# Large System Analysis of Linear Multistage Parallel Interference Cancellation

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***Abstract*—In this paper, we derive an expression for the signal to interference-plus-noise ratio of a linear multistage parallel interference cancellation receiver. We focus on a linear multistage receiver which converges to the linear minimum mean-squared error receiver as the number of stages increases. The signal to interference-plus-noise ratio is given in terms of the system loading, the partial cancellation factor, the number of stages, and the signal-to-noise ratio. Our expression also allows a simple approximation for the bit error rate at each stage. Finally, we perform a numerical optimization to maximize the signal to interference-plus-noise ratio expression with respect to the partial cancellation factor of the resulting linear multistage receiver.**

***Index Terms*—Large system analysis, linear minimum mean-squared error, multiuser detection, parallel interference cancellation, random spreading.**

## I. Introduction

MULTIUSER receivers can dramatically outperform conventional single-user matched filter (SUMF) code-division multiple-access (CDMA) receivers by exploiting the structure of multiple-access interference (MAI). Unfortunately, the optimal multiuser receiver [maximum-likelihood (ML)] is prohibitively complex for the system sizes of interest [1]. The linear minimum mean-squared error (LMMSE) receiver can outperform the conventional SUMF CDMA receiver, and can do so with less computational load than the optimal multiuser receiver [2]. Our paper focuses on a linear multistage parallel interference cancellation (IC) receiver which can achieve near-LMMSE performance with even less computational complexity.

In this paper, we consider a $K$ user-synchronous CDMA system with a processing gain $N$ employing long spreading sequences. Such sequences are much longer than the symbol interval. When long spreading sequences are used, (such as in IS-95 and allowed for in IMT-2000), the LMMSE receiver is different for every symbol interval, and has to be recalculated at a cost of $\mathcal{O}(K^3)$ per symbol. In this case, multistage receivers can reduce the computational cost to $\mathcal{O}(mK^2)$ or $\mathcal{O}(mKN)$ per symbol (depending on the implementation), where $m$ is the number of stages which does not depend on the system size.

The well-known IC receiver of [3] may be regarded as one of the first examples of a multistage IC receiver for CDMA. Each stage estimates and cancels MAI from the hard decisions of filtered measurements at the output of the preceding stage. However, incorrect decisions can propagate from one stage to the next, depending on the level of MAI. It is this problem that encouraged the development of numerous linear and nonlinear IC algorithms [2], [4]–[17]. An example is that of partial parallel IC (PIC) which was centered on analyzing the decision statistics at the output of an arbitrary stage of a multistage PIC receiver, where all user filter measurements are estimated simultaneously [8]–[10]. The amount of MAI cancelled is controlled by what we call partial cancellation factors. Design of the partial cancellation factors is crucial, since a careless choice of these parameters can adversely affect the performance. We shall focus on a receiver that resembles a linear multistage partial PIC receiver, which is linear in the sense that we utilize linear soft decisions in all stages; this linearity eases the performance analysis.

An alternative interpretation of linear multistage partial PIC receivers is that of linear iterative methods. Recently, a link has been established between linear multistage partial PIC and linear iterative methods which solve systems of linear equations [12], [14], [18], [19]. The mathematical literature of general linear iterative methods contains a host of useful results (based on extreme eigenvalues) for selecting appropriate partial cancellation factors. Using these techniques, we can derive a linear receiver structure where the filter coefficients are computed such that the filtered measurements converge to the decorrelator or LMMSE solution as the number of stages increases. In our case, our linear multistage receiver is based on the first-order stationary linear iterative method [12], [14]. The first-order stationary linear iterative method has only one partial cancellation factor over which to optimize the performance. Our linear receiver resembles a linear multistage partial PIC receiver which converges to the LMMSE receiver as the number of stages increases. We can attain near-LMMSE performance with a finite number of stages (which does not depend on the system size) providing a significant computational saving to that of the direct LMMSE receiver. In what follows, we call this receiver the linear multistage partial PIC (PPIC) receiver.

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In this paper, we analyze the performance of the linear multistage PPIC receiver. To do this, we build upon current results from large system analysis. A large system is defined by taking the system parameters $N$ and $K$ to infinity, but keeping their ratio held constant [20]–[24]. Using large system results, we derive an expression for the limiting output signal to interference-plus-noise ratio (SINR) of our receiver which we call the large system SINR. The large system SINR is a deterministic scalar independent of the eigenvalues of the correlation matrix and the signature sequence realizations. We can express it in terms of the system loading $(\alpha = K/N)$, the number of stages $m$, the partial cancellation factor, and the signal-to-noise ratio (SNR).

Our main contribution in this paper is the derivation for the large system SINR expression of the linear multistage PPIC receiver. We empirically show that the MSE between the SINR and the large system SINR of the linear multistage PPIC receiver decreases with a $1/N$ relationship, as $N$ increases with $\alpha = K/N$ held fixed. We numerically maximize the large system SINR with respect to the partial cancellation factor. We also compare the large system SINR of the optimized linear multistage PPIC receiver with the large system SINR of the multistage receiver given in [4] and [25], and described later on in this paper. The latter receiver is called the optimal linear multistage receiver, since it is optimal in the sense of optimising the large system SINR for a particular stage $m$, requiring $m$ filter coefficients (see [25]). Furthermore, we can use the large system SINR expression of the linear multistage PPIC receiver to approximate its bit error rate (BER) at stage $m$.

This paper is organized as follows. The CDMA system model is described in Section II, and the $N$-dimensional linear multistage PPIC receiver structure is defined in Section III. We derive the large system SINR in Section IV, and we illustrate the use of the linear multistage PPIC receiver's large system SINR expression in Section V. Finally, we conclude and summarize our results in Section VII.

## II. Signal Model

We consider a synchronous direct-sequence CDMA communication system with $K$ users and a processing gain of $N$. We assume that all users employ (baseband) binary antipodal modulation and consider a real baseband model [2].

The $N$-dimensional chip matched filter vector for each symbol interval is given by

$$\mathbf{r} = \sum_{k=1}^{K} \sqrt{P_k} b_k \mathbf{s}_k + \mathbf{n} \tag{1}$$

where $b_k$ is the data bit of user $k$ taking on values plus or minus one with equal probability, $P_k$ is the power of user $k$, $\mathbf{s}_k$ is the $N$-dimensional spreading sequence of user $k$, and $\mathbf{n}$ is additive white Gaussian noise (AWGN) with zero mean and covariance $\sigma^2\mathbf{I}$. The $N \times K$ spreading sequence matrix is $\mathbf{S} = [\mathbf{s}_1, \ldots, \mathbf{s}_K]$, and we define the $N \times (K-1)$ spreading sequence matrix excluding the spreading sequence of user $k$ as $\mathbf{S}_k = [\mathbf{s}_1, \ldots, \mathbf{s}_{k-1}, \mathbf{s}_{k+1}, \ldots, \mathbf{s}_K]$. We assume a random spreading model where the elements of $\mathbf{S}$ are independent and identically distributed (i.i.d.) random variables taking values of $\pm(1/\sqrt{N})$ with an equal probability. The random spreading assumption is needed to allow us to apply large system analysis. We also assume that the spreading sequences are known at the receiver, the received user powers are equal with common power $P_k = P$ (for $k = 1, \ldots, K$), and we have perfect estimation of the received user powers and the noise variance $\sigma^2$.

At the receiver, a linear filter for user $k$ produces the soft estimate $\hat{b}_k = \mathbf{c}_k^T \mathbf{r}$ for the filter coefficients $\mathbf{c}_k \in \Re^N$. The SINR for this estimate is [2]

$$\mathsf{SINR}_k = \frac{P\left(\mathbf{c}_k^T \mathbf{s}_k\right)^2}{\mathbf{c}_k^T \left(P\mathbf{S}_k\mathbf{S}_k^T + \sigma^2\mathbf{I}\right)\mathbf{c}_k}. \tag{2}$$

In this paper, we analyze the SINR in the particular case where the filter coefficients are selected using a linear iterative method based on the LMMSE solution. The resulting receiver is discussed in Section III.

## III. Receiver Structure

We start by considering the LMMSE receiver for which there are several equivalent implementations, including

$$\hat{\mathbf{b}} = P^{-1/2}\mathbf{S}^T \left(\mathbf{S}\mathbf{S}^T + \frac{\sigma^2}{P}\mathbf{I}\right)^{-1} \mathbf{r} \tag{3}$$

$$\hat{\mathbf{b}} = P^{-1/2} \left(\mathbf{S}^T\mathbf{S} + \frac{\sigma^2}{P}\mathbf{I}\right)^{-1} \mathbf{S}^T\mathbf{r} \tag{4}$$

where the SNR is $P/\sigma^2$. The LMMSE receiver in (3) involves inverting an $N \times N$ matrix, while (4) involves a $K \times K$ matrix inversion. It is straightforward to show that both produce identical estimates. The first form is more convenient for the SINR analysis. To assist us in writing an expression for the SINR of user $k$ in this case, we first use the matrix-inversion lemma [2] to see that

$$\mathbf{c}_k = \kappa \left(\mathbf{S}_k\mathbf{S}_k^T + \frac{\sigma^2}{P}\mathbf{I}\right)^{-1} \mathbf{s}_k \tag{5}$$

where

$$\kappa = \frac{1}{\sqrt{P}\left(1 + \mathbf{s}_k^T \left(\mathbf{S}_k\mathbf{S}_k^T + \frac{\sigma^2}{P}\mathbf{I}\right)^{-1} \mathbf{s}_k\right)}.$$

Now, the SINR for user $k$, denoted $\mathsf{SINR}_k$, is

$$\mathsf{SINR}_k = \mathbf{s}_k^T \left(\mathbf{S}_k\mathbf{S}_k^T + \frac{\sigma^2}{P}\mathbf{I}\right)^{-1} \mathbf{s}_k. \tag{6}$$

In this paper, we focus our attention on multistage receiver implementations which are computationally less intensive than (3), but still converge asymptotically (in the number of stages) to the SINR of (6) for each user. The first-order stationary linear iterative method we are considering is

$$\mathbf{x}_{k,m} = \tau\mathbf{r} + \left(\mathbf{I} - \tau\left(\mathbf{S}_k\mathbf{S}_k^T + \frac{\sigma^2}{P}\mathbf{I}\right)\right)\mathbf{x}_{k,m-1} \tag{7}$$

and

$$\hat{b}_{k,m} = \mathbf{s}_k^T \mathbf{x}_{k,m}$$

where $\mathbf{x}_{k,m}$ is an $N$-dimensional (measurement) vector for user $k$ after the $m$th stage of filtering and cancelling (of other users interference). The initial $N$-dimensional (measurement) vector for user $k$ is $\mathbf{x}_{k,0} = \tau \mathbf{r}$. Also, $\hat{b}_{k,m}$ is the soft estimate for user $k$ at stage $m$, and $\tau$ a real valued scalar which we call the partial cancellation factor. This linear iterative method will converge to the LMMSE receiver for user $k$ (5), provided that $\tau$ is in the range $0 < \tau < 2/(\lambda_{\max} + (\sigma^2/P))$, where $\lambda_{\max}$ is the maximum eigenvalue of $\mathbf{S}_k\mathbf{S}_k^T$ [19]. The convergence properties for other forms of linear/nonlinear IC receivers have also been studied in [8] and [13].

Rewriting (7) in a direct form gives the linear multistage PPIC receiver after $m$ stages

$$\mathbf{c}_{k,m} = \tau \left( \sum_{i=0}^{m} \left[ \mathbf{I} - \tau \left( \mathbf{S}_k\mathbf{S}_k^T + \frac{\sigma^2}{P}\mathbf{I} \right) \right]^i \right) \mathbf{s}_k. \qquad (8)$$

Note that this receiver structure differs slightly from what we call the "standard" linear multistage PPIC which has a direct form after $m$ stages of [12], [14]

$$\mathbf{c}_{k,m} = \tau \left( \sum_{i=0}^{m} \left[ \mathbf{I} - \tau \left( \mathbf{S}\mathbf{S}^T + \frac{\sigma^2}{P}\mathbf{I} \right) \right]^i \mathbf{s}_k \right). \qquad (9)$$

This is an important distinction, since in the large system SINR analysis, we are interested in key terms of the form $\mathbf{s}_k^T(\mathbf{S}_k\mathbf{S}_k^T)^i\mathbf{s}_k$ which are easier to analyze than $\mathbf{s}_k^T(\mathbf{S}\mathbf{S}^T)^i\mathbf{s}_k$, because $\left(\mathbf{S}_k\mathbf{S}_k^T\right)^i$ is independent of $\mathbf{s}_k$.

Having defined the $N$-dimensional filter coefficients of the linear multistage PPIC receiver as $\mathbf{c}_{k,m}$, ((8) for stage $m$ and user $k$), the large system SINR for this linear receiver will now be derived in Section IV.

## IV. Analysis of the Large System SINR

In this section, we derive an expression for the large system SINR of the linear multistage PPIC receiver in terms of the system loading $\alpha$, the number of stages $m$, the partial cancellation factor $\tau$, and the SNR $P/\sigma^2$.

In our analysis of the large system SINR, a key term is a random variable of the form $\mathbf{s}_k^T(\mathbf{S}_k\mathbf{S}_k^T)^i\mathbf{s}_k$ for $(0 \le i \le 2m+1)$. Consequently, the following lemma and corollary will be useful:

*Lemma 1:* If we take $N \to \infty$ with $\alpha = K/N$ fixed, the random variable $\mathbf{s}_k^T(\mathbf{S}_k\mathbf{S}_k^T)^i\mathbf{s}_k$ converges in probability to the deterministic moment

$$\psi_i(\alpha) = \int \lambda^i dG(\lambda)$$

where $G(\lambda)$ is the limiting empirical distribution function of the eigenvalues of $\mathbf{S}_k\mathbf{S}_k^T$. This limiting distribution is given by (10), shown at the bottom of the page, where $a(\alpha) = (1-\sqrt{\alpha})^2$ and $b(\alpha) = (1+\sqrt{\alpha})^2$.

Note that $\psi_i(\alpha)$ is the $i$th moment of the limiting distribution function.

We shall only give a brief discussion of the proof of *Lemma 1*, however, for more details the reader is referred to [21], [23], and [24]. Now, let $G^{(N)}(\lambda)$ be the empirical distribution function of the eigenvalues of the $N \times N$ matrix $\mathbf{S}_k\mathbf{S}_k^T$. It is well known that with probability one, $G^{(N)}$ converges in distribution to $G$ as $N \to \infty$ with $\alpha = K/N$ fixed (see [21], for example). The idea of the proof is that $\mathbf{s}_k^T(\mathbf{S}_k\mathbf{S}_k^T)^i\mathbf{s}_k$ is close to trace $(\mathbf{S}_k\mathbf{S}_k^T)^i = \int \lambda^i dG^{(N)}(\lambda)$ in a large system, and that this latter quantity converges to $\int \lambda^i dG(\lambda)$. The result can be proved following identical lines to the proof of *Lemma 4.3* in [23] (see also *Lemma 1* in [24]).

*Corollary 1:* The $i$th moment of the limiting distribution function of *Lemma 1* is

$$\psi_i(\alpha) = \frac{1}{i+1}\left[(2i-1)(1+\alpha)\psi_{i-1}(\alpha) - (i-2)(1-\alpha)^2\psi_{i-2}(\alpha)\right] \qquad (11)$$

where $\psi_0(\alpha) = 1$ and $\psi_1(\alpha) = \alpha$. A solution to this difference equation is

$$\psi_i(\alpha) = \sum_{j=0}^{i-1} \frac{1}{j+1} \binom{i}{j} \binom{i-1}{j} \alpha^{j+1}. \qquad (12)$$

*Proof:* For (11) we have

$$\psi_i(\alpha) = \int \lambda^i dG(\lambda) = \int_{a(\alpha)}^{b(\alpha)} \lambda^i g(\lambda) d\lambda$$

where $g(\lambda) = (\sqrt{[\lambda - a(\alpha)][b(\alpha) - \lambda]}/2\pi\lambda)$ and from tables of integrals [26, Table 2.260-1] the result follows. The result of (12) comes from solving the difference equation (11). ■

*Remark:* In studies of the eigenvalues of large sample covariance matrices (such as $\mathbf{S}\mathbf{S}^T$), (12) is usually derived directly from combinatorial arguments, for example, see [21]. The limiting empirical distribution function, $G(\lambda)$, is then determined from the limiting moments.

In the following theorem, we give an expression for the large system SINR of our linear multistage PPIC receiver.

*Theorem 1:* Let $N, K \to \infty$, with $0 < \alpha = K/N < \infty$ held fixed. Then, the SINR of the $m$-stage PPIC receiver converges in probability to a deterministic scalar $\gamma_m$ given by

$$\gamma_m = \frac{\left[\sum_{i=0}^{m} f_i\left(\alpha, \sigma^2, P, \tau\right)\right]^2}{\sum_{i=0}^{m}\left[(i+1)g_i(\alpha, \sigma^2, P, \tau) + (m-i)g_{m+i+1}(\alpha, \sigma^2, P, \tau)\right]} \qquad (13)$$

$$G(\lambda) = \begin{cases} 0, & \lambda < 0 \\ \max(1-\alpha, 0), & 0 \le \lambda < a(\alpha) \\ \max(1-\alpha, 0) + \int_{a(\alpha)}^{\lambda} \frac{\sqrt{[x-a(\alpha)][b(\alpha)-x]}}{2\pi x} dx, & a(\alpha) \le \lambda < b(\alpha) \\ 1, & b(\alpha) \le \lambda \end{cases} \qquad (10)$$

where $f_i(\alpha,\sigma^2,P,\tau)$, $g_i(\alpha,\sigma^2,P,\tau)$ are defined below, and the moment $\psi_j(\alpha)$ is given in *Corollary 1*

$$f_i(\alpha,\sigma^2,P,\tau) = (-\tau)^i \sum_{j=0}^{i} \left(\frac{\sigma^2}{P}-\frac{1}{\tau}\right)^{i-j} \cdot \binom{i}{j} \psi_j(\alpha) \tag{14}$$

$$g_i(\alpha,\sigma^2,P,\tau) = (-\tau)^i \sum_{j=0}^{i} \left(\frac{\sigma^2}{P}-\frac{1}{\tau}\right)^{i-j} \cdot \binom{i}{j} \left(\psi_{j+1}(\alpha)+\frac{\sigma^2}{P}\psi_j(\alpha)\right). \tag{15}$$

The proof of *Theorem 1* is given in the Appendix. Our large system SINR expression is independent on the eigenvalues of the cross-correlation matrix and the realizations of the spreading sequences. It only depends on the system loading, the partial cancellation factor, the number of stages, and the SNR.

### A. Convergence to the Large System LMMSE SINR

The large system SINR $(\gamma_m)$ of *Theorem 1* for stage 0 is equal to the SUMF receiver's large system SINR $(\mathsf{SINR}_{\rm SUMF})$. As the number of stages increases $(m\to\infty)$, $\gamma_m$ converges to the LMMSE receiver's large system SINR $(\mathsf{SINR}_{\rm LMMSE})$.

First, observe that at stage 0 we have

$$\gamma_0 = \frac{\psi_0(\alpha)^2}{\left[\psi_1(\alpha)+\frac{\sigma^2}{P}\psi_0(\alpha)\right]} = \frac{P}{P\alpha+\sigma^2} = \mathsf{SINR}_{\rm SUMF}. \tag{16}$$

This is the $\mathsf{SINR}_{\rm SUMF}$ for equal power users [22], [23].

Let

$$\eta_m = \sum_{i=0}^{m} f_i(\alpha,\sigma^2,P,\tau)$$

and

$$\nu_m = \sum_{i=0}^{m}\left[(i+1)g_i(\alpha,\sigma^2,P,\tau)+(m-i)g_{m+i+1}(\alpha,\sigma^2,P,\tau)\right].$$

Then

$$\begin{aligned}\lim_{m\to\infty}\eta_m &= \lim_{m\to\infty}\sum_{i=0}^{m}(-\tau)^i\sum_{j=0}^{i}\left(\frac{\sigma^2}{P}-\frac{1}{\tau}\right)^{i-j} \cdot \binom{i}{j}\left(\int \lambda^j dG(\lambda)\right)\\ &= \lim_{m\to\infty}\int \frac{1-[1-\tau(\lambda+\frac{\sigma^2}{P})]^m}{\tau(\lambda+\frac{\sigma^2}{P})}dG(\lambda).\end{aligned} \tag{17}$$

Provided $0<\tau<2/(b(\alpha)+(\sigma^2/P))$ and $\sigma^2/P>0$, we can move the limit inside the integral using the dominated convergence theorem to give

$$\lim_{m\to\infty}\eta_m = \frac{1}{\tau}\int \frac{1}{\lambda+\frac{\sigma^2}{P}}dG(\lambda). \tag{18}$$

Similarly, we can show that with $0<\tau<2/(b(\alpha)+(\sigma^2/P))$

$$\begin{aligned}\lim_{m\to\infty}\nu_m &= \int \lim_{m\to\infty}\frac{(1-[1-\tau(\lambda+\frac{\sigma^2}{P})]^{(m+1)})^2}{\tau^2(\lambda+\frac{\sigma^2}{P})}dG(\lambda)\\ &= \left(\frac{1}{\tau}\right)^2\int\frac{1}{\lambda+\frac{\sigma^2}{P}}dG(\lambda).\end{aligned} \tag{19}$$

Combining the above results gives us $\lim_{m\to\infty}\gamma_m = \int 1/(\lambda+(\sigma^2/P))dG(\lambda)$. This is precisely the large system SINR for the LMMSE receiver [23], and can be evaluated in closed form as

$$\mathsf{SINR}_{\rm LMMSE} = \frac{P(1-\alpha)}{2\sigma^2}-\frac{1}{2} + \sqrt{\frac{P^2(1-\alpha)^2}{4\sigma^4}+\frac{P(1+\alpha)}{2\sigma^2}+\frac{1}{4}}. \tag{20}$$

We have shown that $\gamma_m$ of our linear multistage PPIC receiver converges to $\mathsf{SINR}_{\rm LMMSE}$ as $m$ increases.

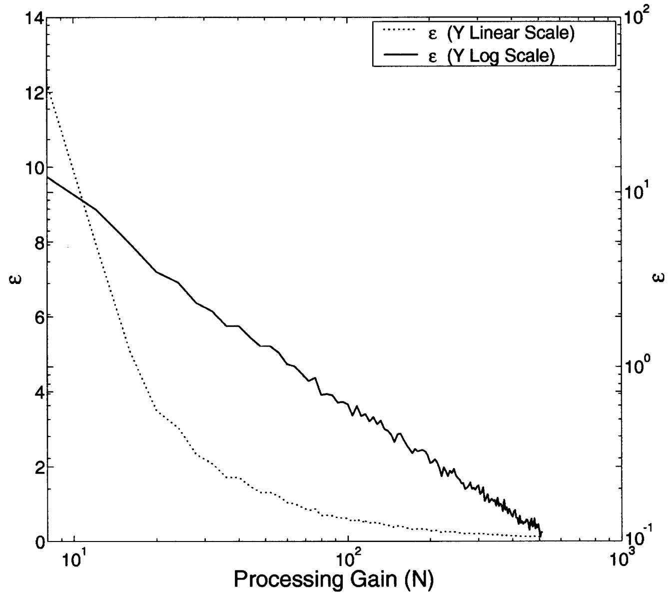



Fig. 1. Empirical MSE versus $N$ ($8\le N\le 512$, $\alpha=0.75$, and $\mathrm{SNR}=12$ dB).

## V. Numerical Studies

In this section, we shall empirically show that the MSE between the SINR, $\gamma_m^{(N)}$, and the large system SINR, $\gamma_m$, of the linear multistage PPIC receiver decreases proportionally with a $1/N$ relationship as $N$ increases with $\alpha=K/N$ held fixed. We perform a numerical optimization of our linear multistage PPIC receiver to maximize $\gamma_m$ with respect to the partial cancellation factor, and compare the result with the large system SINR of the optimal linear multistage receiver. Finally, we use the large system SINR to approximate the BER of the linear multistage PPIC receiver.

### A. Convergence in $N$

We proved in *Theorem 1* that the SINR of the linear multistage PPIC receiver, $\gamma_m^{(N)}$, converges in probability to the large system SINR of the linear multistage PPIC receiver, $\gamma_m$, as $N$, $K$ increases with $\alpha=K/N$ held fixed. This convergence is now shown in another manner by examining the empirical MSE between $\gamma_m$ and $\gamma_m^{(N)}$, which is defined as $\epsilon^{(N)}=\overline{(\gamma_m-\gamma_m^{(N)})^2}$, where $\overline{(\cdot)}$ denotes the mean. This expression allows us to empirically give an indication that the variance of the SINR decreases with a $1/N$ relationship around the large system SINR limit as $N$, $K$ increase with $\alpha=K/N$ held fixed. This convergence is captured in a plot $\epsilon^{(N)}$ versus $N$ shown in Fig. 1. In this plot, we have plotted $\epsilon^{(N)}$, where we have used 1000 realizations of $\gamma_m^{(N)}$ for each value of $N$ in the range $8\le N\le$

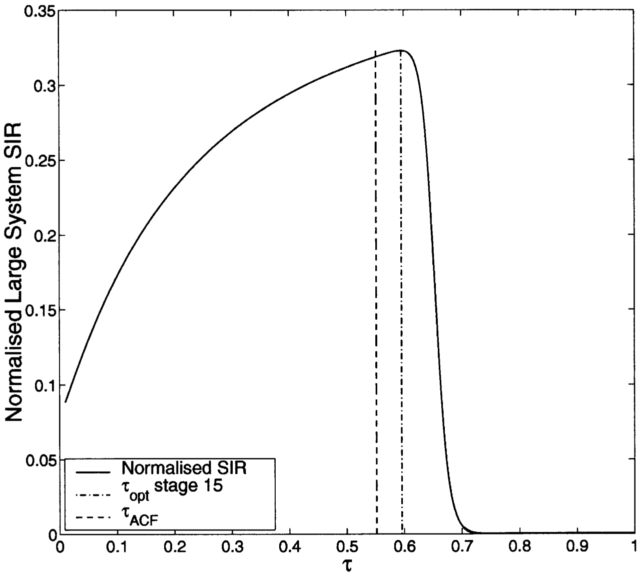



Fig. 2. Large system SINR versus $\tau$ ($\alpha = 0.75$, $m = 15$, and $\mathrm{SNR} = 12$ dB).

512 (incrementing in steps of four), $m = 10$, $\alpha = 0.75$, and $\mathrm{SNR} = 12$ dB. We show both the linear plot, (dotted line, left $Y$ axis), and the log plot (solid line, right $Y$ axis). We found that $\epsilon^{(N)} \approx \hat{\epsilon}^{(N)} = (10^{2.106})N^{-1.120}$, (taken from the empirical data), which indeed shows that $\epsilon^{(N)}$ decreases with a $1/N$ relationship. This is a significant result, as it has only recently been shown specifically for the LMMSE and decorrelating receivers that the variance of the SINR distribution decreases with a $1/N$ relationship [27].

### B. Optimization of the Linear Multistage PPIC Receiver

We shall now optimize $\gamma_m$ in terms of the single partial cancellation factor $\tau$ for a finite number of stages. We applied a simple numerical method to maximize $\gamma_m$ with respect to $\tau$. The stopping threshold for the $(i+1)$th estimate of $\tau_{\mathrm{opt}}$ was $|e_{i+1}| = |\hat{\tau}_{i+1} - \hat{\tau}_i| < 10^{-5}$. The best practical estimate of $\tau$ giving the maximum $\gamma_m$ is denoted $\tau_{\mathrm{opt}}$, as shown in Fig. 2. We have normalized the SINR (SINR/SNR) for this plot, which uses $\mathrm{SNR} = 12$ dB, $\alpha = 0.75$, and $m = 15$. We show $\tau_{\mathrm{opt}}$ and $\tau^*_{\mathrm{ACF}}$, (which is discussed below), as dashed-dotted and dashed vertical lines, respectively. After extensive numerical investigations, we have found that reasonable SINR performance can occur for $0 < \tau \leq \tau_{\mathrm{opt}}$, however, a severe SINR degradation occurs for $\tau > \tau_{\mathrm{opt}}$.

*1) Asymptotic Convergence Factor (ACF) and $\tau^*_{\mathrm{ACF}}$:* The ACF (the spectral radius of the iteration matrix [12], [19]) is used to optimize the first-order stationary linear iterative method. It is minimized when

$$\tau = \tau_{\mathrm{ACF}} = \frac{2}{\mu_{\min} + \mu_{\max}}. \tag{21}$$

where, in our case, $\mu_{\min}/\mu_{\max}$ are the minimum/maximum eigenvalues of $(\mathbf{S}_k\mathbf{S}_k^T + (\sigma^2/P)\mathbf{I})$. Using this expression for $\tau_{\mathrm{ACF}}$ ensures convergence to a solution as $m \to \infty$. However, it is not certain how this relates to a finite number of stages. For a discussion on the ACF, refer to [12] and [19].

If we consider a large system where $N \to \infty$ with $0 < \alpha = K/N \leq 1$ is held fixed, then the extreme eigenvalues of the matrix $(\mathbf{S}_k\mathbf{S}_k^T + (\sigma^2/P)\mathbf{I})$ converge with probability one to two deterministic scalars

$$\mu_{\min} \to \frac{\sigma^2}{P} \text{ and } \mu_{\max} \to (\sqrt{\alpha}+1)^2 + \frac{\sigma^2}{P} \tag{22}$$

see [12] and the references therein.

This means that in a large system (with random spreading) we have approximately $\tau^*_{\mathrm{ACF}} = 2/((\sqrt{\alpha}+1)^2 + (2\sigma^2/P))$ with minimal ACF, where $*$ indicates the large system result. The partial cancellation factor $\tau^*_{\mathrm{ACF}}$ is a function only of the system load ($\alpha = K/N$) and the SNR ($P/\sigma^2$). In small-sized systems, e.g., $N = 32$, and for a finite number of stages, we have previously found that backing off from the value of $\tau = \tau^*_{\mathrm{ACF}}$ was needed to give reasonable performance. For more details, see [14].

An extensive numerical investigation has indicated that $\tau = \tau^*_{\mathrm{ACF}}$ gives good robust performance for a finite number of stages for the system parameters tested within the range $0 < \alpha \leq 1$ and $0 \leq \mathrm{SNR} \leq 18$ dB for this implementation of the linear multistage PPIC receiver.

### C. Comparison With the Optimal Linear Multistage Receiver

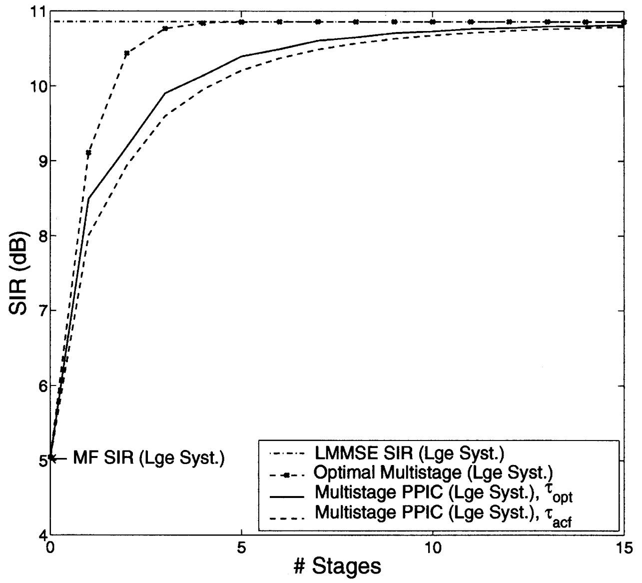
In this subsection, we shall compare $\gamma_m$ with the large system SINR of the optimal linear multistage receiver. The optimal linear multistage multiuser receiver is given by [4], [17], [25], and

$$\mathbf{C}_m = \sum_{i=0}^{m} a_i \left(\mathbf{S}^T\mathbf{S}\right)^i \tag{23}$$

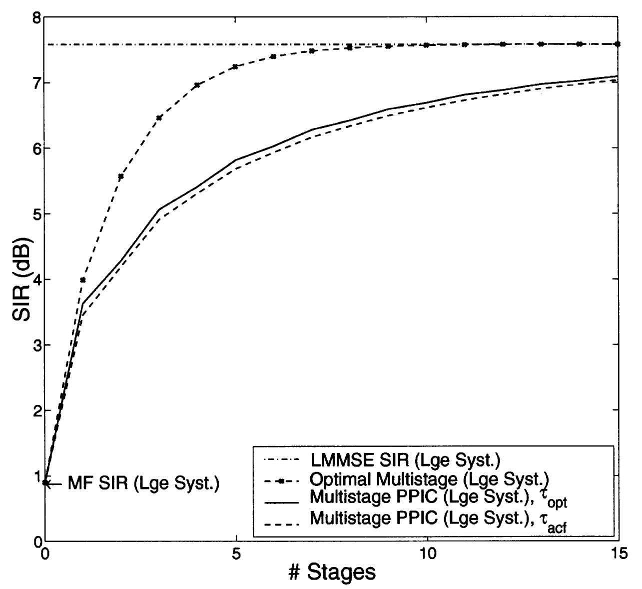
where, briefly, $\mathbf{C}_m$ forms a polynomial expansion in terms of the $K$-dimensional cross-correlation matrix $\mathbf{S}^T\mathbf{S}$, $a_i$ are the polynomial coefficients, and $m$ is the number of stages. This receiver is optimal in the sense that the polynomial coefficients are optimized to minimize the MSE [4], [17], or maximize the SINR [17], [25], for a particular stage $m$. The optimization involves an $m$-dimensional matrix inversion at stage $m$, and for $m = K - 1$, gives LMMSE performance with the computational load of the LMMSE receiver. We call this receiver the optimal linear multistage receiver, since it is "optimal" in the sense that the polynomial coefficients are chosen to maximize the large system SINR for a particular stage $m$ [17], [25].

We show the large system SINR performance of the linear multistage PPIC receiver and the optimal linear multistage receiver as $m$ increases in Fig. 3(a) and (b). We have plotted $\gamma_m$ with increasing stages when $\tau = \tau_{\mathrm{opt}}$ (the solid line) optimized at stage 15, and $\tau = \tau^*_{\mathrm{ACF}}$ (the dashed line). The SUMF receiver's large system SINR is indicated by an arrow, and the large system LMMSE SINR [$\mathrm{SINR}_{\mathrm{LMMSE}}$, given in (20)] is indicated by a horizontal dashed-dot line. The optimal linear multistage receiver's large system SINR is indicated by the dashed line with $\times$ marks. The optimal $m$-parameter receiver clearly converges faster than the simpler one-parameter receiver.

*1) Complexity Comparison:* As mentioned in the introduction, the computational complexity of these linear multistage receivers is $\mathcal{O}(mK^2)$ and $\mathcal{O}(mKN)$ for $m$ stages. For a similar level of performance, the one-parameter linear multistage PPIC receiver requires more stages than the $m$-parameter optimal linear multistage receiver, and, in this sense, is computationally more expensive. However, let us concentrate on the




(a)




(b)

Fig. 3. Large system SINR versus stages (and $\mathrm{SNR} = 12$ dB). (a) $\alpha = 0.25$. (b) $\alpha = 0.75$.

complexity of the coefficient (parameters or partial cancellation factors) calculation needed for these receivers.

For optimal large system SINR performance at the output of stage $m$, the complexity of the coefficient calculation for this receiver is $\mathcal{O}(m^3)$ floating point operations (FLOPS). Although the linear multistage PPIC receiver has a lower performance for the same number of stages, it only needs one parameter, $\tau$, to be calculated. Further, we have shown that $\tau = \tau^*_{\mathrm{ACF}}$ is an excellent, near-optimal choice for this single parameter, and is a simple function of the system load and the SNR. The computational load for the calculation of the single parameter is, thus, $\mathcal{O}(1)$ for the simpler receiver.

In summary, the slower convergence of the linear multistage PPIC receiver is natural, since it has one degree of freedom, as opposed to the optimal linear multistage receiver, which has $m$.

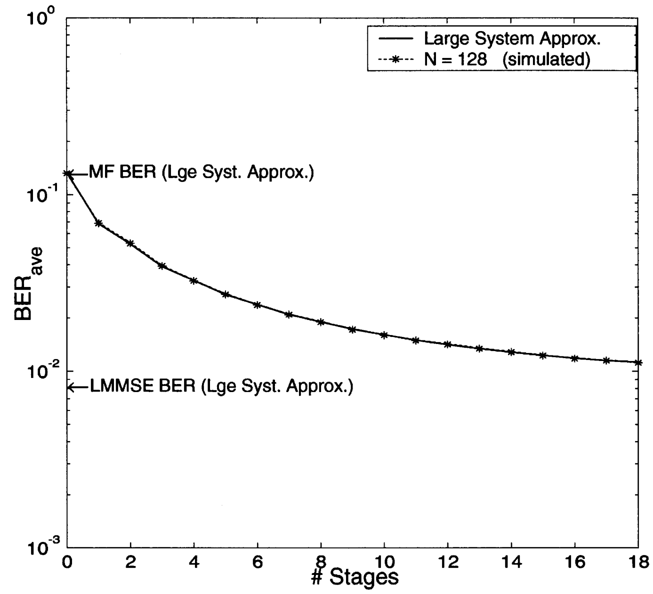



Fig. 4. BER versus stages ($\alpha = 0.75$ and $\mathrm{SNR} = 12$ dB).

The advantage of the simpler receiver is the ease in which the single design parameter can be calculated.

*2) Related Linear Multistage Receivers:* In this subsection, we have focused on a comparison of the linear multistage PPIC receiver and the optimal linear multistage receiver. In related work, the linear multistage receiver of [11] was designed to monotonically decrease the MSE at the output of each stage. This linear multistage receiver has a similar structure to the linear multistage PPIC receiver. It is instead based on the steepest descent method, where a different partial cancellation factor is applied in each stage. When the linear multistage receiver of [11] is appropriately optimized, it would achieve the same performance as the optimal linear multistage receiver given in (23) or [17] and [25]. This is not a surprising result, it has recently been shown that the optimal linear multistage receiver, at stage $m$, maximizes the SINR and minimizes the MSE with respect to its $m$ polynomial coefficients [17]. In the equal power case, the algorithm used in [11] for calculating the $m$ partial cancellation factors involves, among other things, computing the finite $m$th order moments of the cross-correlation matrix, an $m$-dimensional matrix inversion for solving, and a sorting routine. This coefficient calculation is of the same order of complexity as for the optimal linear multistage receiver $(\mathcal{O}(m^3))$.

### *D. BER Approximation*

The BER of the SUMF and LMMSE receivers for binary antipodal modulation may be approximated using $P_b = Q(\sqrt{\mathsf{SINR}})$, where it is assumed that the interference is a Gaussian random variable (the Standard Gaussian approximation), and $Q(x) = 1/(\sqrt{2\pi}) \int_x^\infty e^{(-z^2/2)} dz$. We can use this expression to approximate the BER of the linear multistage PPIC receiver by assuming $\mathsf{SINR} = \gamma_m$, giving the BER for the $m$th stage as $P_{b,m} \approx Q(\sqrt{\gamma_m})$.

We illustrate the use of this approximation in Fig. 4. For this plot, we collected the average BER $(\mathrm{BER}_{\mathrm{ave}})$ from Monte Carlo simulations of the linear multistage PPIC receiver on an AWGN channel. The plot uses $N = 128$, $\mathrm{SNR} = 12$ dB, and $\alpha = 0.75$ for up to 15 stages. We select the partial cancellation factor to be $\tau = \tau^*_{\mathrm{ACF}}$. We have plotted the approximate BER of the SUMF and LMMSE receivers using their large system SINRs from (16) and (20), their BER is indicated by an arrow. The linear multistage PPIC receiver's approximate BER is plotted using $\gamma_m$, and is indicated by the solid line. We have conducted extensive numerical investigations, and the BER approximation using $\gamma_m$ for these system loads gives an excellent BER estimate of the simulated $\mathrm{BER}_{\mathrm{ave}}$ of the linear multistage PPIC receiver for a practical $N$ and $K$.

## VI. Impact of Unequal Power Users

Our discussion until now has focused on a model where the users have equal received powers. We now briefly discuss the impact of unequal power users on the performance of the standard linear multistage PPIC receiver structure (for more details, see [16]).

For the received signal model of (1), the linear MMSE receiver produces the estimate

$$\hat{\mathbf{b}} = \mathbf{P}^{-1/2}(\mathbf{S}^T\mathbf{S} + \sigma^2\mathbf{P}^{-1})^{-1}\mathbf{S}^T\mathbf{r}.$$

The ($K$-dimensional) standard linear multistage PPIC receiver structure that aims to iteratively implement the above receiver is

$$\hat{\mathbf{b}}_{m+1} = \hat{\mathbf{b}}_m + \tau[\mathbf{y} - (\mathbf{S}^T\mathbf{S} + \sigma^2\mathbf{P}^{-1})]\hat{\mathbf{b}}_m$$

where $\mathbf{y} = \mathbf{S}^T\mathbf{r}$ is the $K \times 1$ vector of the received matched filtered signal and $\mathbf{P} = \mathrm{diag}[P_1, \ldots, P_K]$ is a $K \times K$ diagonal matrix of received user powers. When $\tau$ is appropriately chosen, then this receiver will converge to the LMMSE receiver estimate given above (scaled by $\mathbf{P}^{1/2}$).

Denote $\mu^{UEP}_{\min}/\mu^{UEP}_{\max}$ as the minimum/maximum eigenvalues of $(\mathbf{S}^T\mathbf{S} + \sigma^2\mathbf{P}^{-1})$. We use the superscript $^{UEP}$ to indicate that unequal power users are assumed. It can be inferred from [19, *Theorem C.1*], that for any real and symmetric matrix with maximum diagonal element $q_{\max}$, and maximum eigenvalue $\mu_{\max}$, then $\mu_{\max} \geq q_{\max}$. This gives the inequality $\mu^{UEP}_{\max} \geq 1 + (\sigma^2/P_{j,\min})$, where $P_{j,\min}$ is the minimum received power. Since it is possible for $P_{j,\min}$ to approach 0, then it follows that $\mu^{UEP}_{\max}$ can get very large. In this case, if $\tau$ is chosen to minimize the ACF, then $\tau = \tau^{UEP}_{\mathrm{ACF}} = 2/\left(\mu^{UEP}_{\min} + \mu^{UEP}_{\max}\right)$ will be very small, which will result in extremely slow convergence (in the number of stages) to the LMMSE solution.

One solution to this problem is to use the more general iterative receiver

$$\hat{\mathbf{b}}_{m+1} = \hat{\mathbf{b}}_m + \tau\mathbf{U}[\mathbf{y} - (\mathbf{S}^T\mathbf{S} + \sigma^2\mathbf{P}^{-1})]\hat{\mathbf{b}}_m \tag{24}$$

where $\mathbf{U}$ is a nonsingular preconditioning matrix [19]. In this case, the key iteration matrix becomes $\mathbf{U}(\mathbf{S}^T\mathbf{S} + \sigma^2\mathbf{P}^{-1})$ and appropriate choice of $\mathbf{U}$ can minimize the eigenvalue spread. Our investigations suggest that a good choice is to set $\mathbf{U}$ to the diagonal matrix $[\mathbf{I} + \sigma^2\mathbf{P}^{-1}]^{-1}$. The large system analysis of this modified receiver is the subject of current research.

## VII. Conclusions

In this paper, we derived an expression for the large system SINR of the linear multistage PPIC receiver. The large system SINR only depends on the number of stages, the system loading, the partial cancellation factor and the SNR. We have empirically shown that the MSE between the realized SINR and the large system SINR of the linear multistage PPIC receiver decreases proportionally with $1/N$ as the system parameters $N$ and $K$ increase with their ratio held fixed. Using simple numerical methods, the large system SINR can be optimized in terms of the single partial cancellation factor as the system loading varies. However, it has been observed that it is even simpler to choose $\tau = \tau^*_{\mathrm{ACF}}$ for a robust performance near the maximum achievable large system SINR, for the parameter ranges investigated. This reduces the computational load needed to find the filter coefficients of the linear multistage PPIC receiver. Finally, we have shown that the large system SINR can be used in an approximation of the BER of the linear multistage PPIC receiver.

## Appendix

In this appendix, we give the derivation for the large system SINR, denoted $\gamma_m$ (for stage $m$), of the linear multistage PPIC receiver from *Theorem 1*.

*Proof:* Let $\gamma^{(N)}_{k,m}$ be the random SINR for user $k$, at stage $m$ and processing gain $N$ of the linear multistage PPIC receiver. Then, substituting (8) in (2) gives

$$\gamma^{(N)}_{k,m} = \frac{\left(\mathbf{s}_k^T \sum_{i=0}^{m} \left(\mathbf{I} - \tau\left(\mathbf{S}_k\mathbf{S}_k^T + \frac{\sigma^2}{P}\mathbf{I}\right)\right)^i \mathbf{s}_k\right)^2}{\mathbf{s}_k^T \sum_{i=0}^{m}\sum_{j=0}^{m} \left(\mathbf{I} - \tau\left(\mathbf{S}_k\mathbf{S}_k^T + \frac{\sigma^2}{P}\mathbf{I}\right)\right)^{i+j}\left(\mathbf{S}_k\mathbf{S}_k^T + \frac{\sigma^2}{P}\mathbf{I}\right)\mathbf{s}_k} \tag{25}$$

where $P/\sigma^2$ is the SNR and $\tau$ is a real valued scalar. Now, consider[1]

$$\begin{aligned}\eta^{(N)}_{k,m} &= \mathbf{s}_k^T \sum_{i=0}^{m}\left(\mathbf{I} - \tau\left(\mathbf{S}_k\mathbf{S}_k^T + \frac{\sigma^2}{P}\mathbf{I}\right)\right)^i \mathbf{s}_k \\ &= \sum_{i=0}^{m}(-\tau)^i \sum_{j=0}^{i}\left(\frac{\sigma^2}{P} - \frac{1}{\tau}\right)^{i-j} \\ &\quad \times \binom{i}{j}\mathbf{s}_k^T\left(\mathbf{S}_k\mathbf{S}_k^T\right)^j \mathbf{s}_k.\end{aligned}$$

The limit, as $N \to \infty$ with the ratio $\alpha = K/N$ held fixed shall be taken. Note, from *Lemma 1*, the sequence of random variables (rvs) $\mathbf{s}_k^T(\mathbf{S}_k\mathbf{S}_k^T)^j\mathbf{s}_k \xrightarrow{p} \psi_j(\alpha)$ for $0 \leq j \leq m$, where $\psi_j(\alpha)$ is the $j$th moment given in *Corollary 1*, and $\xrightarrow{p}$ denotes convergence in probability. Then, the sequence of rvs $\eta^{(N)}_{k,m}$ also converges in probability to $\eta_m = \sum_{i=0}^{m} f_i(\alpha, \sigma^2, P, \tau)$, where

[1] In the case when $\sigma^2/P = 1/\tau$, then $\eta^{(N)}_{k,m}$ has a simplified form and the binomial expansion is not necessary.

$f_i(\alpha, \sigma^2, P, \tau) = (-\tau)^i \sum_{j=0}^{i} (\sigma^2/P - 1/\tau)^{i-j} \begin{pmatrix} i \\ j \end{pmatrix} \psi_j(\alpha)$ for $0 \le \psi_j(\alpha) < \infty$. We call $\eta_m$ the large system limit of the sequence of rvs $\eta_{k,m}^{(N)}$.

Now, consider

$$\begin{aligned}\nu_{k,m}^{(N)} =& \mathbf{s}_k^T \sum_{i=0}^{m} \sum_{j=0}^{m} (-\tau)^{i+j} \left( \mathbf{S}_k \mathbf{S}_k^T + \left( \frac{\sigma^2}{P} - \frac{1}{\tau} \right) \mathbf{I} \right)^{i+j} \\ & \cdot \left( \mathbf{S}_k \mathbf{S}_k^T + \frac{\sigma^2}{P} \mathbf{I} \right) \mathbf{s}_k \\ =& \sum_{i=0}^{m} \sum_{j=0}^{m} (-\tau)^{i+j} \sum_{l=0}^{i+j} \left( \frac{\sigma^2}{P} - \frac{1}{\tau} \right)^{i+j-l} \begin{pmatrix} i+j \\ l \end{pmatrix} \\ & \cdot \left[ \mathbf{s}_k^T \left( \mathbf{S}_k \mathbf{S}_k^T \right)^{l+1} \mathbf{s}_k + \frac{\sigma^2}{P} \mathbf{s}_k^T \left( \mathbf{S}_k \mathbf{S}_k^T \right)^{l} \mathbf{s}_k \right].\end{aligned}$$

Similarly, the limit as $N \to \infty$ with the ratio $\alpha = K/N$ held fixed is taken, and from *Lemma 1*, the sequence of rvs $\mathbf{s}_k^T (\mathbf{S}_k \mathbf{S}_k^T)^j \mathbf{s}_k \xrightarrow{p} \psi_j(\alpha)$ for $0 \le j \le 2m+1$. Then, the sequence of rvs $\nu_{k,m}^{(N)}$ also converges in probability to

$$\begin{aligned}\nu_m =& \sum_{i=0}^{m} \sum_{j=0}^{m} g_{i+j}(\alpha, \sigma^2, P, \tau) \\ =& \sum_{i=0}^{m} \big[ (i+1) g_i(\alpha, \sigma^2, P, \tau) \\ & + (m-i) g_{m+i+1}(\alpha, \sigma^2, P, \tau) \big]\end{aligned}$$

where $g_i = (-\tau)^i \sum_{j=0}^{i} (\sigma^2/P - 1/\tau)^{i-j} \begin{pmatrix} i \\ j \end{pmatrix} \left( \psi_{j+1}(\alpha) + (\sigma^2/P) \psi_j(\alpha) \right)$ for $0 \le \psi_j(\alpha) < \infty$. We call $\nu_m$ the large system limit of the sequence of rvs $\nu_{k,m}^{(N)}$ for stage $m$. Since, $\eta_{k,m}^{(N)} \xrightarrow{p} \eta_m$ and $\nu_{k,m}^{(N)} \xrightarrow{p} \nu_m$ then

$$\begin{aligned}\gamma_{k,m}^{(N)} & \xrightarrow{p} \gamma_m \\ &= \frac{(\eta_m)^2}{\nu_m} \\ &= \frac{\left[ \sum_{i=0}^{m} f_i\left( \alpha, \sigma^2, P, \tau \right) \right]^2}{\sum_{i=0}^{m} \left[ (i+1) g_i(\alpha, \sigma^2, P, \tau) + (m-i) g_{m+i+1}(\alpha, \sigma^2, P, \tau) \right]}.\end{aligned} \tag{26}$$

■

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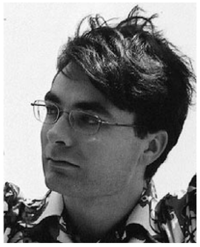

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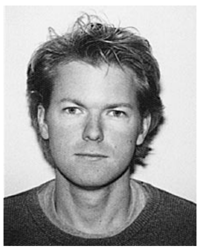

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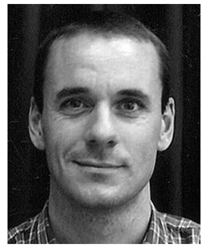

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