An SNR Approximation for Distributed Massive MIMO With Zero Forcing

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***Abstract*—In this letter, we present a new theoretical analysis of the performance of a massive MIMO network with distributed receive antennas. We consider the uplink of a cooperative cellular network that jointly detects signals from multiple transmitters with a large number of distributed antennas. Applying zero-forcing (ZF) across the receivers, we present our accurate approximation for the signal-to-noise ratio (SNR) of an arbitrary user with Rayleigh fading, path loss and shadowing. We further derive easy-to-evaluate closed-form expressions for the symbol error probability (SEP) and the achievable rate of an arbitrary user. Numerical examples highlight that our novel results provide accurate approximations for the performance of a distributed massive MIMO network.**

***Index Terms*—Zero forcing, massive MIMO, distributed antenna arrays.**

I. Introduction

Multiple-input multiple-output (MIMO) technology is an integral part of wireless standards such as IEEE 802.11 and 4G LTE due to the resultant improvement in capacity and reliability [1]. Recently, there has been great interest in massive MIMO which scales up the number of transmitters or receivers by many orders of magnitude compared to the current state-of-the-art [2]. The basic premise behind massive MIMO is to gain the benefits of conventional MIMO on a much greater scale. Asymptotic arguments based on random matrix theory demonstrate that the effects of uncorrelated noise and small-scale fading are smoothed out because of the vast spatial diversity. Furthermore, the required transmit energy per bit diminishes significantly as the number of antennas in a cell grows large [3]. More attractively, simple linear signal processing approaches can be used to realize the advantages of massive MIMO.

In a massive MIMO deployment, each base station is typically envisioned to comprise hundreds of co-located antennas simultaneously serving tens of users [2], [3]. Due to the practical limitations of large co-located antenna arrays and the problem of pilot pollution, the discussion is trending towards distributed antenna arrays to achieve improved performance [4]. It is likely that massive MIMO will not be used in isolation but will be combined with distributed antennas where antenna arrays are spread out within a single cell. Massive MIMO can also be extended across multiple cells via the emerging concept of base station cooperation.

In this letter, we focus on distributed massive MIMO with base station cooperation where antenna arrays are placed in geographically separated locations within the network. This distributed topology creates a new channel structure which is far more complex than the co-located scenario adopted in most current literature. Considering a zero forcing (ZF) receiver, we derive a simple yet accurate approximation for the signal-to-noise ratio (SNR) of an arbitrary user when the channels are subject to Rayleigh fading, independent path loss and shadowing. Our SNR approximation permits intuitive characterizations of two important performance measures. First, we derive the symbol error probability (SEP) based on the characteristic function (CF) of the received SNR. Second, we derive the achievable rate of an arbitrary user which can be further extended to consider the special case of fully distributed antennas. Extensive simulations show that our simple closed-form expressions accurately approximate the SNR, SEP and achievable rate of an arbitrary user in distributed massive MIMO networks.

II. System Model

We consider the uplink of a distributed massive MIMO network with N transmitters and R cooperating receivers where $R \gg N$. The transmitters and the receivers, all equipped with a single antenna, are randomly distributed across the network. We assume that the receivers are connected to a central base station, via high capacity delay free links, at which all transmitted bits are jointly detected. Therefore, the $\mathcal{C}^{R\times 1}$ received vector at the central base station is given by

$$\mathbf{r} = \mathbf{H}\mathbf{s} + \mathbf{w}, \tag{1}$$

where $\mathbf{s} = (s_1, s_2, \ldots, s_N)^T$ is the $\mathcal{C}^{N\times 1}$ data vector which contains transmitted symbols from the N transmitters normalized such that $E\{|s_n|^2\} = 1, \forall n \in 1, 2, \ldots N$. The variable $\mathbf{w}$ represents the $\mathcal{C}^{R\times 1}$ additive white Gaussian noise vector at the R receive antennas which has independent entries with variance $E\{|w_r|^2\} = \sigma^2, \forall r \in 1, 2, \ldots R$. The $\mathcal{C}^{R\times N}$ channel matrix $\mathbf{H}$ is assumed to have full column rank with independent elements, $H_{rn} \sim \mathcal{CN}(0, P_{rn})$, where $E\{|H_{rn}|^2\} = P_{rn}$ is the received power from transmitter n to receiver r. The geographical spread of transmitters and clusters of receivers creates a channel matrix $\mathbf{H}$, which has independent entries with different P_{rn} values.

Assuming perfect channel state information (CSI) at the receivers, we apply the ZF receiver at the central base station such that the estimated symbol vector is given by,

$$\tilde{\mathbf{r}} = \mathbf{s} + (\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H\mathbf{w}. \tag{2}$$

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Using (2), the output SNR of transmitter i is written as

$$\gamma_i = \frac{1}{\sigma^2}\mathbf{h}_i^H(\mathbf{I}-\mathbf{M})\mathbf{h}_i, \tag{3}$$

where $\mathbf{H} = (\mathbf{h}_1, \mathbf{h}_2, \ldots, \mathbf{h}_N)$ with $\mathbf{h}_i$ representing the i-th column vector in $\mathbf{H}$, and $\mathbf{M} = (\mathbf{H}_{\bar{i}}(\mathbf{H}_{\bar{i}}^H\mathbf{H}_{\bar{i}})^{-1}\mathbf{H}_{\bar{i}}^H)$ with $\mathbf{H}_{\bar{i}}$ representing $\mathbf{H}$ with column vector $\mathbf{h}_i$ removed.

III. Approximation of the Received SNR

In this section, we derive a novel expression for γ_i that provides a very accurate approximation in massive MIMO scenarios. To do that, first we reexpress γ_i in (3) as [5]

$$\gamma_i = \frac{1}{\sigma^2}\mathbf{u}_i^H\mathbf{P}_i^{1/2}(\mathbf{I}-\mathbf{M})\mathbf{P}_i^{1/2}\mathbf{u}_i, \tag{4}$$

where we have substituted $\mathbf{h}_i = \mathbf{P}_i^{1/2}\mathbf{u}_i$ with $\mathbf{P}_i = E\{\mathbf{h}_i\mathbf{h}_i^H\} = \mathrm{diag}(P_{1i},, P_{2i}, \ldots, P_{Ri})$ and $\mathbf{u}_i \sim \mathcal{CN}(0, \mathbf{I})$. Note that $\mathbf{M}$ is an idempotent matrix with trace $N-1$. Since the randomness of fast fading disappears in massive MIMO scenarios [2] we proceed to analyse the mean of $\mathbf{M}$, i.e., $E[\mathbf{M}]$. Whilst not shown here due to space limitations, some simple mathematical manipulations show that $E[\mathbf{M}]$ is a diagonal matrix. Thus, we are motivated to use

$$\bar{\mathbf{M}} = \left(\frac{N-1}{R}\right)\mathbf{I}, \tag{5}$$

as an approximation to $E[\mathbf{M}]$ since $\bar{\mathbf{M}}$ is the simple diagonal matrix with the correct trace. As such, we replace $\mathbf{M}$ by

$$\mathbf{M} = \bar{\mathbf{M}} + \mathbf{E}, \tag{6}$$

where $\mathbf{E}$ is an error matrix. Substituting (6) into (4) we get

$$\gamma_i = \frac{1}{\sigma^2}\mathbf{u}_i^H\mathbf{P}_i^{1/2}\mathbf{D}\mathbf{P}_i^{1/2}\mathbf{u}_i(1-\Psi), \tag{7}$$

where (7) is obtained by substituting $\mathbf{D} = \mathbf{I} - \bar{\mathbf{M}}$ and $\Psi = \left(\mathbf{u}_i^H\mathbf{P}_i^{1/2}\mathbf{E}\mathbf{P}_i^{1/2}\mathbf{u}_i\right) / \left(\mathbf{u}_i^H\mathbf{P}_i^{1/2}\mathbf{D}\mathbf{P}_i^{1/2}\mathbf{u}_i\right)$.

In the following, we argue that the term Ψ in (7) is small in a massive MIMO scenario. Note that the numerator and the denominator in Ψ are quadratic forms and they contain a sum of R terms. Thus, we divide both the numerator and the denominator of Ψ by R and set R to a large number. From the strong law of large numbers for non-identical variables [6] the denominator of Ψ can be written as

$$\frac{(\mathbf{u}_i^H\mathbf{P}_i^{1/2}\mathbf{D}\mathbf{P}_i^{1/2}\mathbf{u}_i)}{R} \approx \frac{E\{\mathbf{u}_i^H\mathbf{P}_i^{1/2}\mathbf{D}\mathbf{P}_i^{1/2}\mathbf{u}_i\}}{R}, \tag{8}$$

where the expectation over the channels of transmitter i results in a non-zero constant.[1] As $\mathbf{E}$ is non-diagonal the numerator is more complex. Therefore, we relax the convergence criteria to mean-square and write expectation of the magnitude square of the numerator term as

$$E\left[\frac{\left|\mathbf{u}_i^H\mathbf{\Delta}\mathbf{u}_i\right|^2}{R^2}\right] = \frac{E\left[\mathrm{tr}(\mathbf{\Delta})\mathrm{tr}(\mathbf{\Delta}^H) + \mathrm{tr}(\mathbf{\Delta}\mathbf{\Delta}^H)\right]}{R^2}, \tag{9}$$

[1]The expectation in (8) contains a summation of R independent, non-identical and non-zero mean random variables.

where $\mathbf{\Delta} = \mathbf{P}_i^{1/2}\mathbf{E}\mathbf{P}_i^{1/2}$. By lengthy but straight-forward mathematical manipulations we can show that the expectation of the diagonal elements in $(\mathbf{\Delta}\mathbf{\Delta}^H)$ can be upper bounded by a quadrature function of order R. Since $\mathrm{tr}(\mathbf{\Delta}\mathbf{\Delta}^H)$ is the sum of diagonal elements in $(\mathbf{\Delta}\mathbf{\Delta}^H)$ we can write $E[\mathrm{tr}(\mathbf{\Delta}\mathbf{\Delta}^H)]/R^2 \to 0$, $R \gg N$. Next, we note that $\mathrm{tr}(\mathbf{\Delta}) = \mathrm{tr}(\mathbf{E}\mathbf{P}_i)$. Since $\mathrm{tr}(\mathbf{M}) = \mathrm{tr}(\bar{\mathbf{M}}) = N-1$ we can also write $\mathrm{tr}(\mathbf{E}) = 0$. Therefore, unless the power profile of transmitters is very uneven, i.e., transmitters are located very close to one base station and far away from other base stations, we claim that $(\mathrm{tr}(\mathbf{\Delta})\mathrm{tr}(\mathbf{\Delta})^H)/R^2 \to 0$, $R \gg N$. This argument is also supported by simulations where, in a typical massive MIMO scenario with 100 distributed antennas and 10 users, we find that $(\mathrm{tr}(\mathbf{\Delta})\mathrm{tr}(\mathbf{\Delta})^H)/R^2 < 0.001$ in more than 99% of simulation trials. The remaining 1% corresponds to cases where transmitter i is strongly aligned with other users. Therefore, we can approximate (9) as

$$E\left[\frac{\left|\mathbf{u}_i^H\mathbf{\Delta}\mathbf{u}_i\right|^2}{R^2}\right] \to 0, \quad R \gg N. \tag{10}$$

Using (10) we further claim that the numerator term $(\mathbf{u}_i^H\mathbf{\Delta}\mathbf{u}_i)/R$ converges to zero in a mean-square sense. Thus, based on (8) and (10) we assume

$$0 < |\Psi| \ll 1. \tag{11}$$

Using (11), we finally approximate γ_i in (7) for $R \gg N$ as

$$\gamma_i \approx \mathbf{u}_i^H\mathbf{P}_i\mathbf{u}_i\left(\frac{R-N+1}{R\sigma^2}\right), \tag{12}$$

as $\mathbf{D} = \left(\frac{R-N+1}{R}\right)\mathbf{I}$. The novel result in (12) provides, in most cases of interest, a very accurate approximation for the ZF SNR of an arbitrary transmitter i in a massive MIMO scenario with distributed antennas. Interestingly, we note that if we fix the ratio between N and R, i.e., $N/R = \delta$, then the mean SNR of the approximation in (12) reduces to $E\{\gamma_i\} \approx \mathrm{tr}(\mathbf{P}_i)\left(\frac{1-\delta}{\sigma^2}\right)$, which is similar to the large system limit in [7, eq. 38] for the co-located massive MIMO scenario with statistically identical channels.

IV. Performance Analysis

In this section, we derive new expressions for the SEP and the achievable rate of transmitter i using the SNR approximation in (12).

A. Symbol Error Probability

To derive the SEP of transmitter i, we first derive the CF of γ_i which can be written as

$$\phi_{\gamma_i}(t) = E\{e^{jt\gamma_i}\} \approx E\{e^{jtC\mathbf{u}_i^H\mathbf{P}_i\mathbf{u}_i}\}, \tag{13}$$

where we have substituted (12) into γ_i and $C = \left(\frac{R-N+1}{R\sigma^2}\right)$. As $\mathbf{u}_i$ contains R independent channel elements the probability distribution function (PDF) of $\mathbf{u}_i$ can be written as $f(\mathbf{u}_i) = \frac{1}{\pi^R}e^{-\mathbf{u}_i^H\mathbf{u}_i}$. Using $f(\mathbf{u}_i)$ and [8, Lemma 2] we can derive the expectation in (13) as

$$\phi_{\gamma_i}(t) \approx \frac{1}{|\mathbf{I} - jtC\mathbf{P}_i|}. \tag{14}$$

Based on (14), we use the moment generating function (MGF) approach in [9] to derive an expression for the MGF of γ_i as

$$\mathcal{M}_{\gamma_i}(s) = \phi_{\gamma_i}(-js) \approx \frac{1}{\prod_{r=1}^{R}(1 - sCP_{ri})}. \tag{15}$$

Now, the SEP of transmitter i with a ZF receiver can be evaluated for M-PSK modulation[2] as

$$P_i = \frac{1}{\pi}\int_0^T \mathcal{M}_{\gamma_i}\left(-\frac{Z}{\sin^2\theta}\right) d\theta, \tag{16}$$

where $Z = \sin^2(\pi/M)$ and $T = \frac{(M-1)\pi}{M}$. Substituting (15) into (16) we obtain

$$P_i \approx \frac{1}{\pi}\int_0^T \frac{\sin^{2R}\theta}{\prod_{r=1}^{R}\left(\sin^2\theta + CZP_{ri}\right)} d\theta. \tag{17}$$

The integral in (17) can be solved in closed form using the tight approximation in [10] to produce

$$P_i \approx \left(\frac{T}{2\pi} - \frac{1}{6}\right)\mathcal{M}_{\gamma_i}(-Z) + \frac{1}{4}\mathcal{M}_{\gamma_i}\left(-\frac{4Z}{3}\right) + \left(\frac{T}{2\pi} - \frac{1}{4}\right)\mathcal{M}_{\gamma_i}\left(-\frac{Z}{\sin^2 T}\right). \tag{18}$$

In Section V, we illustrate that (18) provides a very accurate SEP approximation in a massive MIMO scenario with distributed antennas.

B. Achievable Rate

The achievable rate of transmitter i is derived as

$$R_i = E\left\{\log_2(1+\gamma_i)\right\} = \int_0^\infty \log_2(1+\gamma_i) f_{\gamma_i}(\gamma_i) d\gamma_i, \tag{19}$$

where f_{γ_i} is the PDF of γ_i which can be derived using our results for $\phi_{\gamma_i}(t)$. Let us first reexpress $\phi_{\gamma_i}(t)$ in (14) as

$$\phi_{\gamma_i}(t) \approx \frac{1}{\prod_{r=1}^{R}(1 - jtCP_{ri})}. \tag{20}$$

As such, for the special case of fully distributed antennas with distinct received powers, i.e., $P_{ki} \neq P_{ri}, \forall k \neq r$, we can rearrange (20) as

$$\phi_{\gamma_i}(t) \approx \left[\prod_{r=1}^{R}\vartheta_r\right]\sum_{r=1}^{R}\frac{\eta_{ri}}{(\vartheta_r - jt)}, \tag{21}$$

where $\vartheta_r = 1/(CP_{ri})$ and $\eta_{ri} = 1/\prod_{k\neq r}^{R}(\vartheta_k - \vartheta_r)$. From [9, eq. 9.65], $f_{\gamma_i}(\gamma_i)$ is given by $f_{\gamma_i}(\gamma_i) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\phi_{\gamma_i}(t)e^{-jt\gamma_i}dt$. Substituting (21) into $f_{\gamma_i}(\gamma_i)$ we can derive $f_{\gamma_i}(\gamma_i)$ as,

$$f_{\gamma_i} \approx \left[\prod_{r=1}^{R}\vartheta_r\right]\sum_{r=1}^{R}\eta_{ri}e^{-\gamma_i\vartheta_r}, \tag{22}$$

where we have used the identity [11, eq. 7, 3.382] to solve the integral. Finally, substituting (22) into (19) we derive the achievable rate of transmitter i as

$$R_i \approx \left[\prod_{r=1}^{R}\frac{\vartheta_r}{\ln 2}\right]\sum_{r=1}^{R} -\frac{\eta_{ri}}{\vartheta_r}e^{\vartheta_r}Ei(-\vartheta_r), \tag{23}$$

where (23) is derived using the identity [11, eq. 2, 4.337] and E_i denotes the exponential integral function. The above closed-form expression provides a very accurate approximation for the achievable rate of transmitter i under the special case of fully distributed massive MIMO.

For the general case with some equal received powers at the central processor, we note that the achievable rate is cumbersome to evaluate due to the fact that $\mathbf{u}_i^H\mathbf{P}_i\mathbf{u}_i$ in (12) has a summation of terms that follow an independent non-identical Chi-squared distribution [12]. As such, we present the following tractable solution using Jensen's inequality as

$$\begin{aligned} R_i &\leq \log_2(1 + E\{\gamma_i\}) \\ &\approx \log_2\left(1 + \mathrm{tr}(\mathbf{P}_i)\left(\frac{R-N+1}{R\sigma^2}\right)\right). \end{aligned} \tag{24}$$

In Section V, we illustrate that (24) provides a accurate approximation for the achievable rate of transmitter i in a general massive MIMO scenario with distributed antennas.

V. Numerical Examples

In this section, we present numerical examples to validate the accuracy of our SNR, SEP and achievable rate approximations. We consider a hexagonal cell with $N = 10$ transmitters uniformly located within a 1km cell radius. We assume a distributed massive MIMO scenario with L antenna arrays, uniformly distributed across the cell. The antenna arrays are equipped with an equal number of antennas. We assume log-normal shadow fading with a standard deviation of 8 dB and the path loss exponent is $\alpha = 4$. The power of the transmitters is fixed such that the strongest received SNR among all signals from N transmitters is greater than 3 dB for 95% of the time.

Fig. 1 plots the CDFs of the approximate SNR in (12), alongside CDFs of the exact SNR in (3) with $N = 10$ and $R = 100$. The plot illustrates results for five interesting network scenarios. In scenarios 1 to 3 we set $L = 4$ and change the distribution of the users across the cell. In scenario 1, all the users are randomly placed along the cell edge. In scenario 2, all users are randomly placed around the cell centre. In scenario 3, the users are gathered at a location very close to one antenna array. In scenarios 4 and 5 we fix the users at random locations distributed across the cell and change L. In scenario 4, we set $L = R$ and consider a fully distributed antenna scenario. In scenario 5, we set $L = 1$ and consider a co-located antenna scenario. We observe that our approximation agrees very well with the exact SNR values for all scenarios except for scenario 3. In scenario 3 where the signal of desired user is closely aligned with the other users the approximation is not very accurate. However, when the number of antennas increases and users are uniformly scattered around the network in random locations the probability of such scenarios is very low.

Fig. 2 plots the SEP of a given transmitter for $N = 10$ versus the received SNR of the desired transmitter averaged over all antenna arrays, for $R = 40$, 100 and 200. We assume that the transmitted symbols are modulated using binary phase

[2]Note that it is straightforward to also derive the SEP for M-QAM using the MGF expression according to the approach in [9].

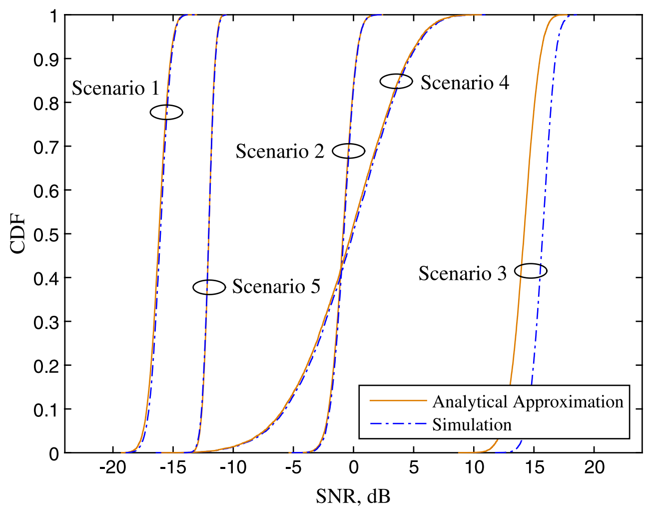

Fig. 1. Approximate and simulated SNR CDF with $N = 10$ and $R = 100$.

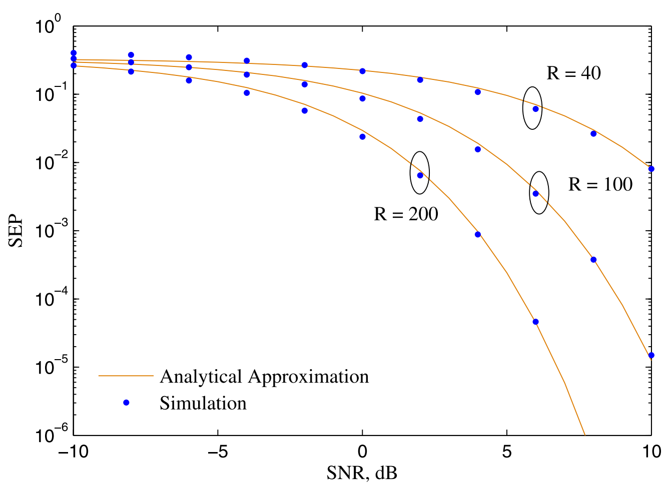

Fig. 2. SEP of the first transmitter vs average received SNR for $N = 10$.

shift keying (BPSK). The analytical approximation is generated using (18) and the simulation points are obtained using Monte Carlo simulation. Note that we have fixed the transmitter locations as R increases. The plot clearly illustrates that our SEP result accurately approximates the exact simulations in a distributed massive MIMO scenario. We also observe that the SEP results improve significantly with increasing R.

Fig. 3 plots the achievable rate of a given transmitter for $N = 10$ versus the average received SNR for $R = 40$, 100 and 200. The analytical approximation is generated using (24) and the simulation points are obtained using Monte Carlo simulation. Again, we observe that our achievable rate approximation is very accurate throughout the full range of SNRs. In fact, at $R = 200$ we observe that the approximation almost coincides with the simulation points.

VI. Conclusion

New expressions for the received SNR, SEP and the achievable rate of a distributed massive MIMO network with ZF are

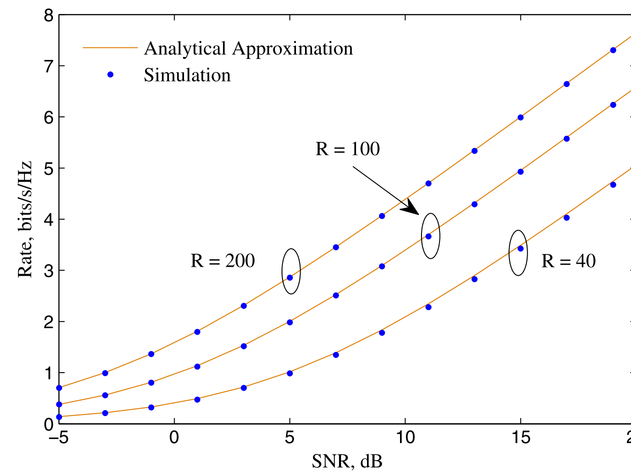

Fig. 3. Achievable rate of the first transmitter for $N = 10$ transmitters.

derived. Our results encompass independent Rayleigh fading, path loss and shadowing. The numerical examples presented illustrate the accuracy of our analysis for interesting network scenarios with a range of users distributions. Furthermore, we highlight the performance gains obtained by increasing the number of distributed antennas in a massive MIMO scenario. The analysis in this paper was based on independent channels. A relevant extension to this letter is to consider spatial correlation that may be present due to the co-located antennas in each distributed array.

References

[1] E. Dahlman, S. Parkvall, J. Skold, and P. Beming, *3G Evolution: HSPA and LTE for Mobile Broadband*, 2nd ed. Amsterdam, The Netherlands: Elsevier, 2008.

[2] T. Marzetta, "Noncooperative cellular wireless with unlimited numbers of base station antennas," *IEEE Trans. Wireless Commun.*, vol. 9, no. 11, pp. 3590–3600, Nov. 2001.

[3] N. Q. Hien, E. G. Larsson, and T. L. Marzetta, "Energy and spectral efficiency of very large multiuser MIMO systems," *IEEE Trans. Commun.*, vol. 61, no. 4, pp. 1436–1449, Apr. 2013.

[4] J. Zhang, C. K. Wen, S. Jin, X. Gao, and K. K. Wong, "On capacity of large-scale MIMO multiple access channels with distributed sets of correlated antennas," *IEEE J. Sel. Areas Commun.*, vol. 31, no. 2, pp. 133–148, Feb. 2013.

[5] M. Kiessling and J. Speidel, "Analytical performance of MIMO zero–forcing receivers in correlated Rayleigh fading environments," in *Proc. IEEE SPAWC*, Rome, Italy, Jun. 2003, pp. 383–387.

[6] P. K. Sen and J. M. Singer, *Large Sample Methods in Statistics*, 1st ed. New York, NY, USA: Chapman & Hall, 1993.

[7] N. Q. Hien, M. Matthaiou, T. Duong, and E. Larsson, "Uplink performance analysis of multicell MU-SIMO systems with ZF receivers," *IEEE Trans. Veh. Technol.*, vol. 62, no. 9, pp. 4471–4483, Nov. 2013.

[8] D. A. Basnayaka, P. J. Smith, and P. A. Martin, "Performance analysis of dual-user macrodiversity MIMO systems with linear receivers in flat Rayleigh fading," *IEEE Trans. Wireless Commun.*, vol. 11, no. 12, pp. 4394–4404, Dec. 2012.

[9] M. K. Simon and M. S. Alouini, *Digital Communication Over Fading Channels*, 2nd ed. Hoboken, NJ, USA: Wiley-Interscience, 2005.

[10] M. McKay, A. Zanella, I. Collings, and M. Chiani, "Error probability and SINR analysis of optimum combining in Rician fading," *IEEE Trans. Commun.*, vol. 57, no. 3, pp. 676–687, Mar. 2009.

[11] I. Grashteyn and I. Ryzhik, *Table of Integrals, Series, and Products*, 7th ed. Burlington, MA, USA: Academic Press, 2007.

[12] R. A. Horn and C. R. Johnson, *Matrix Analysis*. Cambridge, U.K.: Cambridge Univ. Press, 1985.