Joint Allocation of Transmit Power Levels and Degrees of Freedom to Links in a Wireless Network

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Abstract— In this paper, we study a wireless network that has constant or slow-varying data rate requirements. The transmitters transmit to multiple receivers. The allocation of degrees of freedom (orthogonal codes, spectrum or time-slots) is based on a weight vector that is associated with each transmitter. The weight vector will specify how the degrees of freedom should be shared between the receivers associated with the transmitter. This model applies not only to the downlink of a cellular network, but also to more general wireless networks, including multi-hop networks. We devise two distributed algorithms in order to determine the minimal power allocation for the transmitters that will meet the data rate requirement between each transmitter-receiver pair. The main feature of our algorithms is that they can be executed at each transmitter independently with local information.

I. INTRODUCTION

Power control is an important aspect of wireless communications. It is useful both to reduce interference, and as a means of conserving energy. Much early power control work (such as [1]) concerns voice traffic, which is inelastic, and requires a fixed quality of service (QoS) level; power control was used to maintain a signal to interference ratio (SIR) which achieves that QoS.

More recent work [2], [3], [4] has considered power control in conjunction with scheduling and adaptive modulation in the provisioning of elastic data services, where data rates can vary over time. Little work [5], [6] has applied adaptive modulation and adaptive bandwidth allocation to the problem of minimising the transmit power for inelastic traffic, which is the topic of the present paper. We provide a simple, abstract model that allows us to consider a tradeoff between degrees of freedom (spectrum) and signal to interference ratio (spectral efficiency). We develop new power control algorithms for this model, provide convergence analysis, and investigate gains that accrue from the additional degrees of freedom available to the power control algorithm.

One application of our results is to the down link of a CDMA cellular network. Much of the prior work on power control for CDMA networks focuses on the uplink, but the down link is in fact the bottleneck link. Since orthogonality can be more or less preserved on the down link, the base station can vary the number of orthogonal codes between the users in the cell, and at the same time make commensurate adjustments to the signal to interference ratio requirements of each link. Interference to each link arises from the other base stations in the network sending at the same time. Another application

is to the downlink of multi-user OFDM cellular networks, where the number of orthogonal carriers to be allocated to each user is to be determined. Finally, our results also apply to the downlink of a time-slotted cellular network, where the number of slots to be allocated to each user is to be determined. From now on we will abstract the notion of "degrees of freedom" to represent either spectrum, codes or time-slots.

Our results also apply to more general wireless networks, including ad-hoc and multi-hop networks. The algorithms are decentralised; for each communication link in the network, it is required that control information be passed between the receiver and transmitter nodes, but no additional signalling is required. Important modeling assumptions we make are that links emanating from the same node are orthogonal to each other, and that the transmit power density at a node, across the degrees of freedom, is flat. We compare our schemes with schemes that do not make the assumption of a flat spectrum across degrees of freedom.

Our approach differs from the cellular down link power control in [7] in that, while we achieve fixed data rates per link, and use a fixed transmit power level for each transmit node, we use a variable number of degrees of freedom per link, and also variable SIR's. In [7], the power level along each link is determined whilst fixing the SIR levels of the links.

In this work, we develop two distributed algorithms to determine the minimal power allocation for the transmitters. Both algorithms can be run independently at each transmitting node using only local information: the node's transmit power level and the achieved SIR level at each receiver in its cell.

Mobility is not explicitly modelled. Link gains are modelled as static, which implicitly assumes that the algorithms converge fast enough to track changes in link gains. The algorithms are shown to converge from scratch in typically 10 to 100 iterations.

This approach is in contrast to Han and Liu's centralised channel-state-aware scheme [5], which continually varies the allocation of degrees of freedom (time slots) to achieve a target average rate.

The rest of the paper is organized as follows: In section II, we describe the system model and in Section III we formulate the power control problem as an optimization problem. In section IV, we solve the power control problem using two distributed algorithms and provide convergence analysis. Our algorithms are numerically evaluated in section V, and conclusions are drawn in Section VI.

II. Model

A network is modelled as a set of N transmit nodes, denoted by $\mathcal{N} = \{1, 2, \ldots, N\}$, with each transmit node $n \in \mathcal{N}$ having a set $\mathcal{C}_n$ of receive nodes, forming a "cell" around the transmit node. To each transmit node, $n \in \mathcal{N}$, is associated a transmit power level, p_n, and to each receive node $m \in \mathcal{C}_n$ is associated a receiver noise, σ_m^2. The path gain from node k to node m is $\Gamma_{k,m}$. The signal to interference and noise ratio (SIR) at node $m \in \mathcal{C}_n$ is given by

$$\gamma_{n,m}(\boldsymbol{p}) \quad = \quad \frac{\Gamma_{n,m} p_n}{\sum_{k \in \mathcal{N}, k \neq n} \Gamma_{k,m} p_k + \sigma_m^2} \qquad (1)$$

where $\boldsymbol{p} = (p_1, \cdots, p_N)$ is a vector of transmit powers, one for each node in $\mathcal{N}$.

The model above makes an assumption, which we later relax, that transmit power, p_n, is spread uniformly over all degrees of freedom. That is, the transmit power at a particular time and/or frequency does not depend on the receiver, $m \in \mathcal{C}_n$. Signal sets are chosen so that orthogonality is maintained between the links within a cell, but not between cells. If groups of cells are orthogonal, say due to frequency reuse partitioning or scheduling, then (1) may be applied to groups which are mutually non-orthogonal.

The capacity of a link is determined by both the SIR and the number of degrees of freedom available. The degrees of freedom are the carriers, timeslots or spreading codes, depending on whether the system uses (O)FDM, TDM/scheduling or CDMA. It is assumed that the number of degrees of freedom is large compared to the number of receivers per transmitter, allowing a continuous allocation of these resources. The apportionment degrees of freedom in cell n is modelled by a weight vector $\boldsymbol{\phi}_n \in \mathbb{R}^{L_n}$ where $L_n = |\mathcal{C}_n|$; Each transmit node n is assumed to have the same total number of degrees of freedom available, and allocates a proportion $\phi_{n,m}$ to receiver m. Thus $\sum_{m \in \mathcal{C}_n} \phi_{n,m} \leq 1$.

The function $f(\gamma)$ specifies the bandwidth in bits/sec, as a function of the signal to interference ratio, γ, for a link with access to all the degrees of freedom in the system. Thus the bandwidth in bits/sec of the link from transmit node n, to receive node m, $m \in \mathcal{C}_n$, is given by

$$R_{n,m} = \phi_{n,m} f(\gamma_{n,m}) \qquad (2)$$

A specific example is the function $f(\gamma) \equiv W \ln(1+\gamma)$, which applies if the link is optimal with respect to Shannon capacity, and the total available frequency spectrum is W Hz. However, for the sake of generality, we will assume only that $f(\gamma)$ is a continuous, increasing function of γ, with $f(0) = 0$.

One important application of this model is to the downlink of a cellular system. Here, the transmitting nodes are the base stations, the receiver nodes are the mobile terminals, and the "cells" are exactly with the usual "cells" in mobile radio telephony. However, the model is quite general, and is applicable to any wireless network in which each transmit terminal connects to a number of receiving terminals. To maximise spectral efficiency, links originating from the same node are typically orthogonal, but some reuse between nodes is necessary. This gives rise to the mutual interference in the model. The higher-layer resource allocation problem of scheduling or frequency planning is not modelled.

III. The power control problem

We assume that receiver node, m, has a bandwidth requirement of $w_{n,m}$ bits/sec, which, using equations (1) and (2), can be expressed by the constraint:

$$R_{n,m}(\boldsymbol{p}, \phi_{n,m}) \quad \geq \quad w_{n,m}. \qquad (3)$$

where

$$R_{n,m}(\boldsymbol{p}, \phi_{n,m}) \quad \equiv \quad \phi_{n,m}\ f(\gamma_{n,m}(\boldsymbol{p})), \qquad (4)$$

$\boldsymbol{p} \in \mathbb{R}^N$ is the vector of transmit powers of the transmit nodes in the network, and $\phi_{n,m}$ is the fraction of degrees of freedom allocated to link $[n, m]$ in cell C_n. The vector $\boldsymbol{\phi}_n \in \mathbb{R}^{L_n}$ is the vector of degrees of freedom allocated to the different links in cell C_n, and the double array, $\boldsymbol{\Phi}$, is the vector of such vectors.

Additional constraints which must be satisfied are:

$$\begin{aligned} p_n &\geq 0 \\ \phi_{n,m} &\geq 0, \qquad \forall\, m \in C_n,\ \forall n \in \mathcal{N}, \qquad (5)\\ \sum_{m \in C_n} \phi_{n,m} &\leq 1, \qquad \forall\, n \in \mathcal{N}. \end{aligned}$$

We make the following definition.

Definition 1: Let $\mathcal{S}$ be the set of $(\boldsymbol{p}, \boldsymbol{\Phi})$ that satisfy the constraints (3) and (5). Then, any $(\boldsymbol{p}, \boldsymbol{\Phi}) \in \mathcal{S}$ will meet the data rate requirements of the network.

The power control problem we consider is:

$$\min_{(\boldsymbol{p}, \boldsymbol{\Phi}) \in \mathcal{S}} \sum_{n \in \mathcal{N}} p_n. \qquad (6)$$

We note here that the optimal weight vector in terms of minimizing the transmit power level will be achieved with

$$\sum_{m \in C_n} \phi_{n,m} = 1. \qquad (7)$$

Consider a fixed $\boldsymbol{\Phi}$ for the network satisfying the constraints (3) and (5). For a transmitter n to be able to support a receiver $m \in C_n$ at power $\boldsymbol{p}$, we require (from (4) and (3)):

$$\gamma_{n,m}(\boldsymbol{p}) \quad \geq \quad f^{-1}\left(\frac{w_{n,m}}{\phi_{n,m}}\right).$$

Substituting the SIR expression from (1), we get the constraint on the transmit power level for transmitter n to be able to support receiver m:

$$p_n \quad \geq \quad J_{n,m}(\boldsymbol{p}, \phi_{n,m}) \equiv K_{n,m}(\boldsymbol{p}) \times f^{-1}\left(\frac{w_{n,m}}{\phi_{n,m}}\right) \quad (8)$$

where

$$K_{n,m}(\boldsymbol{p}) \quad \equiv \quad \frac{\left[\sum_{k \in \mathcal{N},\, k \neq n} \Gamma_{k,m} p_k + \sigma_m^2\right]}{\Gamma_{n,m}}. \qquad (9)$$

Note that $K_{n,m}(\boldsymbol{p})$ is monotonically increasing in $\boldsymbol{p}$. Thus, for transmitter n to be able to support all receivers $m \in C_n$:

$$p_n \quad \geq \quad \max_{m \in C_n} J_{n,m}(\boldsymbol{p}, \phi_{n,m}) \quad \equiv \quad J_n(\boldsymbol{p}, \boldsymbol{\phi}_n). \qquad (10)$$

The interference function [1] for the network is

$$\boldsymbol{J}(\boldsymbol{p}, \boldsymbol{\Phi}) \quad = \quad (J_1(\boldsymbol{p}, \phi_1), \dots, J_N(\boldsymbol{p}, \phi_N)),$$

i.e., we require that

$$\boldsymbol{p} \quad \geq \quad \boldsymbol{J}(\boldsymbol{p}, \boldsymbol{\Phi}). \tag{11}$$

IV. RESULTS

A. Preliminary results

First, we develop some preliminaries that will assist with the characterization of the solution(s) of the power control problem (6), and which are used in the formulation and analysis of our distributed algorithms. The proofs of the lemmas and theorems can be found in the full version of this paper [8].

Let $\hat{\phi}_{n,m}(q_n, \boldsymbol{p})$ denote the proportion of degrees of freedom required for receiver m to achieve its rate, when the interference is generated by $\boldsymbol{p}$ and the transmit power level at transmitter n is set to q_n (using (8)):

$$\hat{\phi}_{n,m}(q_n, \boldsymbol{p}) \quad \equiv \quad \frac{w_{n,m}}{f\left(\frac{q_n}{K_{n,m}(\boldsymbol{p})}\right)}. \tag{12}$$

Note that when $q_n = p_n$, (12) becomes,

$$\hat{\phi}_{n,m}(p_n, \boldsymbol{p}) \quad = \quad \frac{w_{n,m}}{f\left(\gamma_{n,m}(\boldsymbol{p})\right)}. \tag{13}$$

Define $\sigma_n(q_n, \boldsymbol{p})$ as

$$\sigma_n(q_n, \boldsymbol{p}) \quad \equiv \quad \sum_{m \in C_n} \hat{\phi}_{n,m}(q_n, \boldsymbol{p}). \tag{14}$$

We note that $\sigma_n(q_n, \boldsymbol{p})$ has the following properties:

- $\sigma_n(q_n, \boldsymbol{p})$ is monotonically decreasing in q_n.
- $\sigma_n(q_n, \boldsymbol{p})$ is monotonically increasing in $\boldsymbol{p}$.

Lemma 1: $\sigma_n(q_n, \boldsymbol{p}) \leq 1$ if and only if there is a ϕ_n such that the transmit power level of q_n is sufficient to satisfy each of the receivers $m \in C_n$ when the interference is generated by $\boldsymbol{p}$.

Lemma 2:

$$\sigma_n(\alpha p_n, \alpha \boldsymbol{p}) \quad > \quad \sigma_n(p_n, \boldsymbol{p}), \quad \text{if } \alpha < 1 \tag{15}$$

$$\sigma_n(\alpha p_n, \alpha \boldsymbol{p}) \quad < \quad \sigma_n(p_n, \boldsymbol{p}), \quad \text{if } \alpha > 1. \tag{16}$$

Lemma 3: $\pi(q_n) = \sigma_n(q_n, \boldsymbol{p}) - 1 = 0$ has a unique solution $\bar{q}_n$, and, for an arbitrary ϕ_n satisfying (7), the following holds:

$$\min_{m \in C_n} J_{n,m}(\boldsymbol{p}, \phi_{n,m}) \leq \bar{q}_n \leq \max_{m \in C_n} J_{n,m}(\boldsymbol{p}, \phi_{n,m}). \tag{17}$$

B. Characterization of the minimal solution

To show that the minimization problem (6) has a unique solution, we first consider the problem of finding an optimal weight vector $\boldsymbol{\Phi}$, given a fixed power allocation vector $\boldsymbol{p}$.

For each $n \in \mathcal{N}$, define $\boldsymbol{\Phi}_n$ by

$$\boldsymbol{\Phi}_n = \{\phi_n : \sum_{m \in C_n} \phi_{n,m} = 1 \text{ and } \phi_{n,m} \geq 0, \forall m \in C_n\},$$

which are the available weight vectors for transmitter n. An optimal weight vector $\phi_n^*(\boldsymbol{p})$ to use at transmitter n under $\boldsymbol{p}$ will solve the weight-optimization problem:

$$\min_{\phi_n \in \boldsymbol{\Phi}_n} J_n(\boldsymbol{p}, \phi_n). \tag{18}$$

Lemma 4:   1) If $\bar{\phi}_n$ solves the optimization problem (18), then there exists a $\bar{q}_n$ such that $J_{n,m}(\boldsymbol{p}, \bar{\phi}_{n,m}) = \bar{q}_n, \forall m \in C_n$.
2) $\bar{q}_n$ from 1) satisfies $\sigma_n(\bar{q}_n, \boldsymbol{p}) = 1$.
3) The solution to the optimization (18) is unique.

By Lemma 4, we can define

$$\phi_n^*(\boldsymbol{p}) \quad \equiv \quad \arg \min_{\phi_n \in \boldsymbol{\Phi}_n} J_n(\boldsymbol{p}, \phi_n).$$

Then, the requirement for the network to be able to support the required data rates is given by the vector inequality:

$$\boldsymbol{p} \quad \geq \quad \boldsymbol{J}(\boldsymbol{p}, \boldsymbol{\Phi}^*(\boldsymbol{p})) \quad \equiv \quad \boldsymbol{I}(\boldsymbol{p}). \tag{19}$$

To show that the solution to (6) is unique, we apply the framework for power control developed in [1]. In particular, it can easily be shown that the function $\boldsymbol{I}(\boldsymbol{p})$ defined in (19) is *standard* [1] (see [8] for details).

Theorem 1 in [1] states that if $\boldsymbol{I}(\boldsymbol{p})$ is standard, and if it has a fixed point, then the fixed point is unique. Lemma 1 in [1] shows that if there is any feasible power vector $\boldsymbol{p}$ satisfying

$$\boldsymbol{p} \geq \boldsymbol{I}(\boldsymbol{p}) \tag{20}$$

then $\boldsymbol{I}(\boldsymbol{p})$ has a unique fixed point, which is the *minimal* solution to (20). Applying these results to (19), we have that if $\mathcal{S}$ is nonempty, then there is a unique power allocation $\boldsymbol{p}^*$ which solves (6) and is minimal. Associated with $\boldsymbol{p}^*$ is the unique optimal weight vector $\boldsymbol{\Phi}^*(\boldsymbol{p}^*)$. The following theorem provides a useful characterization of the minimal solution.

Theorem 1: Suppose that $\mathcal{S} \neq \emptyset$. Then, the following statements hold.

1) The function $\boldsymbol{I}(\boldsymbol{p})$ defined in (19) has a unique fixed point $\boldsymbol{p}^*$. Furthermore,

$$\sigma_n(p_n^*, \boldsymbol{p}^*) \quad = \quad 1, \quad \forall n \in \mathcal{N}. \tag{21}$$

2) $(\boldsymbol{p}^*, \boldsymbol{\Phi}^*(\boldsymbol{p}^*))$ is the unique solution to (6).
3) If a power vector $\boldsymbol{p}^\dagger$ satisfies,

$$\sigma_n(p_n^\dagger, \boldsymbol{p}^\dagger) \quad = \quad 1, \quad \forall n \in \mathcal{N}, \tag{22}$$

then, $\boldsymbol{p}^\dagger = \boldsymbol{p}^*$.

C. Algorithm 1: 'Standard' algorithm

Our first distributed algorithm to solve the minimization problem (6) utilizes the fact that $\boldsymbol{I}(\boldsymbol{p})$ is standard. In [1], Theorem 2, it is shown that if the interference function is standard, and if the power control problem has feasible solutions, then the *standard power control algorithm* [1] will converge to the minimal solution. The standard power control algorithm is simply the iteration $p_n(t+1) = I_n(\boldsymbol{p}(t))$.

The only difficulty in applying the standard power control algorithm to the present case is that the interference function $\boldsymbol{I}(\boldsymbol{p})$ is not directly measurable. The value of $I_n(\boldsymbol{p})$ is obtained by solving the optimization (18). Rewriting (18) by expanding for $J_n(\boldsymbol{p}(t), \phi_n)$ by (10) and (8), it becomes:

$$\min_{\phi_n \in \boldsymbol{\Phi}_n} \max_{m \in C_n} \left\{ K_{n,m}(\boldsymbol{p}(t)) \times f^{-1}\left(\frac{w_{n,m}}{\phi_{n,m}}\right) \right\}.$$

We note that (9) can be rewritten as

$$K_{n,m}(\boldsymbol{p}(t)) = \frac{p_n(t)}{\gamma_{n,m}(\boldsymbol{p}(t))}$$

where $\gamma_{n,m}(\boldsymbol{p}(t))$ is the achieved SIR at receiver m at time t. Therefore the nonlinear optimization within (18) can be locally solved at each transmitter knowing only its own transmit power level and the achieved SIR levels at the receivers it transmits to.

Lemma 4 provides an effective means to determine the solution to problem (18). The solution can be determined by solving $\sigma_n(q_n, \boldsymbol{p}) = 1$ for q_n using standard methods such as Newton's method and bisection method described in [9]. The initial estimate for the Newton's method can be any $q_n > 0$. Lemma 3 (Eq. (17)) provides the bounds for the bisection method, that can be used to bracket the solution.

We describe the algorithm here:

Algorithm 1:

- **Initialization:** Start with an initial transmit power level $p_n^{(0)}$. Set $k = 0$.
- **Iteration k:**

  1) Using iterative methods, solve $\sigma_n(\bar{q}_n^{(k)}, \boldsymbol{p}^{(k)}) = 1$ for $\bar{q}_n^{(k)}$.
  2) Set $p_n^{(k+1)} = \bar{q}_n^{(k)}$.

The disadvantage of using Algorithm 1 is that it involves infinite computations (solving a nonlinear optimization problem (18) by iterative methods) at each step of the power control iteration. This motivates us to find an algorithm that interleaves the steps of weight vector optimization and the power control iteration.

D. Algorithm 2: Single iteration algorithm

In this section, we develop an algorithm that will only involve a single iteration to solve the minimization problem (6) by interleaving the steps of power control iteration and weight vector optimization.

Algorithm 2:

- **Initialization:** Start with an initial transmit power level $p_n^{(0)}$. Set $k = 0$.
- **Iteration k:**

  1) Compute $\phi_n^{(k)}$: Using (13) and (14),

  $$\phi_{n,m}^{(k)} = \frac{\hat{\phi}_{n,m}(p_n^{(k)}, \boldsymbol{p}^{(k)})}{\sigma_n(p_n^{(k)}, \boldsymbol{p}^{(k)})}, \quad \forall m \in C_n.$$

  2) Compute the transmit power level requirement $q_{n,m}^{(k)}$ of the receivers $m \in C_n$:

  $$q_{n,m}^{(k)} = J_{n,m}(\boldsymbol{p}^{(k)}, \phi_{n,m}^{(k)}), \quad \forall m \in C_n.$$

  3) Compute the transmit power level for the next iteration $p_n^{(k+1)}$:

  $$p_n^{(k+1)} = \begin{cases} \min_{m \in C_n} q_{n,m}^{(k)} & \text{if } \sigma_n(p_n^{(k)}, \boldsymbol{p}^{(k)}) > 1 \\ \max_{m \in C_n} q_{n,m}^{(k)} & \text{if } \sigma_n(p_n^{(k)}, \boldsymbol{p}^{(k)}) \leq 1 \end{cases}$$

Each iteration of Algorithm 2 can be considered as a mapping T from $\boldsymbol{p}^{(k)}$ to $\boldsymbol{p}^{(k+1)}$. The mapping T is continuous

and consists of two components $\boldsymbol{\Phi}$ and T': the weight vector update $\boldsymbol{\Phi}^{(k)} = \boldsymbol{\Phi}(\boldsymbol{p}^{(k)})$ and the power update $\boldsymbol{p}^{(k+1)} = T'(\boldsymbol{p}^{(k)}, \boldsymbol{\Phi}^{(k)})$. First, note that the function $\boldsymbol{\Phi}(\boldsymbol{p})$ is *not* the solution, $\boldsymbol{\Phi}^*(\boldsymbol{p})$, to the problem (18); it is defined above in step 1) of Algorithm 2. Second, note that the mapping, T, is not standard, and hence the results of [1] do not apply.

Before proceeding with the convergence analysis of Algorithm 2, we describe some useful properties of the mapping T that will be applied in the convergence analysis.

When the transmitters use $(\boldsymbol{p}, \boldsymbol{\Phi}(\boldsymbol{p}))$ for transmission, a transmitter $n \in \mathcal{N}$ can be classified by on whether $\sigma_n(p_n, \boldsymbol{p})$ is greater than, less than or equal to 1, as follows.

1) Its transmit power level p_n is insufficient to meet the data rates of any of the receivers $m \in C_n$ when

$$\sigma_n(p_n, \boldsymbol{p}) > 1. \tag{23}$$

This follows from the fact that a receiver m requires a proportion $\hat{\phi}_{n,m}(p_n, \boldsymbol{p})$ of degrees of freedom to achieve its data rate requirement when the power vector is $\boldsymbol{p}$, and $\phi_{n,m}(\boldsymbol{p}) < \hat{\phi}_{n,m}(p_n, \boldsymbol{p})$ when (23) holds. Consequently, we have $p_n < \min_{m \in C_n} J_{n,m}(\boldsymbol{p}, \phi_{n,m}(\boldsymbol{p}))$, i.e.,

$$p_n < T_n(\boldsymbol{p}). \tag{24}$$

Furthermore,

$$\sigma_n(T_n(\boldsymbol{p}), \boldsymbol{p}) \geq 1, \tag{25}$$

since $T_n(\boldsymbol{p})$ is only just sufficient to satisfy the receivers(s) with the minimum transmit power level requirement, when the interference is provided by $\boldsymbol{p}$.

2) Its transmit power level p_n is exactly sufficient to achieve all the data rates when

$$\sigma_n(p_n, \boldsymbol{p}) = 1. \tag{26}$$

Similarly,

$$p_n = T_n(\boldsymbol{p}) \tag{27}$$

whence

$$\sigma_n(T_n(\boldsymbol{p}), \boldsymbol{p}) = 1. \tag{28}$$

3) Its transmit power level p_n is more than sufficient when

$$\sigma_n(p_n, \boldsymbol{p}) < 1. \tag{29}$$

Since in this case $p_n \geq \max_{m \in C_n} J_{n,m}(\boldsymbol{p}, \phi_{n,m}(\boldsymbol{p}))$, we have

$$p_n > T_n(\boldsymbol{p}). \tag{30}$$

Furthermore,

$$\sigma_n(T_n(\boldsymbol{p}), \boldsymbol{p}) \leq 1, \tag{31}$$

since $T_n(\boldsymbol{p})$ is sufficient to satisfy each of the receivers $m \in C_n$.

Although T does not satisfy the key monotonicity condition required in Yates's framework [1], the following convergence analysis exploits some key inequalities related to the mapping T. These properties imply that to any sequence, $(\boldsymbol{p}^{(k)})_{k=0}^{\infty}$, generated by Algorithm 2, a monotonically non-increasing upper bounding sequence, and a monotonically non-decreasing

lower bounding sequence, can always be found with the property that both bounding sequences provably converge to the solution to (6).

The following definitions will be used to quantify the bounds on the transmit power levels.

Definition 2:

$$\alpha(\boldsymbol{p}) \quad \equiv \quad \min_{n \in \mathcal{N}} \frac{p_n}{p_n^*} \quad \text{and} \quad \beta(\boldsymbol{p}) \equiv \max_{n \in \mathcal{N}} \frac{p_n}{p_n^*}.$$

These definitions immediately imply that

$$\alpha(\boldsymbol{p})p_n^* \le p_n \ \le \ \beta(\boldsymbol{p})p_n^*, \quad \forall n \in \mathcal{N}. \tag{32}$$

The following lemma and its corollary provide the main properties of T that we need to use in the convergence analysis.

Lemma 5:   1) If $\sigma_n(p_n, \boldsymbol{p}) > 1$, then

$$\sigma_n(T_n(\boldsymbol{p}), \boldsymbol{p}) \quad \ge \quad 1, \tag{33}$$
$$\alpha(\boldsymbol{p})p_n^* \quad \le \quad p_n \quad < \quad T_n(\boldsymbol{p}). \tag{34}$$

2) If $\sigma_n(p_n, \boldsymbol{p}) = 1$, then

$$\sigma_n(T_n(\boldsymbol{p}), \boldsymbol{p}) \quad = \quad 1, \tag{35}$$
$$\alpha(\boldsymbol{p})p_n^* \quad \le \quad p_n \quad = \quad T_n(\boldsymbol{p}) \ \le \ \beta(\boldsymbol{p})p_n^*. \tag{36}$$

3) If $\sigma_n(p_n, \boldsymbol{p}) < 1$, then,

$$\sigma_n(T_n(\boldsymbol{p}), \boldsymbol{p}) \quad \le \quad 1, \tag{37}$$
$$T_n(\boldsymbol{p}) \quad < \quad p_n \quad \le \ \beta(\boldsymbol{p})p_n^*. \tag{38}$$

4) If $\sigma_n(p_n, \boldsymbol{p}) \ge 1$ and $\beta(\boldsymbol{p}) \ge 1$, then

$$T_n(\boldsymbol{p}) \quad \le \quad \beta(\boldsymbol{p})p_n^*. \tag{39}$$

5) Part 4) also holds with strict inequalities.
6) If $\sigma_n(p_n, \boldsymbol{p}) \le 1$ and $\alpha(\boldsymbol{p}) \le 1$, then

$$\alpha(\boldsymbol{p})p_n^* \quad \le \quad T_n(\boldsymbol{p}). \tag{40}$$

7) Part 6) also holds with strict inequalities.
8) If $\boldsymbol{p} \ge \boldsymbol{p}^*$ and $\sigma_n(p_n, \boldsymbol{p}) \le 1$, then $T_n(\boldsymbol{p}) \ge p_n^*$.
9) If $\boldsymbol{p} \le \boldsymbol{p}^*$ and $\sigma_n(p_n, \boldsymbol{p}) \ge 1$, then $T_n(\boldsymbol{p}) \le p_n^*$.

Corollary 1:   1) If $\boldsymbol{p} \ge \boldsymbol{p}^*$ then $T(\boldsymbol{p}) \ge \boldsymbol{p}^*$,
2) If $\boldsymbol{p} \le \boldsymbol{p}^*$ then $T(\boldsymbol{p}) \le \boldsymbol{p}^*$,
3) If $\alpha(\boldsymbol{p}) \le 1$, then, $T(\boldsymbol{p}) \ge \alpha(\boldsymbol{p})\boldsymbol{p}^*$, or equivalently, $\alpha(T(\boldsymbol{p})) \ge \alpha(\boldsymbol{p})$,
4) Part 3) also holds with strict inequalities,
5) If $\beta(\boldsymbol{p}) \ge 1$, then $T(\boldsymbol{p}) \le \beta(\boldsymbol{p})\boldsymbol{p}^*$, or equivalently, $\beta(T(\boldsymbol{p})) \le \beta(\boldsymbol{p})$,
6) Part 5) also holds with strict inequalities.

This gives rise to the following main result.

Theorem 2:   If $\mathcal{S} \ne \emptyset$, then, Algorithm 2 converges to the solution of the power control problem (6).

Proof: Consider the following cases.

1) If $1 \le \alpha(\boldsymbol{p}) \le \beta(\boldsymbol{p})$, then, $\boldsymbol{p}^* \le \alpha(\boldsymbol{p})\boldsymbol{p}^* \le \boldsymbol{p} \le \beta(\boldsymbol{p})\boldsymbol{p}^*$. By Corollary 1(5), we have $T(\boldsymbol{p}) \le \beta(\boldsymbol{p})\boldsymbol{p}^*$, and by Corollary 1(1), we have $T(\boldsymbol{p}) \ge \boldsymbol{p}^*$. It follows that

$$1 \quad \le \quad \alpha(T(\boldsymbol{p})) \ \le \ \beta(T(\boldsymbol{p})) \ \le \ \beta(\boldsymbol{p}).$$

Applying this argument inductively to the sequence $\boldsymbol{p}^{(k)} \equiv T^k(\boldsymbol{p})$, we have:

$$\boldsymbol{p}^* \quad \le \quad \boldsymbol{p}^{(k+1)} \le \beta(\boldsymbol{p}^{(k)})\boldsymbol{p}^* \text{ and}$$
$$1 \quad \le \quad \alpha(\boldsymbol{p}^{(k+1)}) \ \le \ \beta(\boldsymbol{p}^{(k+1)}) \ \le \ \beta(\boldsymbol{p}^{(k)}), \quad \forall k.$$

and thus $\beta(\boldsymbol{p}^{(k)}) \downarrow \beta^* \ge 1$ as $k \to \infty$. Note also that $\boldsymbol{p}^* \le \boldsymbol{p}^{(k)} \le \beta(\boldsymbol{p}^{(0)})\boldsymbol{p}^*$, $\forall k$ and hence $(\boldsymbol{p}^{(k)})_{k=0}^\infty$ is bounded. Let $(\boldsymbol{p}^{(k_i)})_{i=0}^\infty$ be a subsequence converging to an accumulation point $\boldsymbol{p}^\dagger$. By continuity of β,

$$\beta(\boldsymbol{p}^{(k_i)}) \quad \to \quad \beta(\boldsymbol{p}^\dagger)\beta^* \text{ as } i \to \infty$$
$$\beta(T(\boldsymbol{p}^{(k_i)})) \quad \to \quad \beta(T(\boldsymbol{p}^\dagger))\beta^* \text{ as } i \to \infty \tag{41}$$

By the continuity of T, $\boldsymbol{p}^\dagger$ must satisfy

$$\boldsymbol{p}^* \le T(\boldsymbol{p}^\dagger) \le \beta(T(\boldsymbol{p}^\dagger))\boldsymbol{p}^* = \beta^*\boldsymbol{p}^* \tag{42}$$

If $\beta^* > 1$, then by Corollary 1(6), $\beta(T(\boldsymbol{p}^\dagger)) < \beta^*$ which is a contradiction. Hence, $\beta^* = 1$ and $\beta(\boldsymbol{p}^{(k)}) \downarrow 1$ as $k \to \infty$. Therefore, $\boldsymbol{p}^{(k)} \to \boldsymbol{p}^*$ from (42).

2) If $\alpha(\boldsymbol{p}) \le \beta(\boldsymbol{p}) \le 1$, then, $\boldsymbol{p}^{(k)} \to \boldsymbol{p}^*$ follows from an analogous argument to that used in case 1, but with α's replacing β's, and using Corollary 1, parts 2, 3 and 4, to obtain an increasing sequence of $\alpha(\boldsymbol{p}^{(k)})$'s, in place of a decreasing sequence of $\beta(\boldsymbol{p}^{(k)})$'s.

3) If $\alpha(\boldsymbol{p}^{(k)}) < 1 < \beta(\boldsymbol{p}^{(k)})$, $\forall k$, then parts 4 and 6 of Corollary 1 imply that $(\alpha(\boldsymbol{p}^{(k)}))_{k=0}^\infty$ is a strictly increasing sequence and bounded above by 1, and $(\beta(\boldsymbol{p}^{(k)}))_{k=0}^\infty$ is a strictly decreasing sequence and bounded below by 1. Let the limits be α^* and β^* respectively. Then, $\alpha^* \le 1 \le \beta^*$. Note also $\alpha(\boldsymbol{p}^{(0)})\boldsymbol{p}^* \le \boldsymbol{p}^{(k)} \le \beta(\boldsymbol{p}^{(0)})\boldsymbol{p}^*$, $\forall k$. Therefore, the sequence $(\boldsymbol{p}^{(k)})_{k=0}^\infty$ has accumulation points. Let $(\boldsymbol{p}^{(k_i)})_{i=0}^\infty$ be a subsequence of points converging to an accumulation point $\boldsymbol{p}^\dagger$. By the continuity of α,

$$\alpha(\boldsymbol{p}^{(k_i)}) \quad \to \quad \alpha(\boldsymbol{p}^\dagger)\alpha^* \text{ as } i \to \infty$$
$$\alpha(T(\boldsymbol{p}^{(k_i)})) \quad \to \quad \alpha(T(\boldsymbol{p}^\dagger)) = \alpha^* \text{ as } i \to \infty \tag{43}$$

and similarly, by the continuity of β, (41) holds. Therefore,

$$\alpha(\boldsymbol{p}^\dagger) \quad = \quad \alpha(T(\boldsymbol{p}^\dagger)) \ = \ \alpha^* \ \le \ 1,$$
$$\beta(\boldsymbol{p}^\dagger) \quad = \quad \beta(T(\boldsymbol{p}^\dagger)) \ = \ \beta^* \ \ge \ 1$$

and $\alpha^*\boldsymbol{p}^* \le T(\boldsymbol{p}^\dagger) \le \beta^*\boldsymbol{p}^*$. If $\alpha^* < 1$, then by Corollary 1(4), $\alpha(T(\boldsymbol{p}^\dagger)) > \alpha^*$, which is a contradiction. Hence $\alpha^* = 1$. Similarly, if $\beta^* > 1$, then by Corollary 1(6) $\beta(T(\boldsymbol{p}^\dagger)) < \beta^*$, which is a contradiction. Hence $\beta^* = 1$. We conclude that $\boldsymbol{p}^\dagger = \boldsymbol{p}^*$.

4) There exists a $\kappa \in \mathbb{Z}_+$ such that $\alpha(\boldsymbol{p}^{(k)}) < 1 < \beta(\boldsymbol{p}^{(k)})$ for $k = 0, 1, \ldots, \kappa - 1$ and, $\alpha(\boldsymbol{p}^{(\kappa)}) \ge 1$ or $\beta(\boldsymbol{p}^{(\kappa)}) \le 1$. Then, at iteration $k = \kappa$, it falls under case 2) if $\beta(\boldsymbol{p}^{(\kappa)}) \le 1$ or case 1) if $\alpha(\boldsymbol{p}^{(\kappa)}) \ge 1$.

■

V. NUMERICAL RESULTS

To evaluate our algorithms, we compared their speed of convergence and the resulting power consumption with the following two algorithms:

Algorithm 3: Simple power control For each transmitter, ϕ_n is set proportional to the rate requirement of the receivers. Then, the transmit power level is repeatedly set to the level required to satisfy the data rate requirement of all receivers, given the current interference [1]. This is the simplest of the four algorithms.
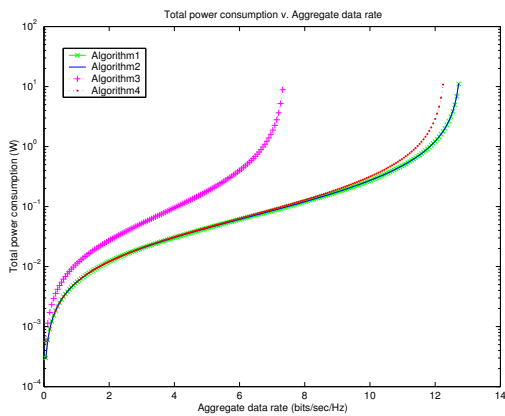
Fig. 1.   Total power consumption vs. Data rate (bits/sec/Hz) for each algorithm

Fig. 2.   Iteration count to within 1% of convergence for each algorithm

Algorithm 4: Per-link power control Again ϕ_n is proportional to the data rates. As in [7], the power level for each receiver is set separately to achieve its required rate. To calculate the interference, the transmit power of each node is assumed to be the same for all degrees of freedom. This models CDMA with independent codes at each transmitter.

We use a network that consists of 10 transmitters and 30 receivers, uniformly randomly distributed in an area of 100 m $times$ 100 m. Each receiver is assigned to the closest transmitter. Log-distance path loss [10] (with a path loss exponent of 4) and a log-normal shadowing with a mean of 0 dB and a standard deviation of 8 dB was used, with a reference distance of 10 m. The noise is 10^{-10} W at each receiver. The initial transmit power levels are the absolute values of a Gaussian with mean 0 and standard deviation 1.

Figure 1 shows the total power consumption for each algorithm, with varying data rates. Algorithms 1 and 2 use equal power as they solve the same optimisation. Figure 1 confirms that Algorithm 3 has the highest power consumption, as it has the fewest degrees of freedom to optimise. It must allocate each transmitter enough power for the worst link, rather than being able to balance the links, either by different symbol rates (as Algorithms 1 and 2) or powers (Algorithm 3). This leads to "power warfare" occurring at lower data rates. Algorithm 4 has a similar performance to Algorithms 1 and 2 for lower data rates, indicating that fine-grained power control provides almost as much benefit as coarse-grained power control with symbol rate adaptation.

Figure 2 shows the number of iterations each algorithm takes to converge to within 1%. Algorithm 2 requires only a few more iterations than Algorithm 1, despite using a sub-optimal symbol rate allocation at each step. Algorithm 3 takes longest to converge, since the power level it must reach is higher than that of the other algorithms. Algorithm 4 is again similar to Algorithms 1 and 2 for lower data rates.

VI. CONCLUSIONS

This paper has considered the impact on power consumption of symbol rate adaptation in systems in which one transmitter mu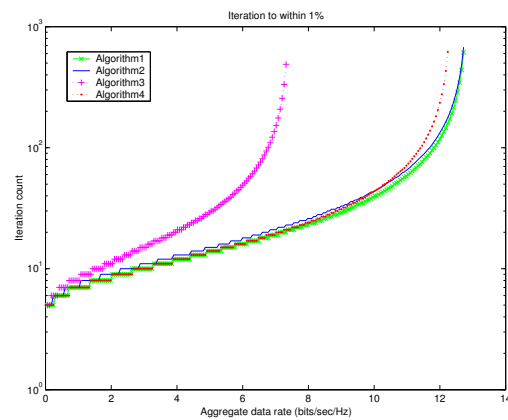st transmit to multiple receivers. It was assumed that the total symbol rate for the transmitter is fixed, but the proportion of symbols that are allocated to each receiver can be varied.

Two distributed algorithms have been proposed for allocating rates, and their convergence proven. The number of power control iterations required for convergence is only slightly higher when the rate allocation problem is only partially solved at each iteration, in the single iteration algorithm.

Optimally allocating rates while maintaining uniform powers provides a slight increase in capacity relative to the standard heuristic of allocating symbol rates in proportion to users' required data rates and then adapting users' powers individually. Which is the better method is not yet clear, and may depend on other factors beyond the scope of the present paper. Future work will examine the relative merits of our new approach, versus the standard approach, taking into account other metrics, such as power efficiency, peak to average power consumption, and decoding complexity.

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