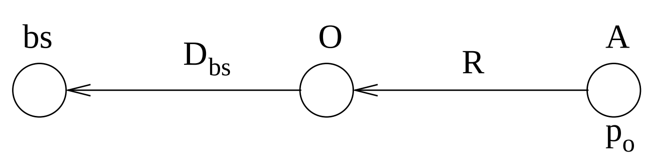

Fig. 4. Two-hop path

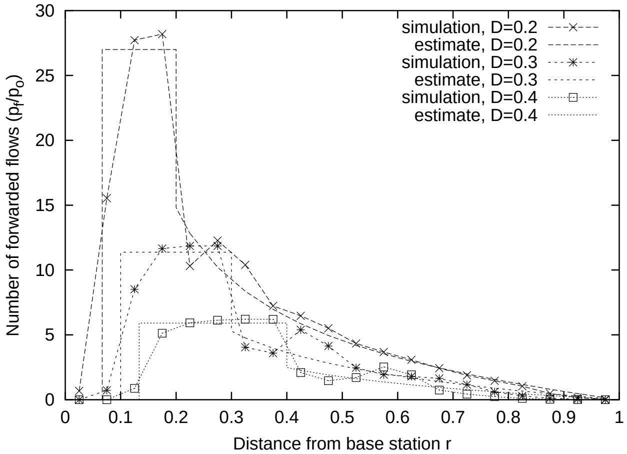

Fig. 5. Forwarding flows, $D_{bs} = D_{ms}$, with $\lambda = 1000$ per cell

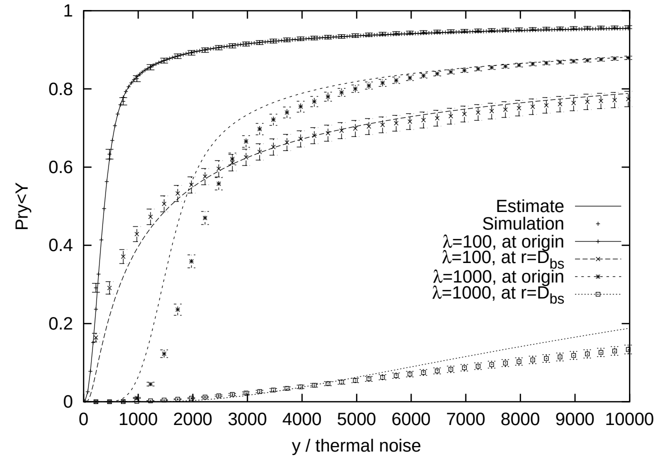

Fig. 6. Cumulative probability distribution of the received interference power y

By refering to Fig. 1, we make an assumption that all packets originating outside a circle of radius r must cross the circle; thus, the number of forwarded flows at r is proportional to the area outside the circle. The number of available relays also decreases as r decreases, which may result in an increase of forwarding load. Transmit probability is then $p_t(r) = p_f(r) + p_o$, where

$$p_f(r) = \frac{\kappa(r_{max}^2 - r^2)}{2r}. \qquad (4)$$

κ is the constant of proportionality (influenced by R), which needs to be found empirically, see Appendix.

All traffic generated in ad-hoc region must be forwarded to the base station by the relays inside base station coverage. Due to the flow load-balanced property of the routing algorithm, it can be seen that the number of forwarded flows tend to be constant at inside base station coverage ($r < D_{bs}$) and gradually drops as r decreases due to the drop on the number of potential relays. We take piecewise approximation of p_f and the width of relay region inside base station coverage is approximately R. The peak of p_f at $r < D_{bs}$ can be determined by distributing all traffic from ad-hoc region to the last relays to base station, i.e. $p_f(D_{bs}^2 - (D_{bs} - R)^2) = p_o(r_{max}^2 - D_{bs}^2)$. Thus,

$$p_f(r) = \frac{p_o(r_{max}^2 - D_{bs}^2)}{R(2D_{bs} - R)}. \qquad (5)$$

We refer $p_f(r)$ in 4 as p_f^A and in 5 as p_f^O. From Fig. 5, the piecewise approximation results are close with simulation results. It can be observed that the shape of $p_t(r)$ can vary by varying R and D_{bs}.

C. Statistics of Interference

To compute p_s, the statistics of interference at the receivers need to be computed. Since the nodes are governed by Poisson random process on the plane A, then transmitting nodes are also Poisson random process X_t with intensity $\lambda_t(r) = p_t(r)\lambda$. With each sample function of X_t, at each receiving point, we can associate the random variable of the received interference power $Y = \sum g(x_i)$, $g(x_i) = d_i^{-n}$, where d_i is the distance of the i^{th} point in the sample function to the receiving point. The characteristic function of Y (Campbell's theorem [18]),

$$E(\exp(zY)) = \exp\Bigg(\int_A \Big(\exp(zg(dA)) - 1\Big)\lambda_t(dA)\Bigg) \qquad (6)$$

When λ_t is constant as in spatially uniform traffic, the results are presented in [10], and for path loss exponent $n = 4$, a closed form for cdf of Y is given by

$$F_Y(y) = erfc\Big(\frac{\pi^{3/2}\lambda_t}{2\sqrt{y}}\Big). \qquad (7)$$

In our work, $\lambda_t(r)$ is based on $p_t(r)$ derived in previous subsection, and $F_Y(y)$ is estimated at $r = 0$ and $r = D_{bs}$. We numerically computed the characteristic function of 6 by standard standard integration routine for oscillatory integrand [19]. We found that integration is stable for estimation at $r = 0$, but not at $r = D_{bs}$. We'll further investigate this. $F_Y(y)$ is then obtained by numerically inverting the characteristic function by means of fast numerical inversion of [20]. Numerical inversion was found to be stable. For the receiver at $r = D_{bs}$, we adopt locally uniform assumption and compute the average λ_t on the circle with radius R centred at $r = D_{bs}$, and use the equation 7.

p_s can be found from $F_Y(y)$ by setting the threshold y_t for successful reception which depends on target received power and CIR requirement.

Fig. 6 shows the comparison of the estimate $F_Y(y)$ with the one obtained from simulation with the same input parameters. The simulation data is described in Section VII. The estimate is more optimistic than simulation result, and it is due to the estimate of $p_t(r)$ does not look like Fig 5 when p_o is not small. Fig. 7 shows $p_t(r)$ using parameters estimated in Section VII. Generally, it is no longer flat at $r < D_{bs}$, but tend to peaked at $r = D_{bs}$ (more retransmission due to worse wireless link to the base station compared to other transmitters inside D_{bs}). The second peak at $r > D_b s$ is also due to more retransmission to the busiest node at $r = D_{bs}$. Further investigation will be carried out to accurately predict $p_t(r)$.

VII. Numerical Results

Simulation was done by taking average performance over realisations of Poisson process. On any realisation, a tree was

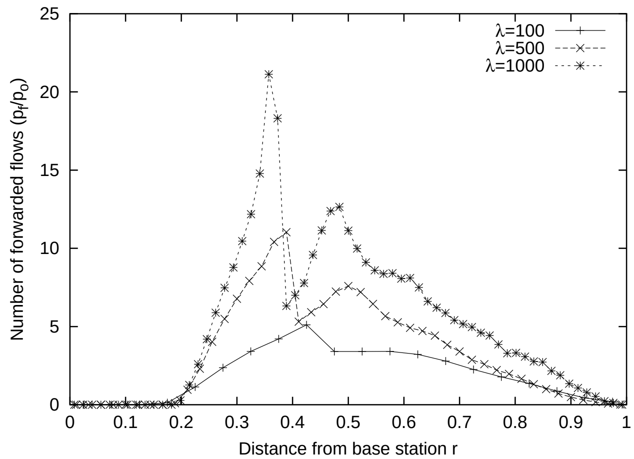

Fig. 7. Forwarding flows using estimated optimal parameters in table II

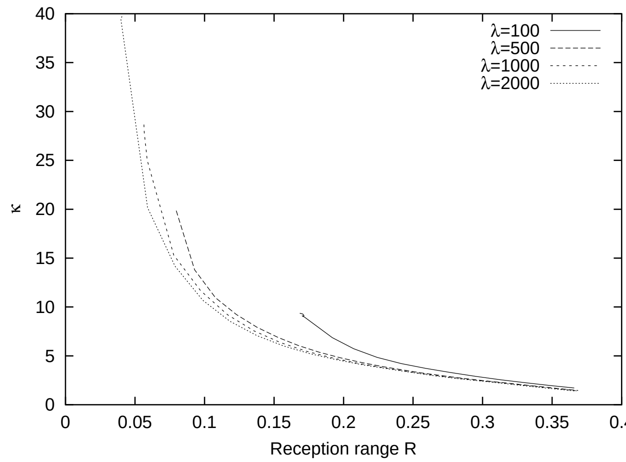

Fig. 8. κ vs Transmission Range R

λ	p_o	D_{ms}	D_{bs}	Two-hop u	Simulation u
100	4.430e-2	2.942e-1	4.542e-1	2.444e-2	2.575E-02
500	1.828e-2	2.084e-1	3.937e-1	1.053e-2	9.987E-03
1000	1.114e-2	1.830e-1	3.727e-1	6.610e-3	4.833E-03

TABLE II
COMPARISON USING THE ESTIMATED PARAMETERS

formed based on algorithm in IV. Fixed-point iteration was done to iteratively update p_t^i and p_s^i of all nodes; hence their throughput. The wireless parameters used are $\alpha = 6.6$ dB,$L = 128$, $N_o = 3.5$ (maximum routing's radius when there are no interfering nodes = 0.5).

A. Statistics of interference

For a collection of nodes with their transmit probability p_t^i, $F_Y(y)$ at any point can be computed directly by numerical inversion of [20], since the Laplace transform for $F_Y(y)$ can be found in closed-form. We found that for a large number of nodes, computing p_s using numerical inversion is more feasible and faster than direct computation using recursive algorithm of [15]. Fig 6 shows the average of $F_Y(y)$ over many realisations.

B. Optimisation

Optimisation was carried out both in simulation and two-hop model. In simulation, exhaustive search was done by bracketing D_{ms} and D_{bs} and one-dimensional search was performed for p_o. It is feasible for small λ, and the optimal parameters are shown in Table I and Fig. 3.

For two-hop model, optimal parameters were estimated by using standard numerical conjugate gradient algorithm in [19]. Table II shows the results obtained by two-hop model and the corresponding results from simulation using the estimated parameters. Even though the throughput values predicted are close, we haven't showed the true optimal parameters for large λ. Nevertheless, the results estimated by two-hop model can be used as a guidance for determining the appropriate ranges. Our work will continue to improve the approximate model, and develop a feasible optimal simulators for large λ as a benchmark.

VIII. CONCLUSION

Reception range is an important property of packet-oriented multihop cellular networks. An improvement in minimum user's throughput can be made by allowing different target range for the base station and other relays. A receiver-based routing algorithm which utilises the concept of reception range is proposed. Furthermore, an approximate model is introduced for fast estimation of the optimal parameters.

ACKNOWLEDGMENT

We thank Taka Sakurai for his assistance. This work was supported by the Australian Research Council.

APPENDIX I
FITTING PARAMETERS FOR THE ESTIMATE TRANSMIT PROBABILITY

κ measures expected number of times the traffic outside r to be forwarded by relays in annulus at r. Fig 8 shows the relationship between κ and R. It can be seen that κ decreases (hence $p_f(r)$) as R increases and vice versa. Thus, κ is fitted as $\kappa = c/R^\theta$. Fig 9 shows the fitting parameters for several values of λ. Asymptotic property was observed for dense networks, and we defer the theoretical study for future work. For the rest of the experiment, we take $c \approx 0.427$ and $\theta \approx 1.424$.

REFERENCES

[1] 3GPP, "Opportunity driven multiple access," Technical Report 3G TR 25.924 v 1.0.0, 3G Partnership Project, Dec. 1999.
[2] A. G. Spilling, A. R. Nix, M. A. Beach, and T. J. Harrold, "Self-organization in future mobile communications," *Electron. Commun. Engin. J.*, vol. 12, no. 3, pp. 133–147, June 2000.
[3] T. Rouse, I. Band, and S. McLaughlin, "Capacity and power investigation of opportunity driven multiple access (ODMA) networks in TDD-CDMA based systems," in *Proc. IEEE Int. Conf. Commun. (ICC)*, 2002, vol. 5, pp. 3202–3206.
[4] Yu-Ching Hsu and Ying-Dar Lin, "Multihop cellular: A novel architectures for wireless data communications," *Journal of Communications and Networks*, vol. 4, no. 1, pp. 30–39, 2002.
[5] Ying-Dar Lin and Yu-Ching Hsu, "Multihop cellular: A new architecture for wireless communications," in *Proc. IEEE INFOCOM*, 2000, pp. 1273–1282.
[6] Ali N. Zadeh, Bijan Jabbari, Raymond Pickholtz, and Branimir Vojcic, "Self-organizing packet radio ad hoc networks with overlay (SOPRANO)," *IEEE Communications Magazine*, vol. 40, no. 6, pp. 149–157, 2002.

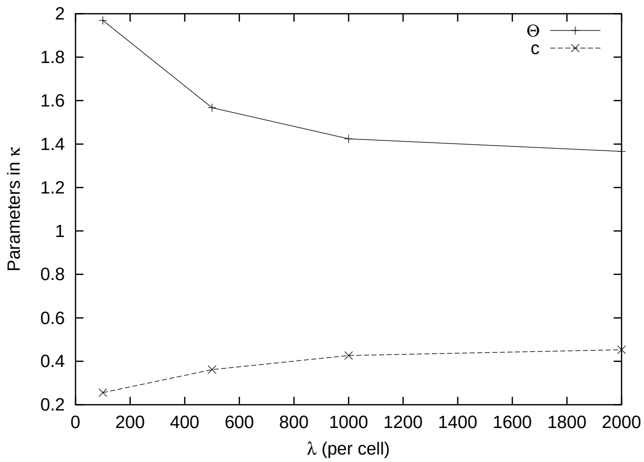

Fig. 9. Parameters in $\kappa = \frac{c}{R^{\theta}}$

[7] A. A. N. Ananda Kusuma and Lachlan L. H. Andrew, “Minimum power routing for multihop cellular networks,” in *Proc. IEEE GLOBECOM*, 2002, pp. 37–41.

[8] L. Kleinrock and J. Silvester, “Optimum transmission radii for packet radio networks or why six is a magic number,” in *Proc. National Telecommunications Conference*, Dec. 1978, pp. 4.3.2–4.3.5.

[9] H. Takagi and L. Kleinrock, “Optimal transmission ranges for randomly distributed packet radio terminals,” *IEEE Trans. Commun.*, vol. 32, no. 3, pp. 246–257, Mar. 1984.

[10] Elvino S. Sousa and John A. Silvester, “Optimum transmission ranges in a direct-sequence spread-spectrum multihop packet radio network,” *IEEE J. Select. Areas Commun.*, vol. 8, no. 5, pp. 762–771, June 1990.

[11] Yukitoshi Sanada and Masao Nakagawa, “Power control techniques in a multihop CDMA packet radio network,” *IEICE Trans. Commun.*, vol. E79-B, no. 9, pp. 1287–1294, Sept. 1996.

[12] Michele Zorzi and Silvano Pupolin, “Optimum transmission ranges in multihop packet radio networks in the presence of fading,” *IEEE Trans. Commun.*, vol. 43, no. 7, pp. 2201–2205, July 1995.

[13] Lang Tong, Qing Zhao, and Gökhan Mergen, “Multipacket reception in random access wireless networks: From signal processing to optimal medium access control,” *IEEE Commun. Mag.*, vol. 39, no. 11, pp. 108–112, Nov. 2001.

[14] M. B. Pursley, “Performance evaluation for phase-coded spread-spectrum multiple-access communication - part 1: System analysis,” *IEEE Trans. Commun.*, vol. 25, no. 8, pp. 795–799, Aug. 1977.

[15] Elvino S. Sousa, John Silvester, and Thomas D. Papavassiliou, “Computer-aided modeling of spread spectrum packet radio networks,” *IEEE J. Select. Areas Commun.*, vol. 9, no. 1, pp. 48–58, Jan. 1991.

[16] Ivan Stojmenovic, “Position-based routing in ad hoc networks,” *IEEE Communications Magazine*, vol. 40, no. 7, pp. 128–134, July 2002.

[17] Yu-Ching Hsu and Ying-Dar Lin, “Base-centric routing protocol for multihop cellular networks,” in *Proc. IEEE GLOBECOM*, 2002.

[18] J.F.C. Kingman, *Poisson processes*, Oxford University Press, 1993.

[19] M. Galassi, J. Davies, J. Theiler, B. Gough, G. Jungman, M. Booth, and F. Rossi, *Gnu Scientific Library*, 2002.

[20] Joseph Abate and Ward Whitt, “Numerical inversion of laplace transforms of probability distributions,” *ORSA Journal on Computing*, vol. 7, no. 1, pp. 36–43, 1995.